PDHonline Course A126 (2 PDH)

Safer Steps from Sloping Sidewalks

Instructor: Kenneth LiDonnici, P.E.

2012

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone & Fax: 703-988-0088
www.PDHonline.org
www.PDHcenter.com

An Approved Continuing Education Provider

Safer Steps from Sloping Sidewalks

Kenneth LiDonnici, P.E.

Stairways "...are the site of accidents resulting in approximately 4,000 deaths and one million injuries requiring hospital treatment annually in the United States," according to *Architectural Graphic Standards, Tenth Edition.* In a properly designed stairway, the dimensions of the risers and treads must conform throughout to certain mathematical relationships which have been developed in order to minimize the danger of tripping. Most notably, these formulas regulate the ratio of the height of the riser to the depth of the tread for the contiguous steps of any given flight of stairs. In general, the higher the riser, the lesser the depth of the tread. Moreover, the ratio of riser to tread must conform to certain design constraints. In addition, various other restrictions are imposed... all with the aim of assuring safe and comfortable stairways.

The reason non-conforming stairways are prone to the creation of tripping hazards results from the disruption of the pedestrian's biomechanical rhythm as he negotiates a stairway. This problem is all too clear to anyone who has tripped over an unexpectedly high riser at the downhill side of such a poorly executed stairway, or to anyone who has ever had to step onto a non-functioning escalator.

Many exterior stairways spill out onto streets that have appreciable slope. In such cases, the riser of the first step up must vary in height, being lesser at the uphill end of the stairway and greater at the downhill end. In such a stairway, there is only one single path up or down where the height of the risers is constant. Short of restricting the width to a single file, there is no way to design a conventional stairway without deviating from the rule that the ratio of rise to run remain constant. However, architects and contractors seem simply to ignore that most fundamental rule of stairway design. Building such stairways by providing a small lower riser at the uphill side, or a high one at the lower end, or both, seems to be a favorite attempt at a solution. This is extremely unfortunate, since the rule that the rise-and-run ratio be uniform is thereby violated. However, perhaps those architects and contractors need to be forgiven, since without the use of the techniques which this new Method teaches, it is not possible to obtain a uniform ratio of riser-to-tread, thereby introducing serious tripping hazards. In any stairway it is most important for that ratio to be consistent. The mind of a pedestrian using a stairway is very quick to implant a biomechanical rhythm, and to predict how high the next step should be. To encounter even a slight difference can result in tripping.

This new Method shows how to compute minor modifications to the normal geometry of the individual steps of stairways that interface with sloping surfaces.

Several reference works offer formulas for configuring safe steps: Architectural Graphic Standards and Marks' Mechanical Engineer's Handbook, to name just two. These formulas differ slightly from one to another, and they usually suggest a slight range of acceptable ratios of rise to run. For purposes of simplification, this Method is based upon the following relationship, which is in quite close agreement with all of those likely to be encountered. That formula is:

2R+T=25 where R=the height of the riser, and T=the depth of the tread
(Note: 25 is the mid-point of an acceptable 24-to-26 range.)

For example: if the riser is chosen to be 7", the ideal tread depth would be 11", since 2x7+11=25. The allowable range allows slight variations in the initial design; however the ratio of riser to tread in any given stairway must **not** vary. (These design parameters assume that the pedestrian climbs or descends the stairway in a direction perpendicular to the risers. At any other angle, of course, the tread distance increases while the riser remains constant, thus changing the ratio.)

The new Method taught here addresses that problem by effecting slight modifications to the individual steps so as to provide a uniform ratio of rise to tread along any trajectory perpendicular to the intersection between the sidewalk with the first riser. This is accomplished by reshaping the steps so that at the uphill side they are less in depth and greater in tread than they are at the downhill side.

Before getting into specifics, however, we must first dispense with any potential confusion over exactly what is meant by rise (or riser) and tread. See Figure 1:

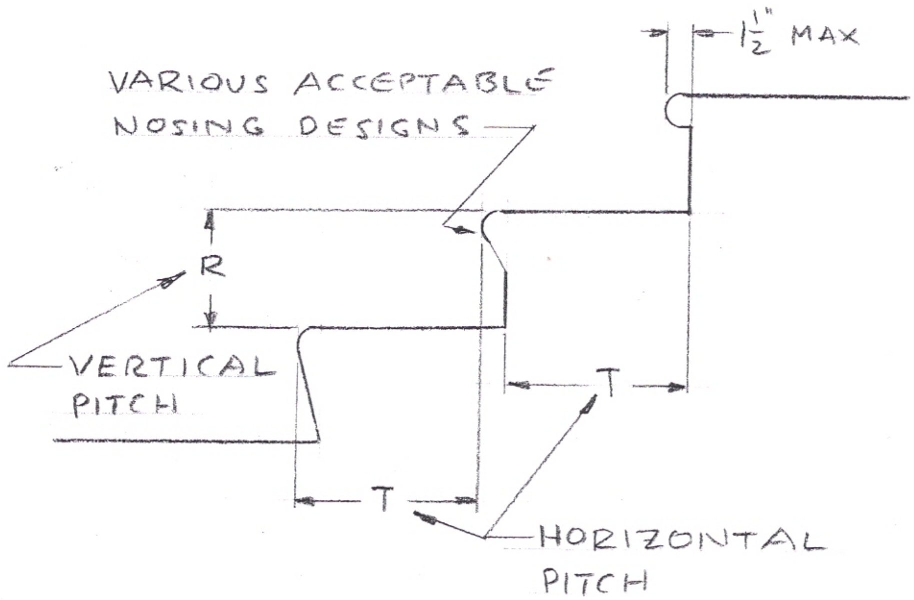

FIGURE 1

Throughout the explanation of this Method, the term "tread" will mean "horizontal pitch," and the term "rise" (or riser) will mean "vertical pitch."

Figure 2 depicts a typical stairway interfacing with a sloping sidewalk.

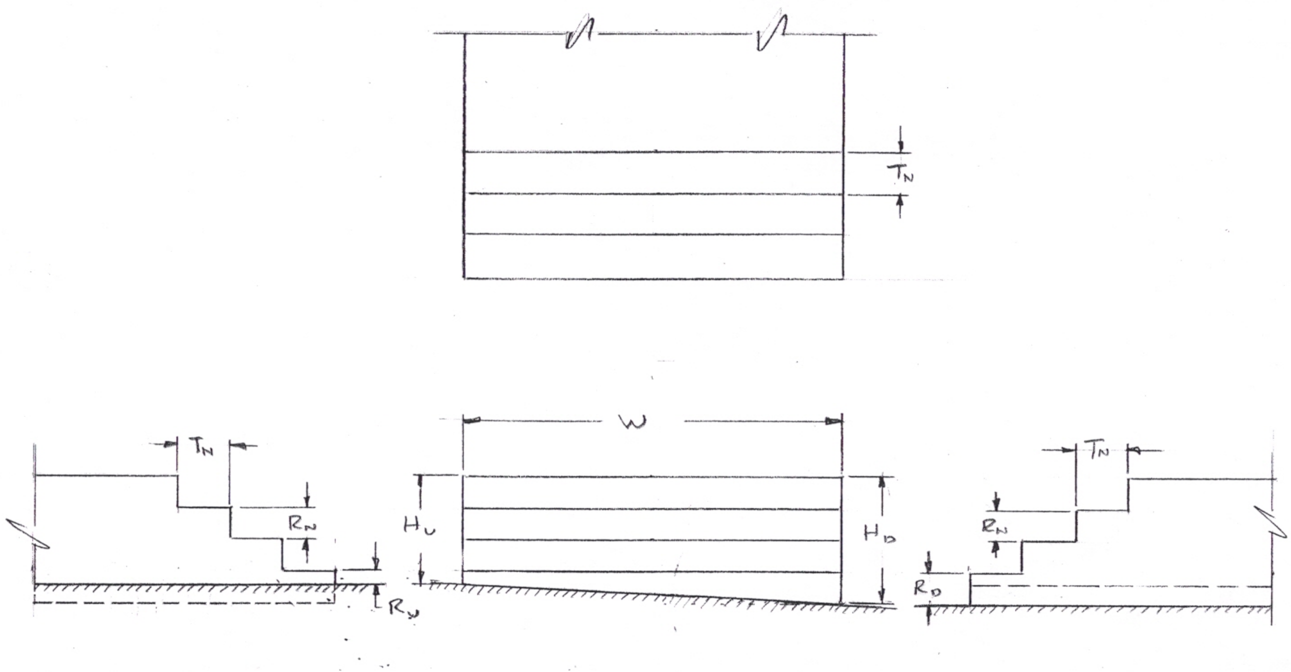

FIGURE 2

H_U = Height of entire flight of steps at the uphill side
H_D = “ “ “ downhill side
W = Width of the stairway
R_U = Lowest riser height at the uphill side
R_D = “ “ downhill side
R_N = “Normal” height of risers above the lowest, for conventional stairways
T_N = “Normal” depth of treads, for conventional stairways

We start with an actual example of a stairway on 1st Avenue (between 55th and 56th streets) in Manhattan, the dimensions of which are given in Figure 3.

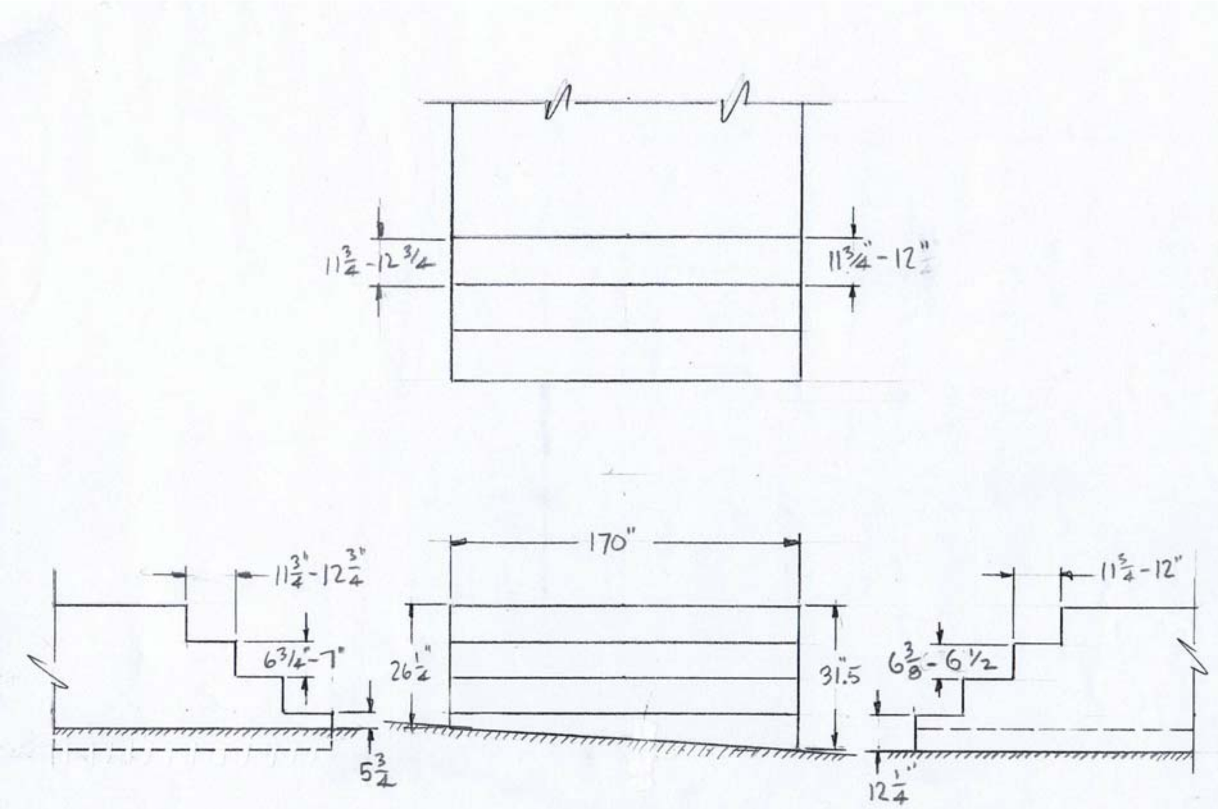

FIGURE 3

H_U = 26-1/4”
H_D = 31-1/2”
W = 170”
R_U = 5-3/4”
R_D = 12-1/4”
R_N = 6-3/4” to 7”
T_N = 11-3/4” to 12-3/4”

The tread depths (horizontal pitch) vary between 11-3/4” and 12-3/4”, which is a result of less-than-precise workmanship. However, Architectural Graphic Standards specifies that: “Tolerances for variation in tread or riser dimension should not exceed 3/16” for adjacent tread width or riser height. The maximum difference between the largest and smallest tread width or riser height for an entire flight of stairs should be 3/8.”

In this instance, the maximum variation of adjacent treads is 1” and that of adjacent risers is 6” on the downhill side, and a 1” variation on the uphill side. One can infer that this disregard for safety is not so much a disrespect of good design, but a result of there being no other viable solution of the problem...until now, that is.

Figure 4 supplies one possible solution.

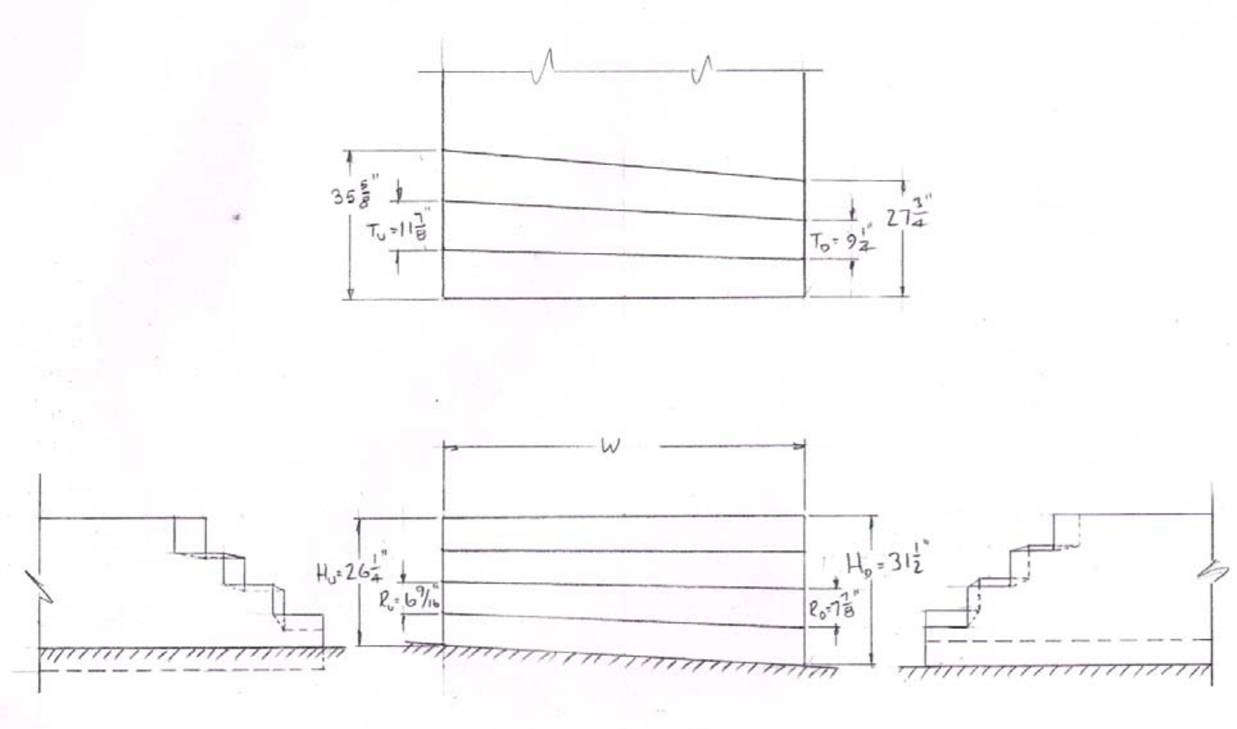

FIGURE 4

We start with a design that uses four risers, as does the actual staircase. The downhill riser dimension is obtained by dividing the downhill height by four, which is the total number of risers. Similarly, the uphill riser height is equal to the uphill height divided by four.

R_D = 31-1/2” / 4 = 7-7/8”
R_U = 26-1/4” / 4 = 6-9/16”

The tread dimensions are obtained from:

Downhill Side (R/H): (2)(7-7/8”) + T_D = 25”
T_D = 9-1/4”

Uphill Side (L/H): (2)(6-9/16”) + T_U = 25”
T_U = 11-7/8”

The downhill risers at 7-7/8” exceed the preferred 7” maximum for outdoor steps. Moreover, the downhill treads would be 9-1/4” deep, which is well under the preferred minimum of 11”. This suggests an improved design utilizing five steps, instead of the original four, and Figure 5 illustrates that configuration.

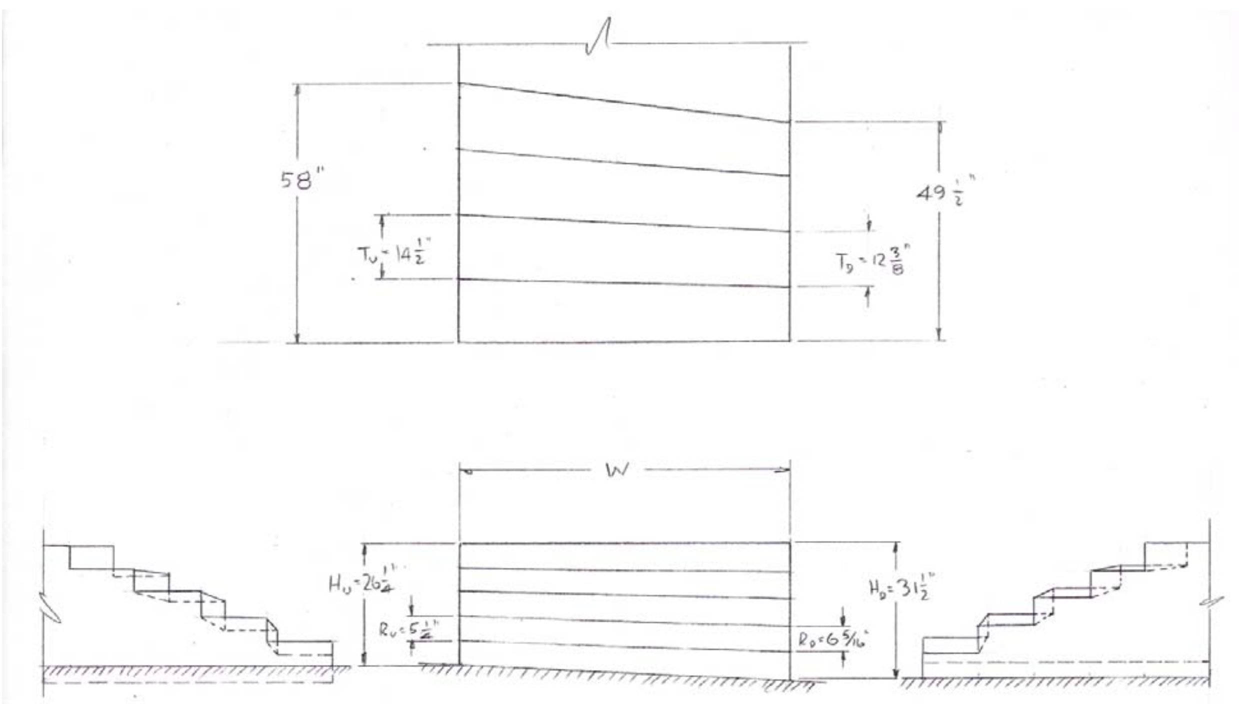

FIGURE 5

Riser dimensions obtained from:

Downhill Side (R/H): 31-1/2" / 5 = 6.3", or 6-5/16"

Uphill Side (L/H): 26-1/4" / 5 = 5-1/4"

And the tread dimensions from:

Downhill Side (R/H): $(2)(6\text{-}5/16") + T_D = 25"$
$T_D = 12\text{-}3/8"$

Uphill Side (L/H): $(2)(5\text{-}1/4") + T_U = 25$
$T_U = 14\text{-}1/2"$

It is important to note that the original steps encroach onto the horizontal upper landing by 36", more or less. Our preliminary solution, also using four steps up, would encroach 35-5/8" onto the landing at the uphill edge; not quite as much as the original. The five-step solution, however, would occupy 58" of landing at the uphill edge, and 49.5" at the downhill edge. This dimensional impact cannot be ignored.

7" as a maximum riser height is a good starting point, and since the maximum elevation of any stairway interfacing with a sloping sidewalk is obviously at the downhill side,

then: $\mathbf{N_R \geq H_D / 7}$

That is, the number of risers should be the whole number that just exceeds that quantity.

So, from: $2R + T = 25$,
$T = 25 - 2R$

And since: $R_U = H_U / N_R$ and: $R_D = H_D / N_R$

Then: $T_U = 25 - (2)(H_U/N_R)$ and $T_D = 25 - (2)(H_D/N_R)$

Or: $T_U = 25 - 2(R_U)$ and $T_D = 25 - 2(R_D)$

And since: $H_D > H_U$ And $R_D > R_U$

Then: $T_D < T_U$

At this point it might be helpful to explain how the configuration of standard pre-cast concrete steps would need to be modified to permit construction of a "Safe-Steps" stairway.

TREAD DIMENSIONS

From the expression T = 25 - 2R: $T_U - T_D = (25 - 2R_U) - (25 - 2R_D)$

Which becomes: $\mathbf{T_U - T_D = 2(R_D - R_U)}$

But since: $R_D = H_D/N_R$ and: $R_U = H_U/N_R$ then: $T_U - T_D = 2(H_D/N_R - H_U/N_R)$

Which becomes: $\mathbf{T_U - T_D = 2[(H_D - H_U)/N_R]}$

so that: **$2[(H_D-H_U)/N_R]$ is the amount that must be removed from the from the tread at its downhill edge.**

RISER DIMENSIONS

The amount that must be removed from the riser at its uphill edge is: $(H_D - H_U)/N_R$

Figure 6 illustrates how a hypothetical "Standard" step would need to be modified in order to create an element of a "Safer Step" stairway.

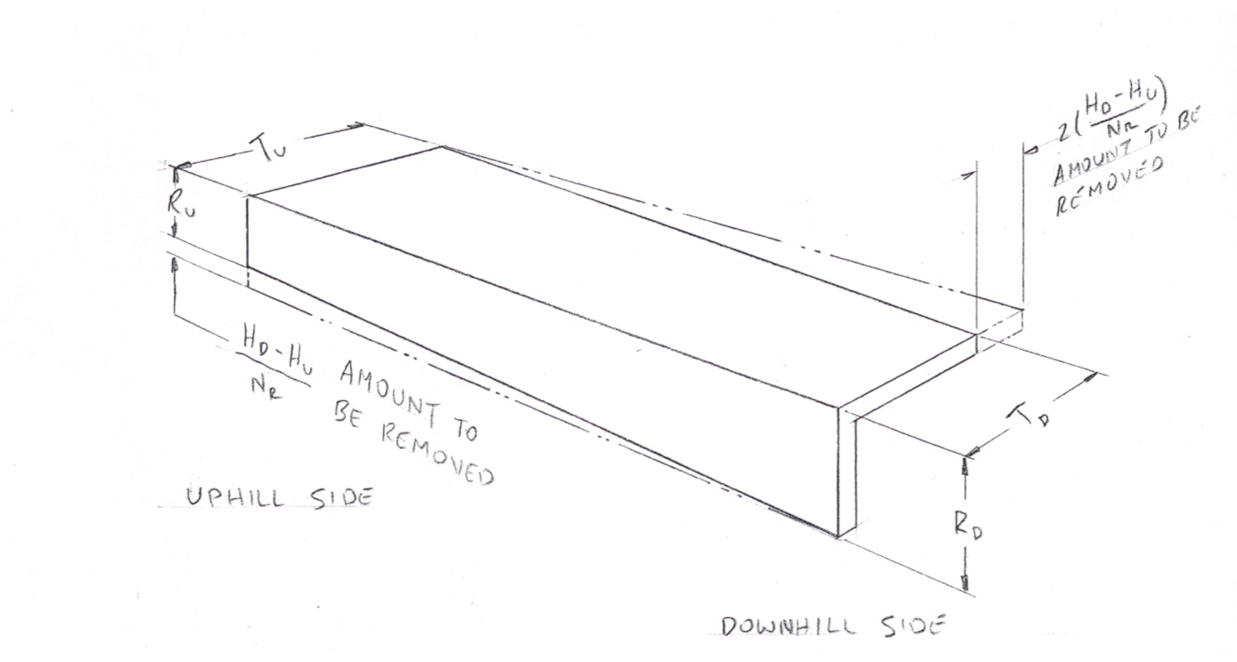

FIGURE 6

Certainly there are cases where the constraints on the designer are such that some flexibility in applying these rules of configuration would seem unavoidable. Here is a rationale for allowing the risers to vary ± ½".

The Marks Mechanical Engineer's Handbook states that T + 2R = 24 to 26, rather than the fixed 25 factor employed by this Method. For example, for a tread depth of 11", this flexibility would allow for the riser to be between 6-1/2" and 7-1/2". This would be acceptable; however, this flexibility is only allowable if **all** risers are equal in height. Remember the stipulation **that the maximum allowable variation of a tread or riser is 3/16" from step to step, and 3/8" for the entire stairway**. Realistically, this minimal allowable variation in dimension can readily be expected to obtain on any job site simply as a result of ordinary workmanship. So, as long as there would be any variation in height between the uphill and downhill risers, this "Safer Step" Method should be employed.

It bears emphasizing that in all of these considerations, we expect that the pedestrian's trajectory will remain essentially perpendicular to the lowest riser of the stairway.

Recapping the "Safer-Step" Method for stairway design:

• Calculate the amount by which the uphill side of the risers should be reduced (as compared with the downhill side) by dividing the difference in the overall height between the uphill and downhill sides by the number of risers.

$$\textbf{Uphill Riser Reduction} = (H_D - H_U)/N_R$$

• Calculate the amount by which the downhill side of the treads should be reduced (as compared with the uphill side) by dividing the difference in the overall height between the uphill and downhill sides by the number of risers, then multiplying by two.

$$\textbf{Downhill Tread Reduction} = 2[(H_D - H_U)/N_R]$$

Now let’s examine a more elegant, if flagrant, example of a violation of the precepts of the Safe-Steps method: the stairway leading into Manhattan’s Trump World Tower, at 1st Avenue, between 47th and 48th Streets. That sidewalk slopes upward from 48th to 47th streets at approximately 2-1/2º (see Figure 7). Note: the dimensions shown are the result of the author’s own cursory measurements and are subject to refinement.

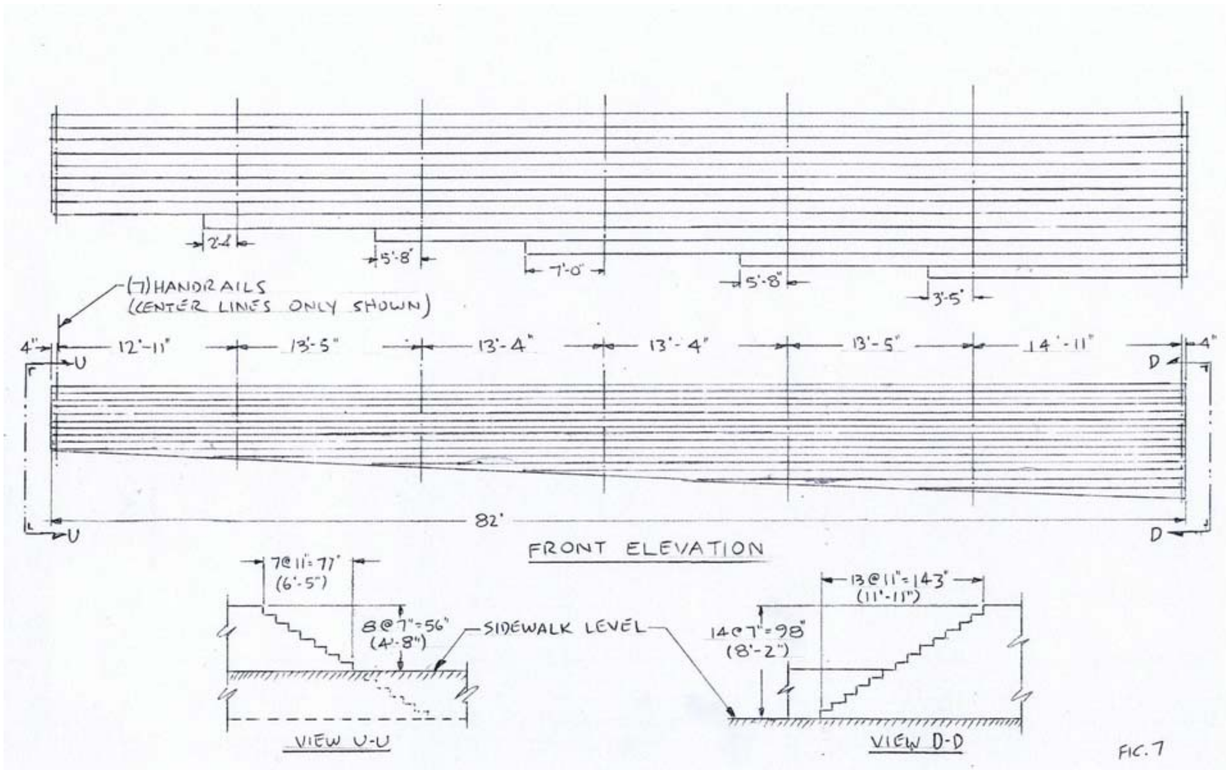

FIGURE 7

At the downhill (north) side, there are fourteen risers and, and there are eight at the uphill end. The height of the stairway at the downhill (north) side is 98 inches. All of the risers start out at 7”; the treads are all 11”. Figure 7 shows how the lower six steps disappear into the rising sidewalk. This illustrates that only at the extreme downhill side of the stairway, and at the five points at which a horizontal tread intersects the sloping sidewalk, is the 7”-rise-to-11”-tread ratio maintained. At every other location on the stairway the biomechanical rhythm of the pedestrian will not remain uninterrupted, thus creating a potential tripping hazard.

So let’s see how this stairway might have been designed using the Safe-Steps method. From NR > HD/7, we get: N_R >98”/7”=14. The riser height at the downhill side, $R_D = H_D/N_R$, is computed as: R_D = 98”/14 = 7”, the original designer’s choice.

The height of the stairway at the uphill end is 56”; therefore, the riser height at that point, $R_U = H_U / N_R$: R_U = 56”/14 = 4”.

From: T = 25-2R, the tread at the downhill end is: T_D = 25-2(7”) = 11”, and at the uphill end: T_U = 25-2(4) = 17”.

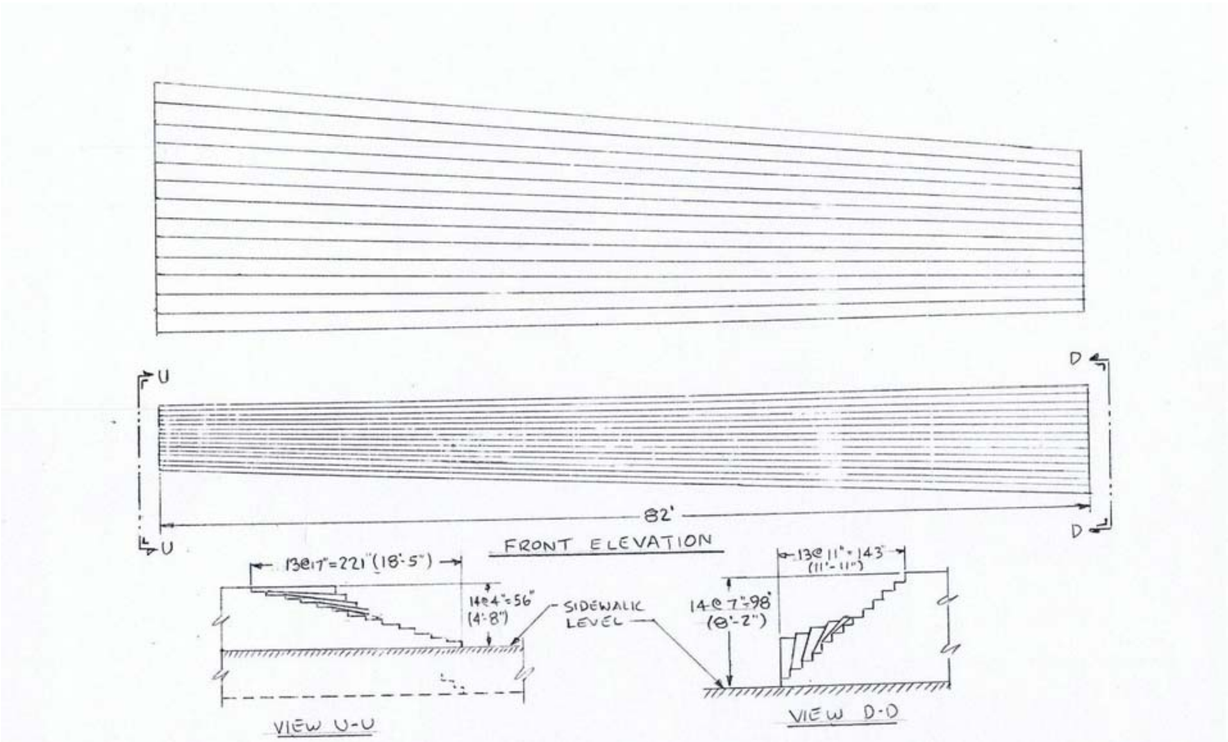

FIGURE 8

From an examination of Figure 8, we can see that the redesigned stairway would need to encroach 18’-5” onto the south end of the upper landing (a much greater distance than the original 11’-11”) and would actually exceed the maximum 18-foot available width. This solution, of course, would not work, and a better solution must be found.

This time we’ll start with the requirement that the encroachment onto the upper landing not exceed the original 11’-11” (143”). In the case of the Trump World Tower, where the building occupies the full width of the block, we might consider altering the pitch of the sidewalk so as to provide a portion of the sidewalk with **no** pitch, and thereby permit construction of a safe stairway of conventional design. See Figure 9.

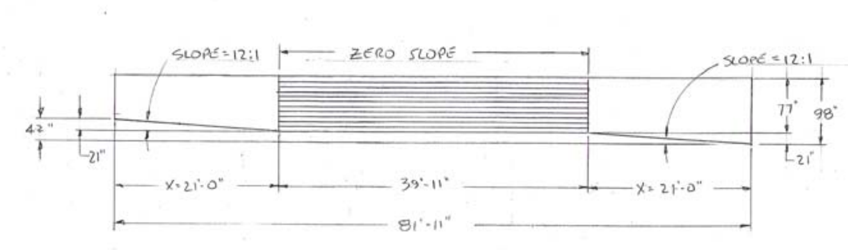

FIGURE 9

The overall change in elevation, from north to south, is 42" (98"-56"). The Americans with Disabilities Act permits providing a maximum slope of 12:1 (approximately 4-3/4º), which over the 82-foot width of the building would allow for a rise of 82/12 = 6.83' (82"), which is almost double the rise actually required**. Note that in order to use this type of solution, the pre-existing slope must be more gradual than 12:1.** This, then, would permit providing a level area at the center of the space, for example, permitting the construction of a normal stairway, free of any sloping surfaces. Figure 9 shows how this can be accomplished. The horizontal width of this conventional stairway is computed to be 40 feet, and could be positioned anywhere along the block.

At the maximum permitted 12:1 slope, the required rise of 42" would consume 42 feet of redesigned slope. The full width of the building being 82 feet allows for a 40-foot width of zero-slope sidewalk, enabling a 40-foot wide stairway of safe, conventional design, which if centered on the building, would require a rise of 77".

Using 2R +T =25 and $N_R > H/7$, we can configure a 77-inch high stairway as follows:

$N_R > 77/7 = 11$, and 77"/11 = 7" as the riser dimension. Then from 2(7) + T =25, we get an 11" tread. The ten 11" treads would encroach 110" onto the upper landing, which is less than the available 11'-11" distance.

Note that this solution is only possible because the original slope of the sidewalk is **less** than the 12:1 maximum slope permitted by the Americans With Disabilities Act. And, of course, the reconfiguration of the sidewalk must retain the original slope at the curbside, but where the sidewalk abuts the building, it must assume the shape shown by the lines delineated by points 6,7,8 and 9 of the Plan view of Figure 10. This newly configured sidewalk might consist of the five plane surfaces designated as I. II. III, IV and V. The only one of these that appears in its true size and shape is plane II, in the Plan view (view B). All the other planes are foreshortened in the Plan view (view B) and the Front (A) and Side (C) Elevations. By means of the auxiliary views, constructed by the use of Descriptive Geometry, the true size and shape of the other planes have been determined.

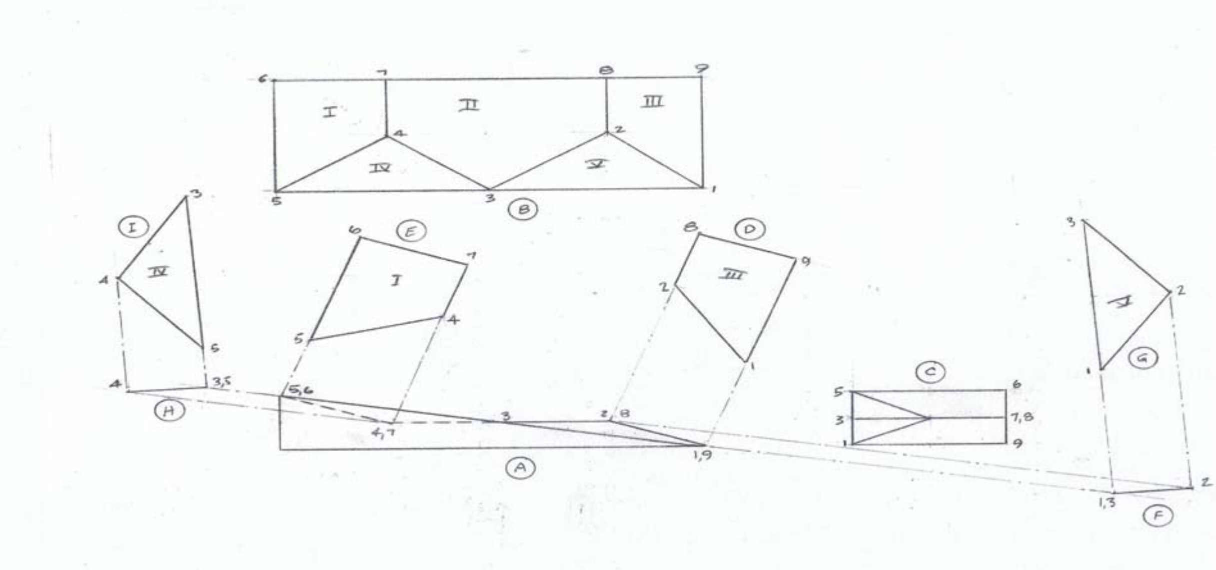

FIGURE 10

And while the mason contractors might wish to exercise their creative proclivities in how the intersections of these planes are blended, they must be aware that the minimum width of the sidewalk where planes I and II (line 4-7, view B) and planes II and III (line 2-8, view B) must be at least 36 inches wide in order to conform with the Americans With Disabilities Act. Moreover, no sloping surface may exceed the 12:1 maximum acceptable ratio.

Figure 11 is an isometric sketch further to illustrate this suggested method. For the sake of clarity, both Figures 10 and 11 exaggerate the slopes.

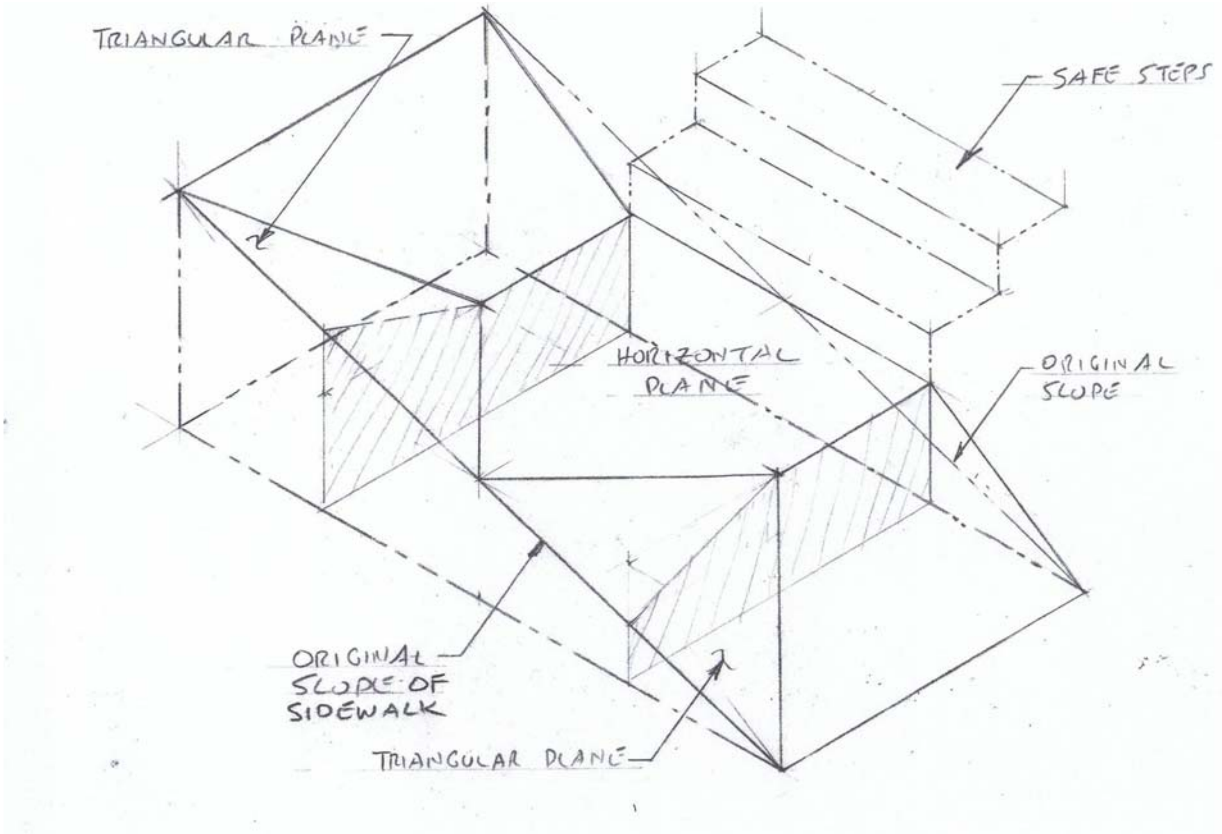

FIGURE 11

Since at the maximum allowable ADA slope of 12:1, one foot of run causes a one inch rise, that amount of the plot width must be reserved for the sloping portion of any newly configured sidewalk. The **remainder** of the plot width is available to provide a horizontal interface with a normal, safely-designed stairway. That is: the maximum width of a conventional stairway is equal to the plot width minus one foot for each inch of rise of the entire plot. See Figure 12.

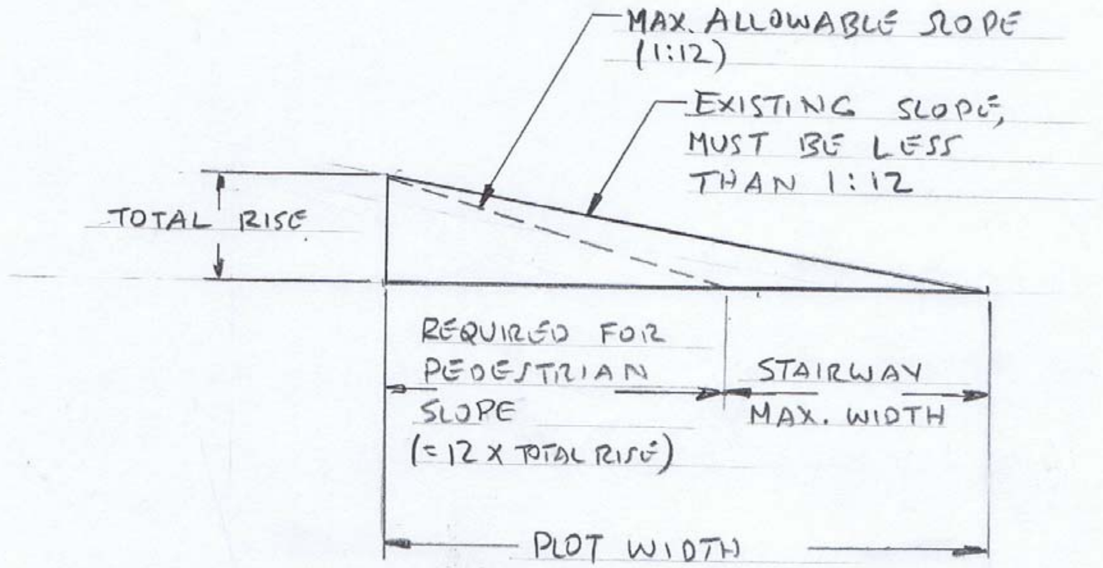

FIGURE 12

Another element in the design of stairways from sloping sidewalks deals with the constraints that determine where best to situate and how best to configure such stairways.

Let’s say an architect is required to design a “Safer-Steps” stairway up from a sidewalk that slopes at an angle of 3º (19:1). She is free to place the steps anywhere along the 20’ front of the 20’-wide property and the steps must rise to a level that is 41” above the downhill side of the property. She is further limited to riser heights of between 6” as a minimum and 7” as a maximum, and the steps may not encroach more than 48” onto the landing. She is asked to choose the maximum width of stairway that will conform to all of these constraints.

The constraint limiting the encroachment onto the upper landing to 48” would allow for four uphill treads of 12” each, or five 9.6” deep treads, or three of 16”.

From the relationship:

T + 2R = 25,

12” treads would require five 6.5” risers (four treads, five risers). 9.6” treads would require six 7.7” risers (which exceeds the 7”-maximum requirement and, consequently ruled out); 16” treads would require four 4.5” risers (also ruled out due to the 6”-minimum requirement).
So five risers it would have to be, and five risers of the maximum allowable 7” would permit a total rise of 35” at the downhill side of the stairway, while five risers of the minimum 6” would yield a rise of 30” at the uphill side. This means that the sidewalk may rise 5” along the width of the stairway.

If we let **W** equal the width of the stairway, then: **W = 5”/tan 3º** and **W=95.4”**
or: **W = 5”(19) = 95”.**

Let’s call it 8’.

Now to determine the location of the downhill end of the stairway: It must be 6” uphill from the lowest point of the property (41 inches minus the 35-inch maximum rise at the downhill side). Let **d** equal the distance from the downhill end of the property to the start of the stairway, then:

d(tan 3º) = 6” so that: **d = 6”/tan 3º = 114.5”**
or: **d = 6”(19) = 114”**

Finally, the stairway starts 114” from the downhill side of the property, is 95” wide and is 31” from the uphill side of the property.

In conclusion, this method could have been put to good use in the construction of many stairways in New York City (and presumably around the world) wherein **no** attention whatever has been paid to the dangerous situations that result from similarly poorly designed stairways.