# Special Foundations - Part I 

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# Special Foundations - Part 1 

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### 1.0 FOREWORD

Any average practicing structural engineer is cognizant of the fact that foundation design is somewhat easier and much less involved than the standard design of the superstructure bearing on it. In other words, that the analysis and sizing of column and wall footings are a straight forward process, provided those footings are concentric and the soil is suitable for the use of conventional spread footings. However, once the design engineer needs to get involved in the analysis of eccentric foundations, or the terrain needs to be helped by implementing soil modification methods, the process may turn more complex very rapidly, demanding special knowledge and experience on his part and may even require his hiring the assistance of a skilled foundation engineer to take the design process to a happy ending.

The foundation system generally accounts for a very small fraction (some $5 \%$ ) of the total construction cost. Its design should be adequate and have a comfortable safety factor, after all, it would not make any sense to set the other $95 \%$ on top of a poor foundation and put all that investment in jeopardy. Engineers, architects and builders should always keep in mind the proverbial story of the Tower of Pisa, now taken by many of our colleagues to the humorous extreme by using jokes and cartoons where the architect and builder, just to save a few barrels of lime/clay mortar and rubble, placed the safety of the entire project in dire straits. We can only imagine how the mere sight of that insidiously progressive leaning tower very likely became their worst nightmare for the rest of their natural lives*.

In this course and all the parts that will follow after this one, we will deal with the use and application of special foundations to satisfy those conditions where conventional foundation design fails to provide the tools and solutions that are necessary for building stability.

Special foundations are more often than not required by prevailing site conditions, whether they are of legal nature or soil generated, such as:
a. buildings where exterior walls were located at or right on the property line(s);
b. buildings erected on the side of a hill with a history of landslides or a site showing the typical characteristics indicating the possibilities of a surface rupture;
c. the presence of poor or unstable soil conditions which required application of one of the many soil modification methods available, as we will see in the following parts under this title; and last but not least,
d. foundations that needed to be erected within existing construction and without the removal of the existing appurtenances and equipment, or without the interruption of
existing manufacturing production work and assembly lines.
*The rest of that story is well covered in Appendix A at the end of this course.

We will have the opportunity to examine real case histories and/or real examples as those which are normally encountered in the course of common, and not so common, engineering practice.

### 2.0 THE NATURE OF SOIL

Although this is not a course in soil mechanics as such, it is not quite possible to discuss the topic of foundations without having to mention the medium which provides support for them. The soil is the ultimate mass of matter where all forces generated by a given structure end up at, so to be dissipated and reach that necessary state of balance and equilibrium.

Despite the fact that there is a lot of material to be covered when it comes to geosciences, we will only touch those areas that are significantly sufficient and necessary for the design engineer to know, and when that is achieved, we will continue with the given topic at hand.

Geotechnical engineers normally describe soils in six broad classifications, such as: rocks, gravel, sand, silt, clay and organic matter. Since rocks are a science of their own with deep roots into geology, we will discuss the remaining five as our subject of interest, since most natural soils are in reality a mix of two or more of those components.

Soil description commonly carries the name of the component that is most prevalent in the mix, while the other component(s) are included in the form of adjectives. For instance, the term sandy clay is one that derives most of its properties from clay but contains, however, a large or significant amount of sand in the mix. This same principle applies to clayey sand, which designation and interpretation works the way around.

In the United States, soil classifications are mostly defined by their grain size, more technically said, by their granulometric characteristics. However, while that is true it does not mean that all which matters is the grain size, there are some other attributes that contribute, such as, gradation, water content, color, odor, density, plasticity, texture and so on, which should be part of a good material description. MIT classifications are widely used since 1931 and generally follow the boundaries shown on the scale part of Figure 2.1 below.

As shown in the same figure, sandy soils have a grain size that varies from 0.06 mm up to 2 mm and depending on grain size they are classified as coarse, medium or fine sand. They also are the preferred foundation soil of the design engineer. They allow effective drainage, and although cohesionless, they are dependable and stable. Clays on the other
hand, have high plasticity and under wet conditions may experience volumetric changes and substantial loss of bearing capacity, and yet, they may achieve high bearing capacity when dry. Those volumetric changes may produce undesirable settlements beyond the adaptability and tolerance of the superstructure, particularly those built out of reinforced concrete.

Silty soils are sometimes difficult to distinguish from clay; however, there are two tests which will tell them apart, they are the dry strength test and the plasticity test. Clay soil test specimens will show a larger dry strength and a much better plasticity.

When it comes to plasticity, there is a "poor man's" test that does not fail when done right: just take a handful of moist soil material, squeeze it and molded it up in your hand, then rolled it up on top of a flat surface down to an eight-of-an-inch diameter thread with a length of at least a foot and hold it from one end, if it holds on without breaking, it is clay. First, you cannot roll silt down to that small diameter and second, much less to expect it to hang up under its own weight.


SOIL CLASSIFICATION SYSTEM
FIGURE 2.1

### 3.0 SOIL PRESSURE

When it comes to soil behavior related to spread footings, it can be said that both, empirical and experimental data agree on the fact that soil pressure, in intensity and distribution, is dependent not only on the size of the load and characteristics of the soil directly below, but also on the width and rigidity of the foundation being used.

Another factor that often must be considered as a fifth variable is the position of the water table with respect to the bottom of the footing. Since wet sand has a loss in bearing capacity of about $50 \%$, the rule of thumb most popular is that when the water table is located to a depth equal or less than B (width of the footing), the ultimate bearing capacity should be reduced to half of that pertaining to dry sand.

Figure 3.1 represents a typical sandy soil pressure distribution for a footing width (B) equal to about 20 inches. Although under a given load the pressure curves travel deep into the subsoil, only the upper three feet are of any significance, from there on down, the calculated pressure is so small that has little importance for the design engineer. From the graph we can deduct that at a depth of one foot below the bottom of footing, the soil pressure is about $80 \%$ of the average calculated pressure of $\mathrm{P} / \mathrm{A}$. The unidentified small curve shown immediately under the bottom of the foundation is
where the maximum pressure takes place, sometimes reaching levels of $200 \%$ of the average pressure, depending on factors such as, foundation width (B), soil density and stiffness of the foundation plate.

Sand relative density is instrumental in the determination of the soil ultimate bearing capacity as well as the potential soil settlement directly under the bottom of footing. The common formula for such important value in foundation engineering design is:

$$
\mathrm{qd}^{\prime}=\left[1 / 2 \mathrm{~B}_{\gamma} \mathrm{N}_{\gamma}\right]+\left[\gamma \mathrm{Df} \mathrm{~N}_{\mathrm{q}}\right]
$$

The equation consists of two terms which we have placed in brackets to make it easier for the description that follows. The allowable soil pressure should be taken as one-third of the ultimate pressure, to allow the recommended safety factor of 3.0.

The corresponding nomenclature is as follows:
$\mathrm{P}=$ column load
$\mathrm{P}^{\prime}=$ column load plus surcharge load plus footing weight
$B=$ footing width
$B_{Y}=a$ factor function of the footing width and the unit weight
$\mathrm{Df}_{\mathrm{f}}=$ depth of foundation, or surcharge over the foundation plane (in feet)
$\mathrm{N}_{Y}=$ a factor function of the angle of internal friction
$\mathrm{N}_{\mathrm{q}}=$ also a factor function of the angle of internal friction
$\mathrm{qd}=$ allowable soil pressure ( $0.333 \mathrm{qd}^{\prime}$ )
$\mathrm{qd}^{\prime}=$ ultimate soil pressure
$\Phi=$ angle of internal friction (in turn a function of the relative density)
$\gamma=$ soil unit weight (in pounds per cubic foot)
Soil engineering practitioners use above equation often because it gives them the opportunity to use both terms as they see fit, since the first term is useful for shallow foundations and the second term for deep foundations where the surcharge above the foundation bottom becomes significant contributor to the design soil pressure.

Since some settlement of the soil directly under a footing should be expected, most known foundation design procedures are based on an expected soil settlement of 1 inch. In our office we often used the quick reference chart shown below as Figure 3.2. This chart proved to be quite handy for the determination of safe soil pressure. However, the applicability of such chart was subject to the following conditions:
a. soil settlements were limited to one inch,
b. water table had to be below the footing base for a depth equal or larger than B. If the water table was any closer, then soil pressure had be cut in half,
c. chart was limited to sandy soils.

Please observe from said chart that the relative density variable has been substituted by
the value N , which is the number of blows on the sampling spoon necessary to achieve one foot of penetration, as part of the standard penetration test (SPT).



FIGITRE 3.2

### 4.0 COMMON AND NOT SO COMMON FOUNDATION CASES

Figure 4.1 shows a two storied corner commercial building located in the downtown area at the intersection of Maple Avenue and Oak Street in a typical American town. It depicts an assumed ground floor column layout carried by a particular mix of concentric and eccentric spread footing conditions which may represent a good probability of occurring situations that the structural or foundation engineer may find in his/her daily routine design work assignments. Please also notice we have assumed a sandy soil with an allowable soil bearing capacity of 3,000 pounds per square foot.

The principle of the concentric footing is more often than not referred to as the occurrence of single column bearing on a single foundation. However, that does not deny the possibility of a plurality of columns bearing on a single or a series of interconnected foundations, as in the case of combined footings.

One debatable item amongst consulting engineers is the fact that he/ she should make a reasonable effort to solve the different problems generated by the particular structural layout as given to him/her. However, in cases such as the one depicted in Figure 4.1 the engineer could decide to suggest his architect to relocate gridline D a few feet in the West direction. That would be entirely his call.

As we make progress on the different parts of this series, we will refer back to this layout from time to time, so as to pinpoint examples and case studies which may be useful to the common practitioner.

When it comes to pile foundations, sheet pilings, helical piers, tie-backs, underpinnings, marine structures, as well as the application of soil modification methods, we have reserved their study for the latter parts of this series.


### 5.0 THE BASIC CONCENTRIC FOOTING

The concept advocating for footings to be sized reflecting areas directly proportional to the loads upon them, and further, adhering to the principle that the centroid of the area of their bases should have coincided with the load center, was first proposed and published in the United States by a Chicago architect in the year 1873, his name was Frederick Baumann (1826-1921). Such daring contentions for that time were part of the contents of his popular book titled "The Art of Preparing Foundations", where he well explained the concept and principles of concentric footing design.

Nowadays, that type of foundation is not only the most common found in practice, but also the easiest to design. Consequently, we will proceed to steadily progress through the labyrinth of available choices by going from the easy concentric footings to the not-so-easy cases of eccentric foundations.

When it comes to Mr. Baumann's plausible effort to define concentric footings, one clarification is in order. As much as his definition was straight down to the point, the pure condition as defined is not commonly found in practice, except in cases where the connection from column to footing is made through a fully operational mechanical hinge. Otherwise, there will always be a transmission of moment descending from the superstructure down into the foundation pad. Such moment (large or small) will affect the stress distribution at the foundation plane. At that point, it would be up to the design engineer to decide what to do. Normally, when the eccentricity generated by the moment is kept within the middle third of the footing dimension, the consequences are of limited effect. However, in cases of large eccentricities the compressive nature of the pressure distribution could be disturbed to the point of having undesirable tension stresses in the soil. In such an event, the design engineer could decide to relocate the centroid accordingly, so to overcome the created eccentricity.

Referring back to Figure 4.1 and looking at the grid system we can see a classic case of concentric footing at location B-4. One might be tempted to also categorize B-2, B-3 and $C-2$ in the same group, as they are shown together in the column load schedule; however, for the puritan engineer they belong apart due to the likely load eccentricity caused by the presence of moments generated by either the wind " $X$ " bracing or the foundation strap-beam.

The reader should also notice that the eccentricity at C-3 has been greatly minimized, or perhaps eliminated altogether, by using an old foundation scheme rarely employed by engineers, this time as a variant of the "double-buttressed" footing which details will be covered somewhere ahead in this part of the series.

Now, let us go back once more to Figure 4.1 and its footing marked B-4 which we will use as a case study for concentric footings. Such condition has been enlarged on Figure 5.1 to show some important considerations that the design engineer should keep in mind while preparing his working details. While dimensions B and Df have been defined above, $t$ is the footing total thickness and $p$ is the space between the footing top and the bottom of the ground slab, which in some instances may become very
significant. Finally, P is the column load excluding the footing weight and surcharge.
Commonly, the mechanical contractor and the plumber will find their way around and in between the footings, however, when bathrooms happen to be adjacent to columns, there must be provisions for the drainage pipelines to be located over the top of the concrete footing, and that is why the dimension $p$ should be appropriately adequate not only to accommodate the pipe diameter but also the required slope as well as room for other crossing pipes. Given such case of so common occurrence, we recommend that dimension to be at least one foot ( $1^{\prime}-0^{\prime \prime}$ ).

Footing analysis and design proceeds in the following manner:
Design Data:
Soil bearing capacity: 3,000 PSF
Concrete ultimate compressive strength: 3,000 PSI
Column load (as taken from Figure 4.1): 215.2 kips
Footing dead weight: $\underline{20.3}$

Total: $\quad 235.5 \mathrm{kips}$

$$
A_{f}=235.5 / 3.0=78.5 \text { sq. } \mathrm{ft} .
$$

Taken from the CRSI tables, an appropriate choice should be: $9^{\prime}-0^{\prime \prime} \times 9^{\prime}-0^{\prime \prime} \times 1^{\prime}-8^{\prime \prime}$ footing with 9 \# 8 bars, each way.

Adding the surcharge load:
Floor slab: 4,010 lbs
Backfill:
8,100 "
$12,110 \mathrm{lbs}$
Recap on soil pressure:

$$
\begin{aligned}
235,500+12,110 & =247,610 \mathrm{lbs} \\
\mathrm{f}_{\mathrm{s}}=247,610 / 81 & =3,057 \mathrm{PSF}
\end{aligned}
$$

Resulting soil overstress: $\quad 3,057 / 3,000=1.019(\approx 2 \%)$ ok


### 6.0 THE DOUBLE-BUTTRESSED FOOTING

This is a most ingenious solution that can be used to turn an eccentric condition into a concentric one by placing the footing contact areas in the right arrangement. Further, it holds pleasant surprises for the algebra inclined as we will see next.

To help with both, the geometric and the algebraic conception, please refer to Figure 6.1 at the end of this section. Before we start with the calculations, let us examine that figure in a more detailed manner. First, there are two C's shown on the figure, one is the grid line and the other one is the side dimension of the column, please do not confuse them. Dimension " $b$ " represents the typical width of any and all buttresses, while " $a$ " is the identical projection of any and all four buttresses.

Also notice that there is a four (4) inch difference between dimensions $b$ and $C$, it has been done that way to allow the column forms to sit even and squarely over a 2 inch shoulder at the intersection of the buttresses. If there is a good reason why that should not be done that way, then make such difference zero and therefore $C$ will equal $b$.

One more pertinent detail regarding this case, as shown on the graphic, buttresses need to be formed on both inner and outer sides. However, the interior form could be saved if the slab is rather made of a variable thickness, tapering from the top of buttress level down to its end thickness " t ".

Although there may be more than one way to determine the net area of foundation ( Af ), we recommend the following straight forward path to save time consuming and unnecessary trial and error procedure.

The first step should be the determination of the area of foundation (Af) which is equal to:

$$
\mathrm{Af}_{\mathrm{f}}=\mathrm{P}^{\prime} / \mathrm{fs}_{\mathrm{s}}
$$

In the same manner, the required foundation area is also shown in a graphic way on the upper right corner of Figure 6.1. In case you have not figured it out already, all that just means: four times the area of the buttresses plus the area of the triangular slabs, plus the common area at the buttress intersection, which can be expressed algebraically as:

$$
\mathrm{Af}_{\mathrm{f}}=\mathrm{a}^{2}+4 \mathrm{ab}+\mathrm{b}^{2}
$$

That is an algebraic expression called a trinomial which you are so familiar with, and the solution to that expression in terms of " $a$ " is:

$$
a=2 b\left[-1+\sqrt{ }\left(A f-b^{2} / 4 b^{2}\right)+1\right]
$$

Once the dimensional value of " a " is determined, the dimensions " m " and " n " should follow through:

$$
\mathrm{m}=1.41 \mathrm{a}
$$

$$
\mathrm{n}=\mathrm{a} / 1.41
$$

Since the soil pressure resultant under the buttress is:

$$
\mathrm{P}_{1}=\mathrm{f}_{\mathrm{s}} . \mathrm{a} \cdot \mathrm{~b}
$$

And under the triangular slab:

$$
\mathrm{P}_{2}=1 / 4\left(\mathrm{f}_{\mathrm{s}} . \mathrm{a}^{2}\right)
$$

Then, the design moment should be:

$$
\mathrm{M}=1 / 2 \mathrm{a}\left(\mathrm{P}_{1}+\mathrm{P}_{2}\right)
$$

Now, we are going to apply those equations to a practical example and using the details and information contained as part of Figures 4.1 and 6.1. These are the necessary values and dimensions:
$\mathrm{P}^{\prime}=170,000 \mathrm{lbs}$
$\mathrm{C}=16 \mathrm{in}$.
b $=20$ in. (1.67)
$\mathrm{fs}_{\mathrm{s}}=3,000 \mathrm{PSF}$
Required foundation area:

$$
\mathrm{Af}_{\mathrm{f}}=\mathrm{P}^{\prime} / \mathrm{f}_{\mathrm{s}}=170,000 / 3,000=56.67 \mathrm{SF}
$$

Also, by solving the trinomial:

$$
\begin{gathered}
a^{2}+4 a b+b^{2}=56.67 \\
a=3.34(-1+\sqrt{ } 5.83)=3.34 \times 1.415=4.725 \mathrm{ft}
\end{gathered}
$$

Verifying the results,

$$
\mathrm{Af}_{\mathrm{f}}=4(4.72 \times 1.67)+(4.725 \times 4.725)+(1.67 \times 1.67)=56.68 \mathrm{SF} \quad(\mathrm{ok})
$$

Now we can deduct the rest of the dimensions:

$$
\begin{aligned}
& \mathrm{n}=\mathrm{a} / 1.41=4.725 / 1.41=3.35 \mathrm{ft} \\
& \mathrm{~m}=1.41 \mathrm{a}=1.41 \times 4.725=6.66 \mathrm{ft}
\end{aligned}
$$

Soil pressure resultant under buttresses:

$$
\mathrm{P}_{1}=\mathrm{fs} \mathrm{a} b=3,000 \times 4.725 \times 1.67=23,672 \mathrm{lbs}=23.67 \mathrm{kips}
$$

Soil pressure resultant under triangular slab:

$$
\mathrm{P}_{2}=1 / 4\left(\mathrm{fs}_{\mathrm{s}} . \mathrm{a}^{2}\right)=0.25\left(3,000 \times 4.725^{2}\right)=16.74 \mathrm{kips}
$$

And the design moment:

$$
\mathrm{M}=1 / 2 \mathrm{a}\left(\mathrm{P}_{1}+\mathrm{P}_{2}\right)=1 / 2 \times 4.725 \times 40.41=95.47 \mathrm{kf}
$$



### 7.0 THE WEDGED FOUNDATION

This form of foundation can be technically described as an inverted truncated pyramid and its application should be reserved for large building or bridge column loads in excess of 800 kips. But on the other hand, it is also recommended for conditions with overturning moments such as guyed towers, TV antennas, cable car towers and elevated water tank foundations.

As many readers may perceive, this type of foundation has in fact been borrowed from the distant past. The idea seems to have originated in the early 1800's and entertained in the minds of engineers and architects up and well until the times of the controversial architect Frank Lloyd Wright.

As it becomes obvious by examination, there is no wasted effort or excavated material since the excavation can be shaped to accurately fit the configuration of the designed foundation, which is one of the reasons why loose soils are not recommended, as you will see below. Further, there are savings in the reinforcing cage since technically due to the absence of primary bending moments (or so as the case may be), this particular foundation shape may not require any reinforcing steel, although, we do recommend that the design engineer use some marginal reinforcing bars as dictated by his local jurisdictional code requirements on temperature stresses, or to fit the requirements of overturning moments, if any.

Although theoretically, this type of foundation could be used in any kind of soil medium, however, for practical reasons we recommend its use to be limited to soils that can be cut in a steeped angle and can develop higher than average soil bearing values, such as:
a. solid rock
b. limestone
c. sandstone
d. clayey gravel
e. sandy clay
f. clayey sand
g. loess
h. tuff

That eliminates from consideration soils such as: loose sand, loose gravel, pure clay, marl, gumbo, silt, peat, muck or any other soil material showing the presence of organic matter.

The accepted analytical procedure assumes a uniform soil pressure distribution at the flat portion of the base as well as on the inclined sides, in such a way that the total load $P^{\prime}$ is equal to $R 1+4 R 2$.

Figure 7.1 depicts a typical profile of the wedge foundation as well as the assumed soil
pressure distribution. Some available test results seem to suggest that soil pressure on the slanted sides tends to sharply diminish towards the upper end as shown with dash lines on the soil pressure diagram.

The following considerations are important before we go any further into this matter:
\#1- On the lower image of Figure 7.1, the area marked EFG would normally by subject to a weak tension. If no reinforcing bars were provided, sharp corner E could eventually crack and fallen free, in anticipation of such event a 6 in. chamfer is recommended all around the upper perimeter of the foundation.
\#2- In the upper left hand side image, angle $\beta$, although similar to, is not the internal friction angle ( $\Phi$ ) but rather the friction angle between concrete and the foundation soil.
\#3- Dimension $\mathrm{D}^{\prime}$ is not taken parallel to the slant side of foundation, but rather normal to the friction plane defined by angle $\beta$. The reason for that becomes evident next.
\#4- Soil pressure on the slanted sides also become influenced by the position of the friction plane, as well as the resultants marked R2.
\#5- Soil pressure on the bottom face of foundation is treated as commonly done, normal to side $\mathrm{B}^{\prime}$, and so its resultant R1.
\#6- Although the conventional bulb of pressures may suggest otherwise, for simplicity the soil pressure is assumed to be the same on the bottom as well as on all lateral sides of the upside-down pyramid.
\#7- This equation is always valid: $\mathrm{P}^{\prime}=\mathrm{R} 1+\mathrm{n} 2$, where n is in this case the number of sides of the truncated pyramid.

The following nomenclature has been maintained to be consistent with the geotechnical terminology:
$\alpha=$ slanted side angle
$\beta=$ friction angle between concrete and foundation soil
$\mathrm{P}=$ column load
$\mathrm{P}^{\prime}=$ column load plus surcharge
$\mathrm{f}_{\mathrm{s}}=$ soil pressure
$\mathrm{k}=$ angular constant
In the normal analytical procedure, all above values are known. However, for the analysis to have a starting point, dimension $\mathrm{B}^{\prime}$ at the base must either be assumed, deducted by trial \& error or obtained from reliable empirical sources. As a point of beginning, some comparative reference values are enclosed in Table 7.1 below:

TABLE 7.1
Design Aid*

| DESIGN PARAMETERS | P' <br> $($ KIP6 $)$ | $B^{\prime * *}$ <br> $(F T)$ |
| :--- | :---: | :---: |
| $f_{S}=6,000$ PSF | 800 | 5 |
| $\alpha=14^{\circ}$ | 1,000 | 6 |
| $\beta=33^{\circ}$ | 1,500 | 7 |
| $=3.45$ | 2,000 | 8 |
|  | 2,500 | 10 |

*Reproduced and adapted from "Beton Und Eisen", Craemer/Enyedi/Frolich, 1935.
**After dimensional equivalence, values were rounded to the nearest foot.

As a case study, we will apply now the concept of the wedged foundation as what could have been a possible solution for the Tower of Pisa. The general information and other known pertinent details on the project have been shown as part of Appendix A at the end of this course. When it comes to the dimension B to be used, the recorded vertical load of 35,000 kips has fallen beyond the limits of Table 7.1, therefore, we will make our own trial and error iteration to establish the best dimensional choice.

In order to have a better idea of the soil at the site, some time ago we made phone contact with a Mr. C. Giordanelli, he was a retired civil engineer who years back practiced geotechnical sciences in the city of Genoa, some 150 kilometers (approximately 94 miles) northwest of Pisa. He affirmed that although the soil at the site had a bad name for centuries, however, consistent with his personal experience, all of the coastal zone facing the Ligurian Sea, and in spite of the muck found in some isolated areas, they do have a calcarean hardpan at depths which varied from 5 to 6 meters $(16-20 \mathrm{ft})$ and that for footing sizing they used an allowable soil bearing capacity of about 3 kilograms per square centimeter, which equates to some 6,000 PSF by our standards. As far as the water table was concerned, he said that consistent with his observations along the Gulf of Genoa, during the rainy season the water table should expected at depths of 10 to 14 feet below grade.

Summarizing, the known values are: the total vertical load ( $\mathrm{P}^{\prime}$ ), the allowable soil bearing pressure ( fs ), the base width ( $\mathrm{B}^{\prime}$ ), and the angles $\alpha$ and $\beta$. On the other hand, the values sought are: the upper base dimension (B) and the foundation depth (D). Such values must be commensurate and not exceeding the design allowable soil pressure.

From equation:
$\operatorname{tg} a=\left(B-B^{\prime}\right) / 2 D$
we can deduct:

$$
D=\left(B-B^{\prime}\right) / 2 \operatorname{tg} a
$$

and since,

$$
\begin{gathered}
R_{1}=\left(B^{\prime}\right)^{2} \cdot f_{s} \\
\sin (\alpha+\beta)=\left(P^{\prime}-R_{1}\right) / 4 R_{2}
\end{gathered}
$$

then,
Now, we need to determine the constant $k$ :

$$
k=\sin (\alpha+\beta)+[\cos (\alpha+\beta)] / \tan \alpha
$$

to define $\mathrm{D}^{\prime}$ :

$$
D^{\prime}=1 / 2 k\left(B-B^{\prime}\right)
$$

As result, we can determine the effective area of every side of the pyramid measured along the friction plane:

$$
\text { Alat }=1 / 4 \mathrm{k}\left(\mathrm{~B}^{2}-\mathrm{B}^{\prime 2}\right)
$$

And finally:

$$
\left.B=\sqrt{ } B^{\prime 2}+\left[\left(P^{\prime} / f_{s}\right)-B^{\prime 2}\right)\right] / k \cdot \sin (\alpha+\beta)
$$

The value of D was already deducted above.
Now, back to the example of the Tower of Pisa:
$\mathrm{P}^{\prime}=35,000 \mathrm{kips}$
$\mathrm{fs}_{\mathrm{s}}=6,000 \mathrm{PSF}$
$\mathrm{B}^{\prime}=50 \mathrm{ft}\left({ }^{*}\right)$
$\alpha=14^{\circ}$
$\beta=33^{\circ}$
$(\alpha+\beta)=47^{\circ}$
$\tan 14^{\circ}=0.25$
$\sin 47^{\circ}=0.73$
$\cos 47^{\circ}=0.68$
$\mathrm{k}=3.45$
*After three consecutive trial \& error cycles, we decided for this value of B' because it had the best conditions to suit the characteristics of the job.

$$
B^{\prime 2}=50^{2}=2,500
$$

That gives us the foundation's upper width as:

$$
B=\sqrt{ }(2500)+(5833-2500) / 2.52=\sqrt{ } 3,823=61.83 \approx 62 \mathrm{ft} .
$$

And the depth as:

$$
D=(62-50) /(2 \times 0.25)=12 / 0.50=24 \mathrm{ft} .
$$

Those values are all represented as part of Figure 7.2 with the suggested foundation profile. Although we realize to be 840 years too late to be useful to the needs of the time, this is just what it is, an academic exercise. Lastly, no wind loadings have been taken into consideration. As we happen to touch the topic of wind, it is interesting to note that the tower in question would constitute a classic example of a good candidate for wind tunnel testing. While on the positive side, it is basically round in a plan view and cylindrical three-dimensionally, the exterior surface is roughened by intricate ornate forms, thin columns and penetrations, the center is by itself a vertical tunnel, or shaft, which would rush wind currents upwards, thus creating hard to visualize and/or predict suctions and pressures every inch of the way.

Figure 7.2 also shows the existing conditions of the site at time of construction, as well as what could have been our suggestions*, could we have had the opportunity to blend time and space to our free will and needs. On the left side of the graphic we show the 20 ft . layer of mucky soil laying over the calcareous hardpan (HP) described by the expert eyewitness. On the right side we show the area PQRS already excavated (all around) and the existing muck replaced by a transported soil mix combination of gravel, sand and caliche (G/S/C), placed in layers of one to two feet (maximum) and compacted to a verified $100 \%$ density. Then, the remaining depth cut through the hard pan to achieve the required foundation depth of 26 ft . ( $\mathrm{D}+2 \mathrm{ft}$ ).

Once the wedged foundation had been poured to the calculated depth of 24 ft and dowels conveniently left projecting out from the first pour, then, a 2 ft . second pour, marked M , consisting of a concrete pad reinforced with a double (top \& bottom) steel mat. Finally, doweling of the tower base to the second pour would have also been a highly desirable consideration.

Although the reinforcement shown in Figure 7.2 seems to suggest a circular distribution of the reinforcing bars, however, for convenience we would certainly favor a square patterned configuration with the main axes oriented in the NS and EW directions.

Unfortunately, the first engaged builder, Mr. Gerardo di Gerardo (or whomever he was) did not have the accumulated knowledge of our times, the materials, nor access to the notable work of talented soil engineers such as Drs. Karl Terzaghi's and Ralph B. Peck's to have taken advantage of, as we have.
*Other than drilling through the hard pan to reach greater depths and have drilled piers cast for an even better and more dependable solution, which needless to say, was an unknown methodology at the time.



### 8.0 CONCLUSION

Although the average architect and his consultants are not often in the position to advise a client before land is purchased. However, it would be wise for the prospective owner(s) to seek expert advice on site and foundation conditions before any purchase is finalized. Soil exploration or even an expert visual examination can reveal before hand, potential dangers that may later translate into unanticipated and burdensome expenses. Very often such an idealistic strategy on our part, gets shortcut and the owner ends up with an expensive solution in his hands which adds up to the price of land, and may ultimately reflect in the overall cost of the improvements.

With all that in mind, it has been the main objective of this course to provide all, the architect, the professional engineer as well as the dedicated engineering student, a condensed review of the available foundation theories and the practical application of those solutions as they may appear in common practice.

We have further tried to improve comprehension and procedural familiarity by presenting pertinent numerical examples with practical results, so as to dissipate any possible doubts which could otherwise always be present when only theoretical principles are enunciated. We certainly hope to have achieved such purpose and objectives.


## APPENDIX A

The Leaning Tower of Pisa ("Torre Pendente di Pisa" in the Italian Language or "La Torre Inclinada de Pisa" in Spanish), has always been a fascinating topic for most of the engineering professionals and enthusiasts, and although the original purpose of the building was just to serve as a bell tower, it became a worldwide well known structure for its peculiar unintended southbound inclination.

It is also a well known fact that its inclination, as it has been in progress for the last sixhundred years, has been caused by its inadequate foundation design in the presence of poor, unstable and erratic subsoil that varies from soft on one end to softer on the other, thus inducing very undesirable differential settlements in the direction of its offplumbness.

Construction of the tower was affected by periods of political upheaval and neglect which made the process to span for a period of one-hundred and ninety nine years, from 1173 to 1372. Builders came and went during that period of time, as it passed from the hands of Gerardo di Gerardo to Giovanni di Simone to Tommaso di Andrea to Guido Speziale to Giovanni Pisano, in an endless parade of contractors.

When it came to the responsible architect, the records are not clear enough to place the blame on a particular person and likely no one was any anxious to become the escapegoat, however, until recently the search has been narrowed down to three possible individuals: Bonanno Pisano, Guglielmo, or Diotisalvi. One of those three is the culprit in such a malpractice case. Unsurprisingly, whoever was the architect, he had more concern for the artistic value of his detailing than the structural stability of the skeleton (read framing) which was to provide support and carry on his work of art.

The available records show that the foundation system for the tower was erected on August 9,1173 and we all know that it was a far cry from what was needed. The massively conceived tower commanded an unfathomable total vertical dead weight of $32,000^{*}$ kips (as reported), that by itself produced a soil pressure of 16,913 PSF (see below) that was some four times in excess of the ultimate bearing capacity of the soil which material has been described at best as "muck with clay intrusions", as it was observed when the foundation was first excavated.
$\mathrm{P}=32,000$ Kips $\quad \mathrm{A}=0.7854\left(53^{2}-20^{2}\right)=0.7854 \times 2,409=1,892 \mathrm{SF}$
Then, $\quad \mathrm{P} / \mathrm{A}=32,000,000 / 1,892=16,913$ PSF
Ignoring live loads, which are indeed an insignificant part of this picture, and instead adding a moderate wind pressure of 60 miles per hour (MPH), plus the moment generated by the eccentricity induced by the tower's inclination:

If:

$$
\begin{gathered}
\mathrm{I}=0.0491 \times 7730481=379,567 \\
\mathrm{c}=26.5 \mathrm{ft}
\end{gathered}
$$

$$
\mathrm{M}_{\mathrm{w}}+\mathrm{Me}_{\mathrm{e}}=6,210,540+224,000,000=230,210,540 \mathrm{ft}-\mathrm{lbs}
$$

Then,

$$
\mathrm{Mc} / \mathrm{I}=(230,210,540 \times 26.5) / 379,567=16,072 \mathrm{PSF}
$$

Which adds another 16,072 pounds and almost doubles up the soil pressure at the footing toe. As we can see, this design was doomed since the beginning and the only factor that helped to keep the tower standing was the length of time it took for the construction process to reach its completion. That fact by itself, allowed the soil to slowly settle and get fairly squeezed and compacted as the erection was on the "stop and go" mode, otherwise the tower would have toppled at an early age and so avoided its fateful fame.

The enclosed figure show two images: an approximate sectional view of the tower and a footprint of the foundation. All dimensions, shapes and details that were necessary for us to reconstruct those views, were gathered from existing reference material available at the archives of the Brooklyn Museum as well as from the drawing files of the consulting firm "CyArk".

It should be noted here, that the position of the tower as shown reflects the conditions so as to the year of 1989. However, subsequent repairs were made in 1990 and 2008 and the engineers in charge have stated at the time of work completion that "the tower [was] is now stable for the next 200 years". Another piece of good news should be added, as part of the restoration efforts the angle of inclination $\delta$ has been reduced and is now just 4 degrees even.
> *Although a total of $32,000 \mathrm{kips}$ is the officially reported dead weight, we took it upon ourselves to verify that figure. We were actually able to verify an accountable weight of 22,000 kips, the rest may well exist and the difference could be in part, due to our relative unfamiliarity with the case. However, even the 22,000 kips would still produce considerably large soil pressure values quite in excess of a reasonably well assumed ultimate capacity.

## END

