PDHonline Course E105 (12 PDH)

Power Systems - Basic Concepts and Applications - Part II

Instructor: Shih-Min Hsu, Ph.D., P.E.

2020

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

An Approved Continuing Education Provider

Power Systems -
Basic Concepts and Applications

Part II

By Shih-Min Hsu, Ph.D., P.E.

MODULE 6: Power System Stability.

Overview

The importance of power system stability is increasingly becoming one of the most limiting factors for system performance. By the stability of a power system, we mean the ability of the system to remain in operating equilibrium, or synchronism, while disturbances occur on the system. There are three types of stability, namely, steady-state, dynamic and transient stability. However, to understand the basic concepts of power systems stability, only the transient stability with simplified system will be presented in this module.

Stability Definitions

In the study of electric power systems, several different types of stability descriptions are encountered. There are three types of stability, namely,

(1) Steady-state stability – refers to the stability of a power system subject to small and gradual changes in load, and the system remains stable with conventional excitation and governor controls.

(2) Dynamic stability – refers to the stability of a power system subject to a relatively small and sudden disturbance, the system can be described by linear differential equations, and the system can be stabilized by a linear and continuous supplementary stability control.

(3) Transient stability – refers to the stability of a power system subject to a sudden and severe disturbance beyond the capability of the linear and continuous supplementary stability control, and the system may lose its stability at the first swing unless a more effective countermeasure is taken, usually of the discrete type, such as dynamic resistance braking or fast valving for the electric energy surplus area, or load shedding for the electric energy deficient area. For transient stability analysis and control design, the power system must be described by nonlinear differential equations.

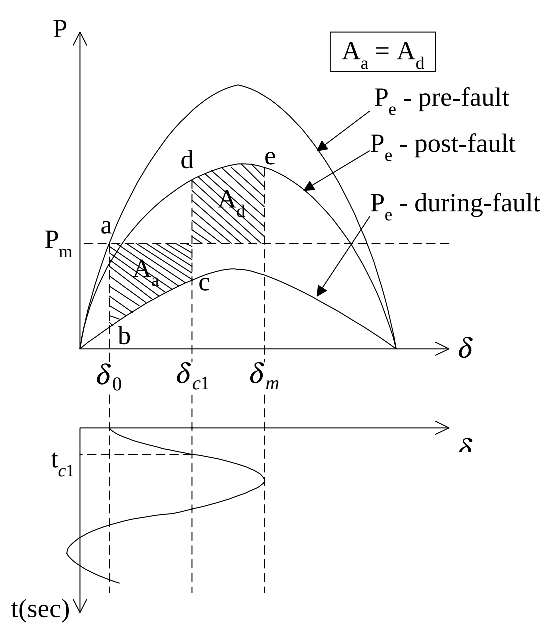
To understand the basic concept of stability of power systems, transient stability is discussed in this module. Examples requiring only simple calculations and computer programming using Matlab will be presented.

Fundamentals of Transient Stability

Transient stability concerns with the matter of maintaining synchronism among all generators when the power system is suddenly subjected to severe disturbances such as faults or short circuits caused by lightning strikes, the sudden removal from the transmission system of a generator and/or a line, and any severe shock to the system due to a switching operation. Because of the severity and suddenness of the disturbance, the analysis of transient stability is focused on the first few seconds, or even the first few cycles, following the fault occurrence or switching operation.

First swing analysis is another name that is applied to transient stability studies, since during the brief period following a severe disturbance the generator undergoes its first transient overshoot, or swing. If the generator(s) can get through it without losing synchronism, it is said to be transient stable. On the other hand, if the generator(s) loses its synchronism and can not get through the first swing, it is said to be (transient) unstable. There is a critical angle within which the fault must be cleared if the system is to remain stable. The equal-area criterion is needed and can be used to understand the power system stability. Before getting into detail discussion of the subject, some simple figures can be utilized to graphically represent the difference between a stable case and an unstable case. In a stable case, as shown in Figure 6-1, if the fault is cleared at t_{c1} second, or at angle δ_{c1}, where the area A_a (area associated with acceleration of the generator) equals the area A_d (area associated with deceleration of the generator). One can see that the angle reaches its maximum δ_m at t_{c1} and never gets greater than this value. In the unstable case, as shown in Figure 6-2, the fault is cleared at t_{c2} second with the area A_a greater than the area A_d. Also, it is very clear that for an unstable case, with the fault cleared at t_{c2} the angle keeps increasing and goes out-of-step, or unstable, as shown in Figure 6-2. More detail discussion on equal-area criterion will be presented later.

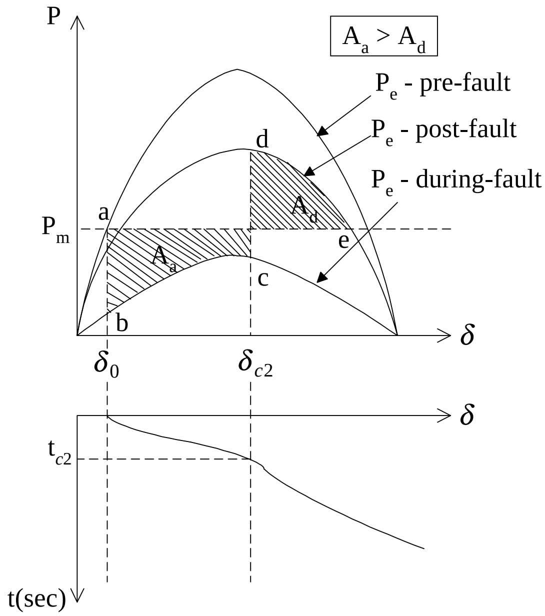
Stable Case



Response to a fault cleared in t_{c1} seconds - stable case

Fig. 6-1. First swing analysis for a stable case.

Unstable Case



Response to a fault cleared
in t_{c2} seconds - unstable case

Fig. 6-2. First swing analysis for an unstable case.

Swing Equation

The moment of inertia and the accelerating torque of a synchronous machine can be related as follows

$$J\frac{d^2\delta_m}{dt^2} = T_a ,$$

where

J = moment of inertia,

δ_m = mechanical angle,

and

$T_a = T_m - T_e$ = accelerating torque = the difference between the mechanical torque and the electromagnetic torque.

In steady-state conditions, $T_m = T_e$ and $T_a = 0$.

The relationship between the mechanical angle and the electrical angle (rotor angle) can be expressed as

$$\delta = \frac{P}{2}\delta_m ,$$

where

P is the number of poles of the machine.

Then, the equation of the accelerating torque can be re-written as

$$J \cdot \frac{2}{P} \cdot \frac{d^2\delta}{dt^2} = T_a .$$

It is reasonable to assume that the machine speed deviates very little from the synchronous speed, ω_s, therefore,

$$\omega_s \cdot J \cdot \frac{2}{P} \cdot \frac{d^2\delta}{dt^2} = \omega_s \cdot T_a = P_a .$$

A commonly used constant, inertia constant H, is defined as the ratio between the stored energy in watt-seconds and VA rating of the machine, namely,

$$H = \frac{\frac{1}{2}J\omega_s}{S} .$$

It can be re-arranged as

$$J\omega_s = \frac{2HS}{\omega_s} .$$

One can relate this equation to the equation for the accelerating power P_a,

$$\frac{2HS}{\omega_s} \cdot \frac{2}{P} \cdot \frac{d^2\delta}{dt^2} = P_a$$

If one defines

$$\omega_0 = \frac{P}{2}\omega_s ,$$

then, the above equation can be expressed as

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \frac{P_a}{S} ,$$

where all quantities are in their actual values.
Finally, the swing equation with the accelerating power in per unit value can be obtained as follows

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_a ,$$

or

$$M\frac{d^2\delta}{dt^2} = P_a\,,$$

where

M is the angular momentum,

and

$$M = \frac{2H}{\omega_0} = \frac{H}{60\pi}\,,$$

for the frequency of 60 Hertz.

Equal-Area Criterion

To understand the basic concepts of transient stability, equal-area criterion is suitable for this. The one-line diagram of a generator connecting to an infinite bus through a GSU, four transmission lines and a system reactance is shown in Figure 6-3. It is assumed that a three-phase fault occurs right at the plant switchyard. Its equivalent circuit can be obtained as shown in Figure 6-4. A classical model consisting of a generator transient reactance and a voltage source is used for the generator.

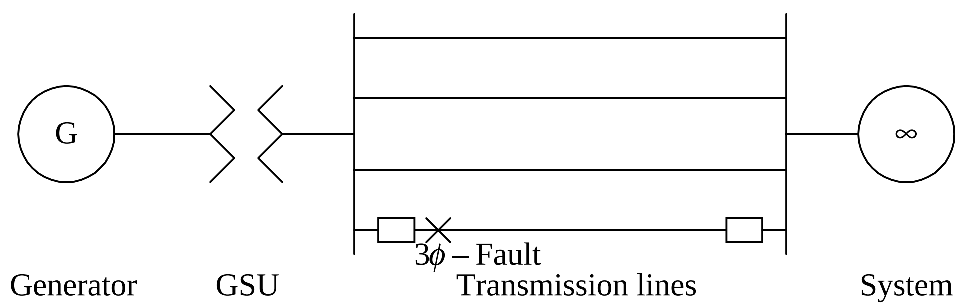


Fig. 6-3. One-line diagram of a one-machine with infinite bus system.

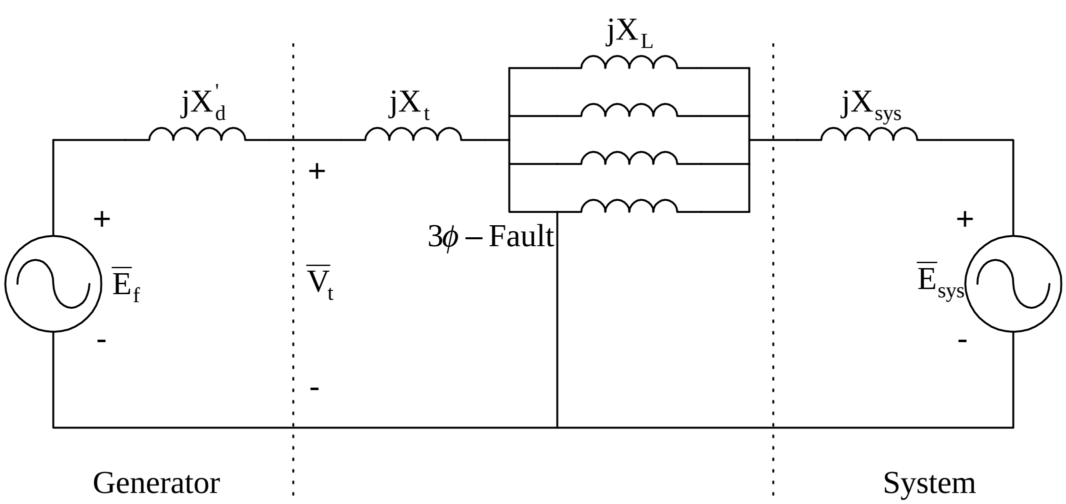


Fig. 6-4. Equivalent circuit of the simple system.

The following parameters are used for this simple system:

$S = 1.0$	: Initial VA output of the machine (in pu)
$pf = -0.8$	: Initial power factor of the machine (lagging)
$\overline{V}_t = 1.0\angle 0°$	: Initial terminal voltage (in pu)
$X_t = 0.1$	: GSU reactance (in pu)
$X_L = 0.4$	: Transmission line reactance (in pu)
$X_{sys} = 0.1$	: System reactance (in pu)
$X_d^{'} = 0.25$	: Generator transient reactance (in pu)
$H = 4$	: Inertia constant of the machine

Assuming a three-phase fault occurs at one of the four identical transmission line, however, some calculations are needed before the fault occurs, namely,
The generator current before the fault

$$\overline{I}_{gen} = \frac{S}{V_t}\angle -\cos^{-1}(pf) = 1\angle -36.87°.$$

The system voltage and the generator internal generated voltage can be calculated as

$$\overline{E}_{sys} = \overline{V}_t - j\left(X_t + \frac{X_L}{4} + X_{sys}\right)\overline{I}_{gen} = 0.82 - j0.24 = 0.8544\angle -16.31°,$$

and

$$\overline{E}_f = \overline{V}_t + \left(jX_d^{'}\right)\overline{I}_{gen} = 1\angle 0° + (j0.25)(1\angle -36.87°) = 1 + (0.15 + j0.2) = 1.1673\angle 9.87°.$$

The initial power angle equals the phase angle between the two voltages, namely,

$$\delta_0 = 9.87° + 16.31° = 26.18°.$$

Then, the maximum power

$$P_{max,0} = \frac{E_f E_{sys}}{X_{total,0}} = \frac{(1.1673)(0.8544)}{0.25 + 0.1 + 0.1 + 0.1} = 1.81335.$$

The initial mechanical power can be obtained

$$P_{mech,0} = P_{max,0}\sin(\delta_0) = (1.81335)\sin(26.18°) = 0.8,$$

which matches the initial power factor of the generator of 0.8.

Since it is a three-phase fault and the location of the fault, the electrical power output of the generator is zero when the fault is on. When the fault is cleared at t_{c1} with the transmission line tripped off, there are only three transmission lines left. The maximum power can be re-calculated as

$$P_{max,f} = \frac{E_f E_{sys}}{X_{total,f}} = \frac{(1.1673)(0.8544)}{0.25 + 0.1 + \frac{0.4}{3} + 0.1} = 1.70973 .$$

The final power angle equals

$$\delta_f = 180° - \sin^{-1}\left(\frac{P_{mech,0}}{P_{max,f}}\right) = 180° - \sin^{-1}\left(\frac{0.8}{1.70973}\right) = 152.1° .$$

The fault may be cleared between the initial angle and the final angle. For instance, if the fault is cleared at $\delta_{c1} = 80°$ as shown in Figure 6-5, then, the areas A_a and A_d can be calculated

$$A_a = \int_{\delta_0}^{\delta_{c1}} (P_{mech,0} - 0) \cdot d\delta = \int_{0.4569}^{1.3963} 0.8 d\delta = 0.7515 ,$$

and

$$A_d = \int_{\delta_{c1}}^{\delta_f} (P_{max,f} \sin\delta - P_{mech,0}) \cdot d\delta = \int_{1.3963}^{2.6547} (1.70973 \sin\delta - 0.8) \cdot d\delta = 0.8011721 .$$

Since $A_a < A_d$, it is stable.

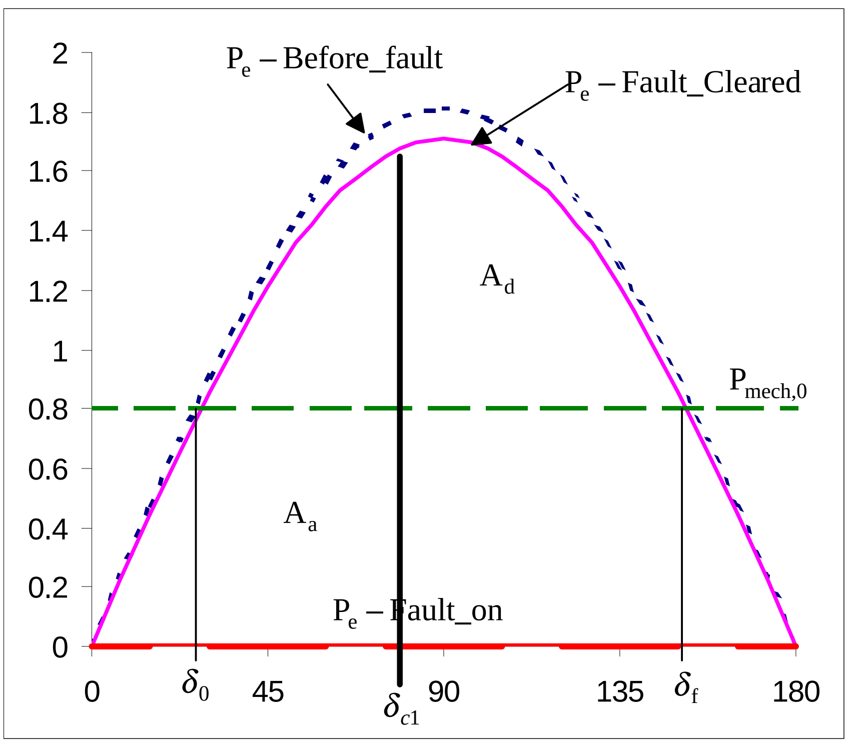


Fig. 6-5. A plot of power vs. angle for a fault cleared at $\delta_{c1} = 80°$ (a stable case).

If the fault is cleared at $\delta_{c2} = 110°$ as shown in Figure 6-6, then, the areas A_a and A_d can be calculated

$$A_a = \int_{\delta_0}^{\delta_{c2}} (P_{mech,0} - 0) \cdot d\delta = \int_{0.4569}^{1.92} 0.8 \cdot d\delta = 1.17037,$$

and

$$A_d = \int_{\delta_{c2}}^{\delta_f} (P_{max,f} \sin\delta - P_{mech,0}) \cdot d\delta = \int_{1.92}^{2.6547} (1.70973 \sin\delta - 0.8) \cdot d\delta = 0.33848.$$

Since $A_a > A_d$, it is unstable.

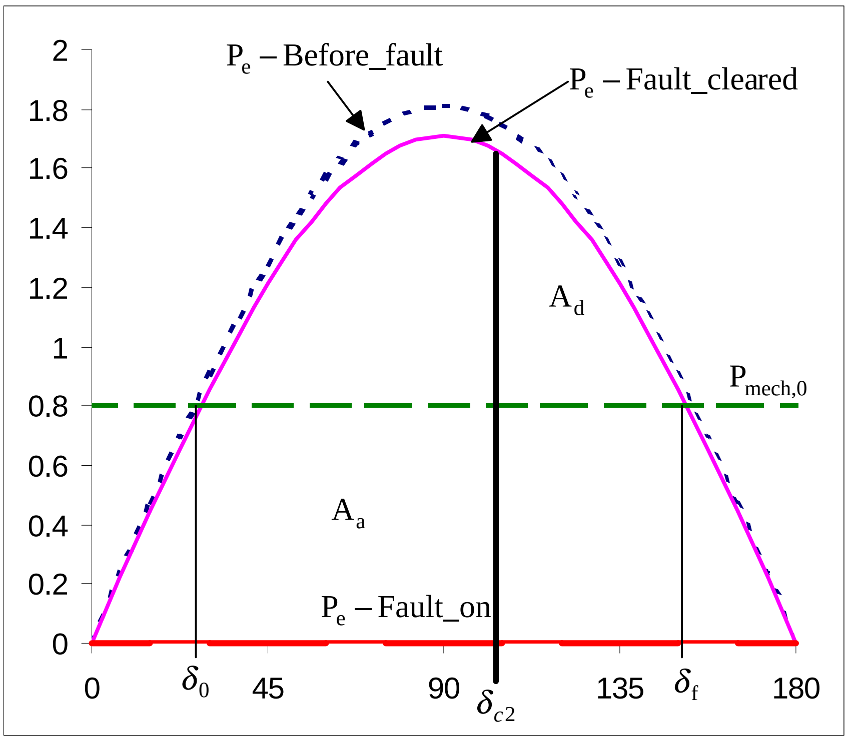


Fig. 6-6. A plot of power vs. angle for a fault cleared at $\delta_{c2} = 110°$ (an unstable case).

It is more practical for engineers to know what the maximum angle to clear the fault and still have a stable case. Such an angle is commonly called the "critical clearing angle," δ_{cc}. If one defines the ratio between the total initial reactance and the final reactance as

$$R_2 = \frac{X_{total,0}}{X_{total,f}}.$$

If the fault cleared at δ_{cc}, Areas $A_a = A_d$, therefore,

$$\int_{\delta_0}^{\delta_{cc}} (P_{mech,0}) \cdot d\delta - \int_{\delta_{cc}}^{\delta_f} (P_{max,f} \sin\delta - P_{mech,0}) d\delta = 0.$$

$$\Rightarrow \int_{\delta_0}^{\delta_{cc}} (P_{max,0} \sin\delta_0) \cdot d\delta + \int_{\delta_{cc}}^{\delta_f} (P_{max,0} \sin\delta_0 - R_2 \cdot P_{max,0} \sin\delta) d\delta = 0.$$

$$\Rightarrow (\delta_{cc} - \delta_0)\sin\delta_0 + \sin\delta_0(\delta_f - \delta_{cc}) + R_2(\cos\delta_f - \cos\delta_{cc}) = 0\,.$$

Therefore, the equation for the critical clearing angle can be expressed as

$$\delta_{cc} = \cos^{-1}\left(\frac{(\delta_f - \delta_0)\sin\delta_0 + R_2\cos\delta_f}{R_2}\right).$$

For the simple system, the critical clearing angle is

$$\delta_{cc} = \cos^{-1}\left(\frac{(2.6547 - 0.4569)\cdot\sin(26.18°) + \left(\frac{0.55}{0.5833}\right)\cdot\cos(152.1°)}{\left(\frac{0.55}{0.5833}\right)}\right) = 81.69°\,.$$

The corresponding critical clearing time, t_{cc}, may be useful for relay engineers to set the relay time. With a three-phase fault on,

$$M\frac{d^2\delta}{dt^2} = P_{mech,0}\,.$$

Then,

$$\frac{d\omega}{dt} = \frac{P_{mech,0}}{M}\,,$$

where

$$\omega = \frac{d\delta}{dt}\,.$$

Finally,

$$\frac{d\delta}{dt} = \frac{P_{mech,0}}{M}t\,.$$

Therefore,

$$\delta = \delta_0 + \frac{1}{2}\frac{P_{mech,0}}{M}t^2\,.$$

At $\delta = \delta_{cc}$, the corresponding critical clearing time can be obtained as

$$t_{cc} = \sqrt{\frac{2M(\delta_{cc} - \delta_0)}{P_{mech,0}}}\,.$$

For the simple system with three-phase fault, the critical clearing time is

$$t_{cc} = \sqrt{\frac{2\left(\frac{4}{60\pi}\right)(1.4257 - 0.4569)}{0.8}} = 0.22671 \text{ second} = 13.6 \text{ cycles.}$$

Therefore, to have a stable case the three-phase fault needs to be cleared within a total of 13.5 cycles.

There is another category of situations where the power out of the generator under evaluation is not zero during the fault on. If a fault other than a three-phase fault occurs at the plant or a three-phase fault occurs at locations away from the plant, for instance somewhere along a transmission line, the power output of the generator will not be zero while the fault is on.

A simple system with two identical transmission lines, as shown in Figure 6-7, is used to illustrate this situation. Since a single-line-to-ground (SLG) fault is assumed in this example, Figure 4-22 may be used to obtain the zero sequence network for the delta-wye grounded GSU. When the fault is on, the connection between the positive, negative and zero sequence networks can be obtained as shown in Figure 6-8. This circuit can be simplified into two voltage sources with a wye-connected reactances between them, as shown in Figure 6-9. The wy- connected reactances can be converted into a delta connection as shown in Figure 6-10.

The following parameters are used for this simple system:

$S = 1.0$: Initial VA output of the machine (in pu)
$pf = -0.8$: Initial power factor of the machine (lagging)
$\overline{V}_t = 1.0\angle 0°$: Initial terminal voltage (in pu)
$H = 4$: Inertia constant of the machine

	Positive sequence	Negative sequence	Zero sequence
X_L	0.4	0.4	1.0
X_{sys}	0.1	0.1	0.05
X_t	0.2	0.2	0.2
X_d'	0.25	0.1	N/A

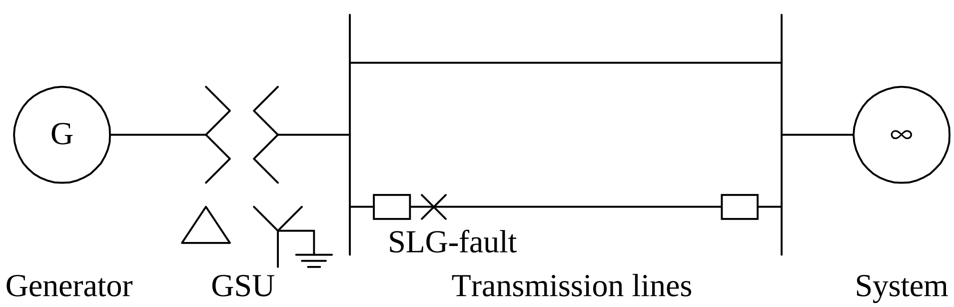


Fig. 6-7. A simple system with a SLG fault at plant switchyard on a transmission line.

Similar to the previous example, some calculations before the fault occurs are needed.
The generator current before the fault

$$\bar{I}_{gen} = \frac{S}{V_t}\angle -\cos^{-1}(pf) = 1\angle -36.87^\circ.$$

The system voltage and the generator internal generated voltage can be calculated as

$$\bar{E}_{sys} = \bar{V}_t - j\left(X_t + \frac{X_L}{2} + X_{sys}\right)\bar{I}_{gen} = 0.7 - j0.4 = 0.8062\angle -29.74^\circ,$$

and

$$\bar{E}_f = \bar{V}_t + j\left(X_d^{'}\right)\bar{I}_{gen} = 1\angle 0^\circ + (j0.25)(1\angle -36.87^\circ) = 1 + (0.25\angle 53.13^\circ) = 1.15 + j0.2 = 1.1673\angle 9.87^\circ$$

The initial power angle equals the phase angle between the two voltages, namely,

$$\delta_0 = 9.87^\circ + 29.74^\circ = 39.61^\circ = 0.6913\text{rad}.$$

Then, the maximum power before the fault occurs

$$P_{max,0} = \frac{E_f E_{sys}}{X_{total,0}} = \frac{(1.1673)(0.8062)}{0.25 + 0.2 + 0.2 + 0.1} = 1.2548.$$

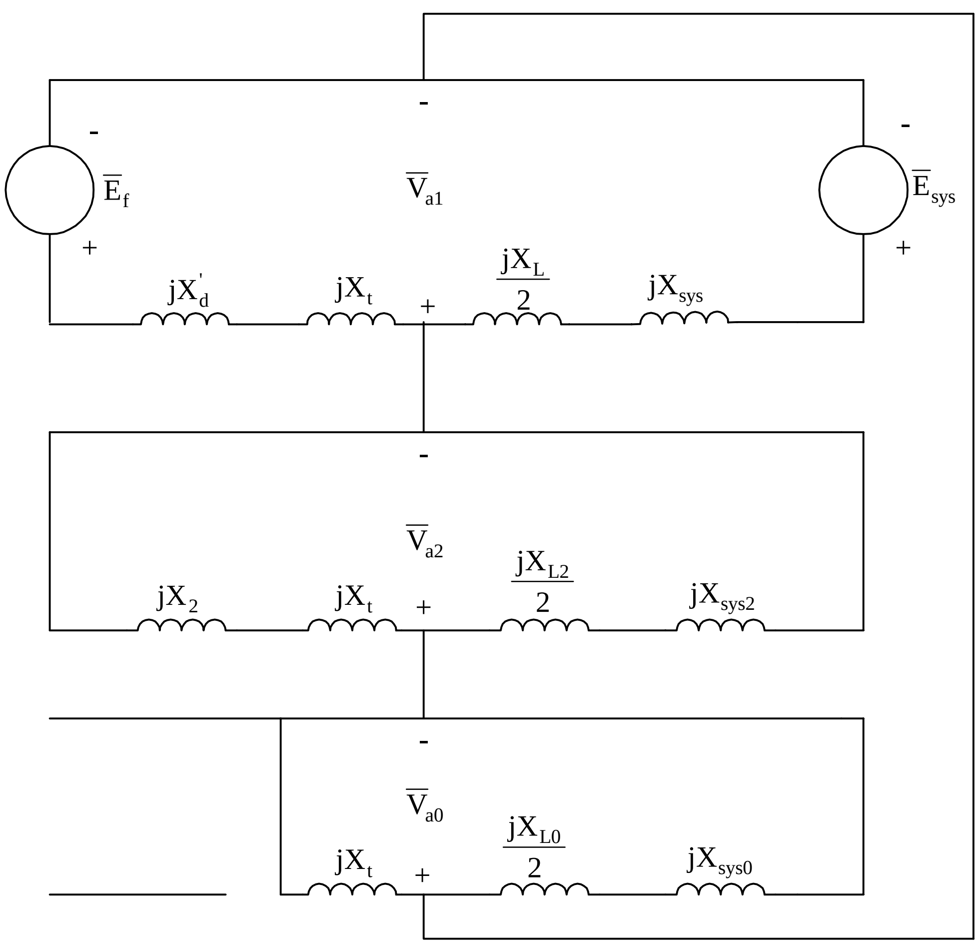


Fig. 6-8. Connection between positive, negative and zero sequence networks for the system with SGL fault.

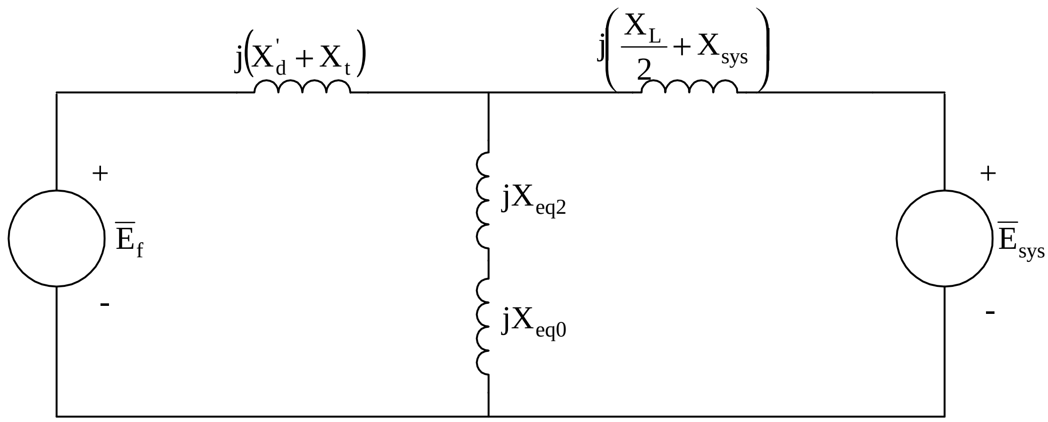


Fig. 6-9. The simplified equivalent circuit for SLG fault.

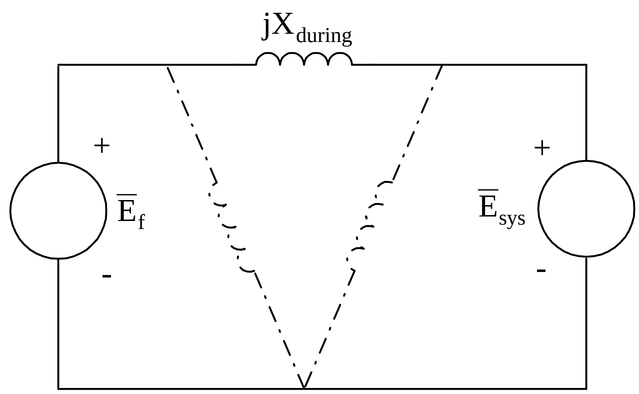


Fig. 6-10. The final equivalent circuit of SLG fault during the fault is on.

The initial mechanical power can be obtained

$$P_{mech,0} = P_{max,0}\sin(\delta_0) = (1.2548)\sin(39.61°) = 0.8,$$

which matches the initial power factor of the generator of 0.8.

Since it is a SLG fault, the electrical power output of the generator will not be zero when the fault is on. To calculate the maximum power during the fault on, Figures 6-8, 6-9 and 10 can be helpful to obtain the reactance needed. The equivalent reactances for the negative and zero sequence networks can be calculated as follows, respectively,

$$X_{eq2} = (X_2 + X_t)//\left(\frac{X_{L2}}{2} + X_{sys2}\right) = (0.1 + 0.2)//(0.2 + 0.1) = 0.15,$$

and

$$X_{eq0} = (X_t)//\left(\frac{X_{L0}}{2} + X_{sys0}\right) = (0.2)//(0.5 + 0.05) = 0.1467.$$

With the wye-delta conversion of the reactances as shown in Figure 6-10, the reactance between the two voltage sources can be obtained, namely,

$$X_{during} = \frac{\left(X_d^{'} + X_t\right)\left(X_{eq2} + X_{eq0}\right) + \left(\frac{X_L}{2} + X_{sys}\right)\left(X_{eq2} + X_{eq0}\right) + \left(X_d^{'} + X_t\right)\left(\frac{X_L}{2} + X_{sys}\right)}{\left(X_{eq2} + X_{eq0}\right)},$$

and

$$X_{during} = \frac{(0.45)(0.2967) + (0.3)(0.2967) + (0.45)(0.3)}{(0.2967)} = 1.2051.$$

Therefore, the maximum power during the fault on can be express as

$$P_{max,during} = \frac{E_f E_{sys}}{X_{during}} = \frac{X_{total,0}}{X_{during}} P_{max,0} = R_1 P_{max,0} = (0.6224)\cdot(1.2548) = 0.7809,$$

where R_1 is defined as

$$R_1 = \frac{X_{total,0}}{X_{during}} = \frac{0.75}{1.2051} = 0.6224.$$

When the fault is cleared at t_{c1} with the transmission line tripped off, there is only one transmission line left. The maximum power can be re-calculated as

$$P_{max,f} = \frac{E_f E_{sys}}{X_{total,f}} = \frac{X_{total,0}}{X_{total,f}} P_{max,0} = R_2 P_{max,0} = (0.7895)\cdot(1.2548) = 0.9906,$$

where R_2 is defined as

$$R_2 = \frac{X_{total,0}}{X_{total,f}} = \frac{0.75}{0.95} = 0.7895.$$

The final power angle equals

$$\delta_f = 180° - \sin^{-1}\left(\frac{P_{mech,0}}{P_{max,f}}\right) = 180° - \sin^{-1}\left(\frac{0.8}{0.9906}\right) = 126.14° = 2.2015\text{rad}.$$

The fault may be cleared between the initial angle and the final angle. For instance, if the fault is cleared at $\delta_{c1} = 80°$ as shown in Figure 6-11, then, the areas A_a and A_d can be calculated

$$A_a = \int_{\delta_0}^{\delta_{c1}} \left(P_{mech,0} - R_1 P_{max,0} \sin\delta\right)\cdot d\delta = \int_{0.6913}^{1.3963} (0.8 - 0.7809 \sin\delta)\cdot d\delta = 0.0979,$$

and

$$A_d = \int_{\delta_{c1}}^{\delta_f} \left(P_{max,f} \sin\delta - P_{mech,0}\right)\cdot d\delta = \int_{1.3963}^{2.2015} (0.9906 \sin\delta - 0.8)\cdot d\delta = 0.1120.$$

Since $A_a < A_d$, it is stable.

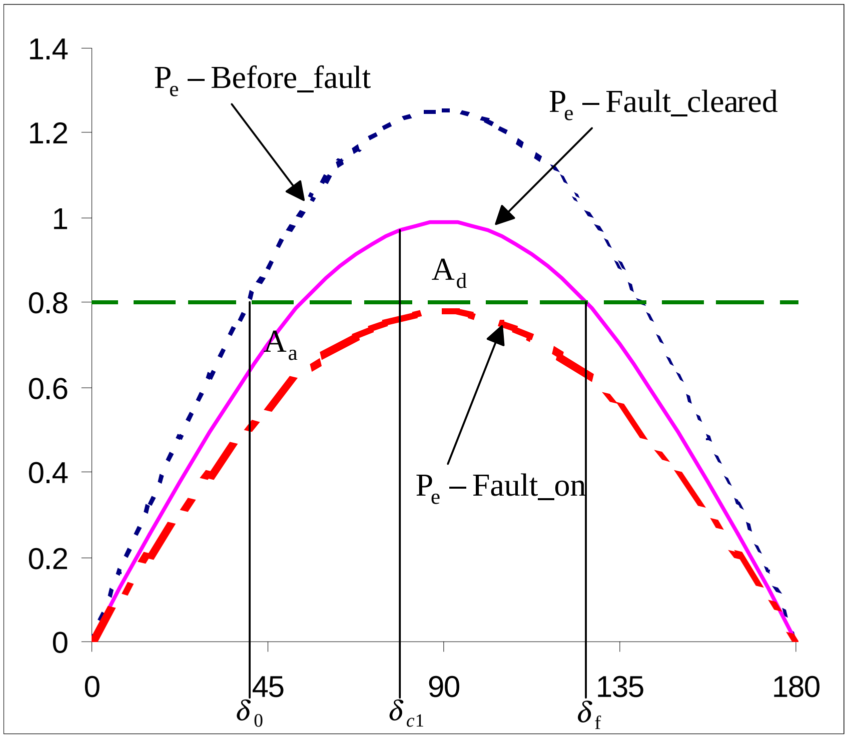


Fig. 6-11. A plot of power vs. angle for a SLG fault cleared at $\delta_{c1} = 80°$ (a stable case).

If the fault is cleared at $\delta_{c2} = 90°$ as shown in Figure 6-12, then, the areas A_a and A_d can be calculated

$$A_a = \int_{\delta_0}^{\delta_{c2}} \left(P_{mech,0} - P_{max,during} \sin\delta\right) \cdot d\delta = \int_{0.6913}^{1.5708} \left(0.8 - 0.7809 \sin\delta\right) \cdot d\delta = 0.1019,$$

and

$$A_d = \int_{\delta_{c2}}^{\delta_f} \left(P_{max,f} \sin\delta - P_{mech,0}\right) \cdot d\delta = \int_{1.5708}^{2.2015} \left(0.9906 \sin\delta - 0.8\right) \cdot d\delta = 0.0796.$$

Since $A_a > A_d$, it is unstable.

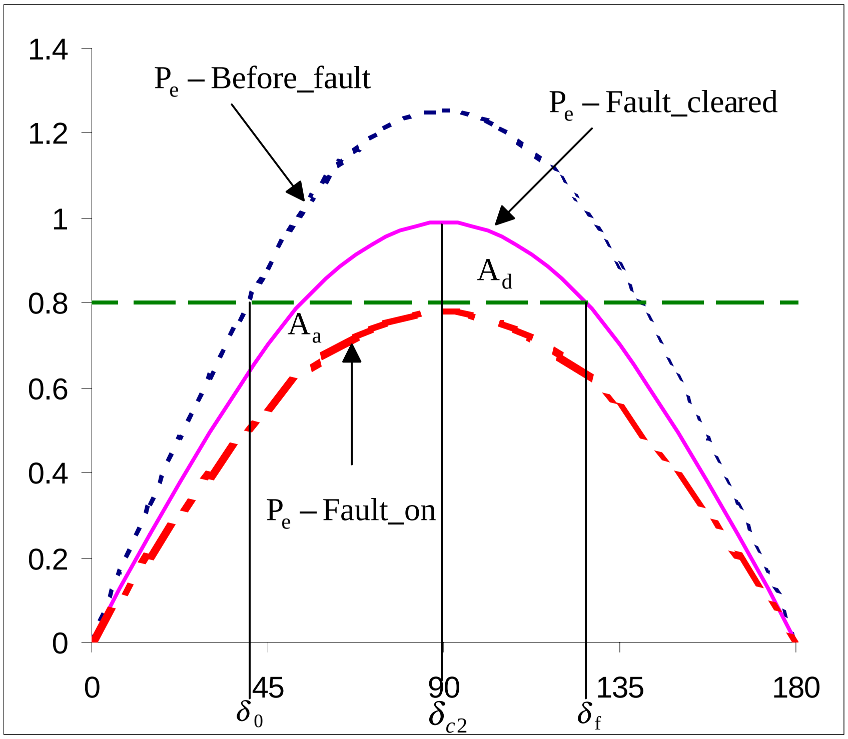


Fig. 6-12. A plot of power vs. angle for a SLG fault cleared at $\delta_{c2} = 90°$ (an unstable case).

Again, it may be more practical for engineers to know what the maximum angle to clear the fault and still have a stable case. If the fault cleared at δ_{cc}, Areas $A_a = A_d$, therefore,

$$\int_{\delta_0}^{\delta_{cc}} \left(P_{mech,0} - P_{max,during} \sin\delta\right)\cdot d\delta - \int_{\delta_{cc}}^{\delta_f} \left(P_{max,f} \sin\delta - P_{mech,0}\right) d\delta = 0 .$$

$$\Rightarrow \int_{\delta_0}^{\delta_{cc}} \left(P_{max,0} \sin\delta_0 - R_1 \cdot P_{max,0} \sin\delta\right)\cdot d\delta + \int_{\delta_{cc}}^{\delta_f} \left(P_{max,0} \sin\delta_0 - R_2 \cdot P_{max,0} \sin\delta\right) d\delta = 0 .$$

$$\Rightarrow (\delta_{cc} - \delta_0)\sin\delta_0 + R_1 \cos\delta_{cc} - R_1 \cos\delta_0 + \sin\delta_0(\delta_f - \delta_{cc}) + R_2 \cos\delta_f - R_2 \cos\delta_{cc} = 0$$

Therefore, the equation for the critical clearing angle can be expressed as

$$\delta_{cc} = \cos^{-1}\left(\frac{(\delta_f - \delta_0)\sin\delta_0 - R_1 \cos\delta_0 + R_2 \cos\delta_f}{R_2 - R_1}\right).$$

For the simple system, the critical clearing angle is

$$\delta_{cc} = \cos^{-1}\left(\frac{(2.2015 - 0.6913)\sin(39.61°) - (0.6224)\cos(39.61) + (0.7895)\cos(126.14°)}{0.7895 - 0.6224}\right) = 83.89°$$

Therefore, for the given simple system with a SLG fault, the fault needs to be cleared before 83.89° to have a stable system. Otherwise, the system will go unstable.

Unlike the previous case, the corresponding critical clearing time, t_{cc}, can not be calculated as discussed earlier. One way to get it is to solve the swing equation. A popular technique is the Runge-Kutta method. However, details on Runge-Kutta method is not the scope of this material. By printing the time steps and the corresponding angles, as shown in Table 6-1, one can easily estimate the critical clearing time, t_{cc}. For instance, the critical clearing time for this simple system with a SLG fault at plant switchyard is between 0.375 and 0.3833 second since its critical clearing angle is 83.89°. Therefore, to have a stable case, the SLG fault needs to be cleared within 22.5 cycles.

Table 6-1. Time steps and corresponding power angles from the example.

Time Steps in Second	Time Steps in cycles	Power Angles in degrees
0	0	39.6107
0.0083	0.5	39.6390
0.0167	1.0	39.7239
•	•	•
•	•	•
0.2917	17.5	68.4727
0.3000	18.0	69.8538
0.3083	18.5	71.2474
0.3167	19.0	72.6524
0.3250	19.5	74.0676
0.3333	20.0	75.4921
0.3417	20.5	76.9248
0.3500	21.0	78.3648
0.3583	21.5	79.8115
0.3667	22.0	81.2640
0.3750	**22.5**	**82.7219**
0.3833	**23.0**	**84.1845**
0.3917	23.5	85.6514
0.4000	24.0	87.1223
0.4083	24.5	88.5970
0.4167	25.0	90.0753
0.4250	25.5	91.5572
0.4333	26.0	93.0428
0.4417	26.5	94.5321

There are two Matlab programs (m-files) for the two examples given in this module. The first one is for the case when the output power from the generator under evaluation is zero while the fault is on. It is given in Appendix 6A. The second m-file is for the case when the output power from the generator while the fault on has a none-zero value. It is given in Appendix 6B. Readers have Matlab may use these two m-files for simple stability evaluations. Figures 6-13 and 6-14 show the power angle curves with the clearing angles given in the two examples.

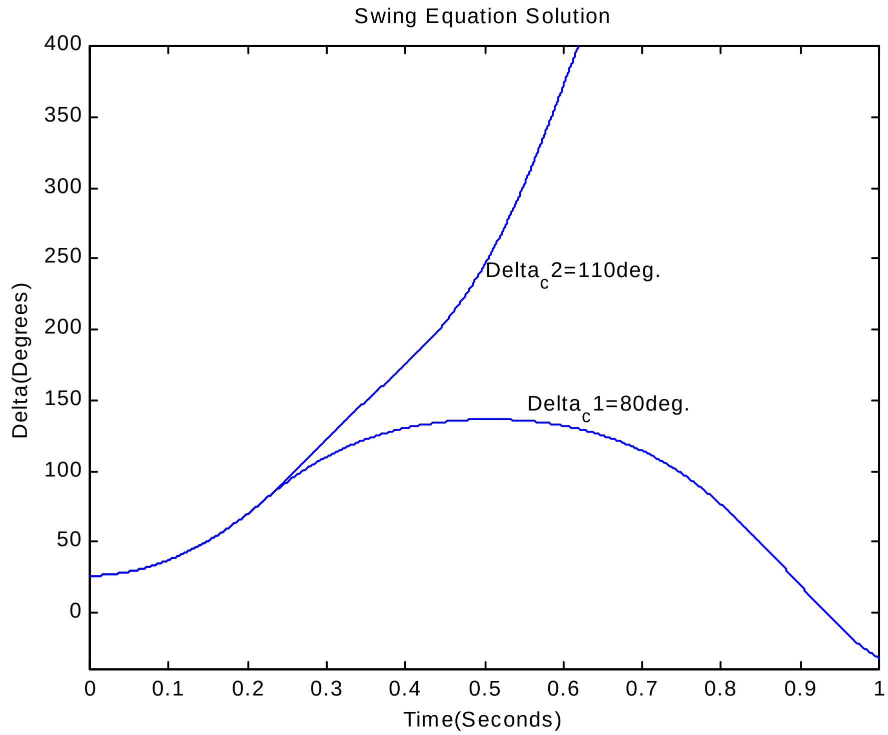


Fig. 6-13. Power angle curves for the example with a three-phase fault.

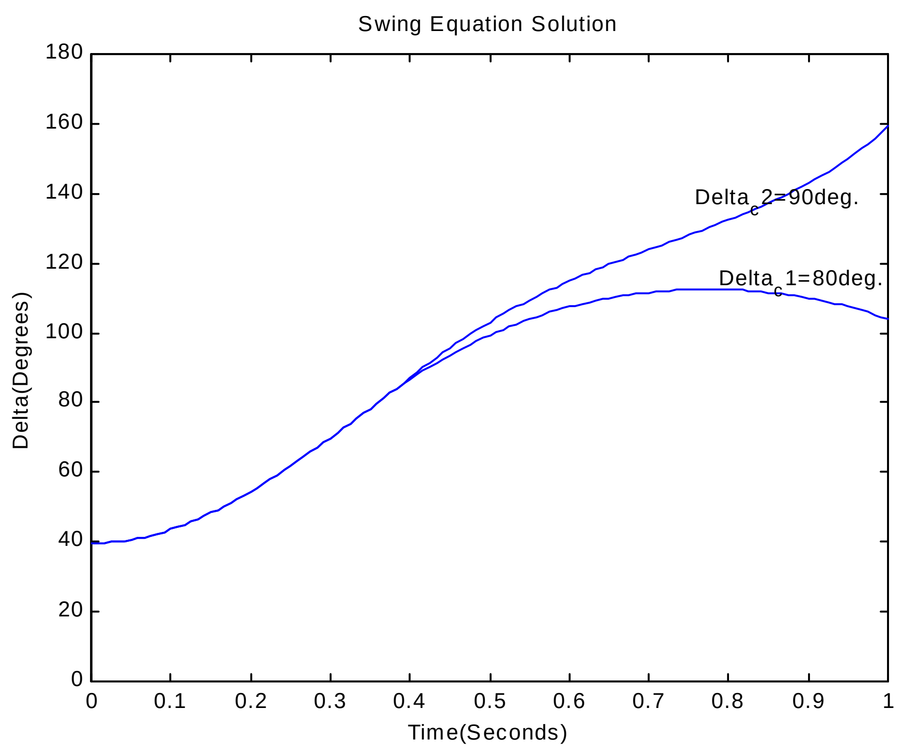


Fig. 6-14. Power angle curves for the example with a SLG fault.

Appendix 6A: Matlab program for a three-phase fault with zero output power from the generator when fault is on

```
% This program computes the swing curve of a generator connected to
% an infinite bus through a step-up transformer of reactance Xt,
% Np transmission lines (each having a reactance XL) and a system
% reactance Xsys.  The generator has reactance Xpd.  The system
% voltage magnitude is Esys and the voltage magnitude behind the
% generator transient reactance is Ef.  The initial generator output
% VA and pf are specified.  Note: The pf is negative if it is lagging.
%
% SMH 8/12/2001 for PDH Center - Power Systems Part II
%

clear all

Np = 4         % Number of transmission lines
S = 1.0;       % Initial VA output of the machine (per unit)
pf = -0.8;     % Initial pf of machine
Vt = 1.0;      % Initial terminal voltage of machine (per unit)
XL = 0.4;      % TL reactance
Xsys = 0.10;   % System reactance
Xt = 0.10;     % Transformer reactance
Xpd = 0.25;     % Transient reactance of generator
rad = 180/pi; % Converts radians to degrees
H = 4;         % Inertia constant of machine
Imag = S/Vt;   % Initial current output of machine (per unit)
global Pmech   % Input power of machine
global Pmax    % Output power of machine
global M       % Machine constant
global Tc      % Clearing time


% Calculate pf angle

if pf < 0.0
   theta = -acos(abs(pf));
else
   theta = acos(abs(pf));
end

% Calculate the complex current outout of the machine
Igen = Imag*(cos(theta)+j*sin(theta))

% Calculate the system voltage
Esysc = Vt - j*(Xt + (XL/Np) + Xsys)*Igen
Esys = abs(Esysc)
Esys_deg=angle(Esysc)*rad

% Calcualte the voltage behind the transient reactance
Efc = Vt + j*Xpd*Igen
Ef = abs(Efc)
Ef_deg=angle(Efc)*(180/pi)

% Calculate the impedance of the system before and after the fault
Xbefore = Xpd + Xt + XL/Np + Xsys
```

```
Xafter = Xpd + Xt + XL/(Np-1) + Xsys

% Calculate the initial value of delta

delta_init = angle(Efc) - angle(Esysc);
delta_init_deg=delta_init*rad

% Calculate the initial Pmax of the machine
Pmax0=(Ef*Esys)/Xbefore

% Calculate the initial input power to the machine
Pmech = Pmax0*sin(delta_init)

% Calculate the ratio of impedance after the fault to before the fault
R2 = Xbefore/Xafter

% Calculate the value of delta after the fault
delta_final = pi - asin((sin(delta_init)/R2));
delta_final_deg=delta_final*rad

M = H/(60*pi);

% Calculate critical clearing time
T = ((delta_final-delta_init)*sin(delta_init)+R2*cos(delta_final))/R2

% Calculate the critical clearing angle
delta_clear = acos(T);
delta_cc_deg=delta_clear*rad
Tcc = sqrt((2*M*(delta_clear - delta_init))/Pmech)

% Calculate final Pmax
% Pmax = (Ef*Esys)/Xafter
Pmax=R2*Pmax0

dt = 0.0001; % step size

period=input('Please input the time period in seconds for the simulation ')
nsteps = 1 + period*1/dt % number of steps in 1 second
% Let clearing time be Tc = 0.22 seconds
% Compute nsteps1 which is the number of steps that fault is on and Pa =
Pmech
input_method=input('please input "1" for clearing time in seconds, OR "2" for
clearing angle in degrees  ');
if input_method == 1
   Tcip=input ('Please input the clearing time in seconds =');

   deltac=delta_init+(1/2)*(Pmech/M)*Tcip*Tcip
   Tc=round(Tcip*1000)/1000
end
if input_method == 2
   deltac_deg=input('please input the clearing angle in degrees =');
   deltac=deltac_deg/rad;
   Tcini = sqrt((2*M*(deltac - delta_init))/Pmech)
   Tc=round(Tcini*1000)/1000
end

% calculate the area of accelerating power Aa
```

```
Aa=Pmech*(deltac - delta_init)

% calculate the area of deccelerating power
Ad=Pmax*(-cos(delta_final)+cos(deltac))-Pmech*(delta_final - deltac)

%Tc=Tc_c/60
nsteps1 = 1 + Tc/dt;

nsteps2 = nsteps1 + 1;

% The following code solves the swing equation by using the 4th order
% Runge-Kutta method
t(1) = 0.0;
delta(1) = delta_init;
w(1) = 0;

for i = 2:nsteps1
   k1 = dt*w(i-1);
   l1 = dt*(Pmech/M);
   k2 = dt*(w(i-1) + l1/2);
   l2 = dt*(Pmech/M);
   k3 = dt*(w(i-1) + l2/2);
   l3 = dt*(Pmech/M);
   k4 = dt*(w(i-1) + l3);
   l4 = dt*(Pmech/M);
   delta(i) = delta(i-1)+(k1+2*k2+2*k3+k4)/6;
   w(i) = w(i-1)+(l1+2*l2+2*l3+l4)/6;
   t(i) = (i-1)*dt;
end


for i = nsteps2:nsteps
   k1 = dt*w(i-1);
   l1 = dt*(Pmech - Pmax*sin(delta(i-1)))/M;
   k2 = dt*(w(i-1) + l1/2);
   l2 = dt*(Pmech - Pmax*sin(delta(i-1)+ k1/2))/M;
   k3 = dt*(w(i-1) + l2/2);
   l3 = dt*(Pmech - Pmax*sin(delta(i-1)+ k2/2))/M;
   k4 = dt*(w(i-1) + l3);
   l4 = dt*(Pmech - Pmax*sin(delta(i-1)+ k3))/M;
   delta(i) = delta(i-1) + (k1+2*k2+2*k3+k4)/6;
   w(i) = w(i-1) + (l1+2*l2+2*l3+l4)/6;
   t(i) = (i-1)*dt;
end

for i = 1:nsteps
   deg(i) = rad*delta(i);
end

plot(t,deg,'b');
xlabel('Time(Seconds)')
ylabel('Delta(Degrees)')
title('Swing Equation Solution')
grid

% results(:,1) = t(:);
% results(:,2) = deg(:); %x1 in degrees
```

```
% results(:,3) = x2(:);
% results
```

Appendix 6B: Matlab program for a SLG fault - none-zero output power from the generator when fault is on

```
% This program computes the critical clearing time of a generator connected
% to an infinite bus through a step-up transformer of reactance Xt, and two
T.L.
% The fault is a line-to-ground (SLG) fault on a line close to the step-up
transformer.
% Np transmission lines (each having a reactance XL) and a system
% with reactance Xsys.  The generator has reactance Xpd.  The system
% voltage magnitude is Esys and the voltage magnitude behind the
% generator transient reactance is Ef.  The initial generator output
% VA and pf are specified.  Note: The pf is negative if it is lagging.
%
% SMH 3/12/2002 for PDH Center - Power Systems Part II
%


Np = 2          % Number of transmission lines
S = 1.0;        % Initial VA output of the machine (per unit)
pf = -0.8;      % Initial pf of machine
Vt = 1.0;       % Initial terminal voltage of machine (per unit)
XL = 0.4;       % positive sequence TL reactance
XL0 = 1.0;      % zero sequence TL reactance
Xsys = 0.10;    % System reactance
Xt = 0.20;      % Transformer reactance
Xpd = 0.25;     % Transient reactance of generator
rad = 180/pi;   % Converts radians to degrees
H = 4;          % Inertia constant of machine
Imag = S/Vt;    % Initial current output of machine (per unit)

% Calculate pf angle

if pf < 0.0
   theta = -acos(abs(pf));
else
   theta = acos(abs(pf));
end

% Calculate the complex current output of the machine
Igen = Imag*(cos(theta)+j*sin(theta));

% Calculate the system voltage
Esysc = Vt - j*(Xt + (XL/Np) + Xsys)*Igen;
Esys = abs(Esysc);
Esys_deg=angle(Esysc)*(180/pi);


% Calculate the voltage behind the transient reactance
Efc = Vt + j*Xpd*Igen;
Ef = abs(Efc);
Ef_deg=angle(Efc)*(180/pi);

M = H/(60*pi);

% Calculate the initial value of delta, i.e., delta0
```

```
delta_init = angle(Efc) - angle(Esysc);
delta_init_deg = rad*delta_init;


% Calculate three impedances between the voltage sources.
% One X is before the fault occurs. This impedance is called Xbefore
% A second X is while the fault is on. This impedance is Xduring
% The third X is after the fault is cleared. Call it Xafter
Xbefore = Xpd + Xt + XL/Np + Xsys;
Xafter = Xpd + Xt + XL/(Np-1) + Xsys;

% Xduring is calculated from a wye-delta transformation. Let c
% be common point in wye. Let 1 be generator internal point,
% Let 2 be infinite bus internal point, let n be neutral point

X1c = Xpd + Xt;
X2c = XL/Np + Xsys;

% Let X2 of generator be 0.10, let X2 of system be 0.10,
% let X0 of system be 0.05
X2gen = 0.10; X2sys = 0.10;  X0sys = 0.05;

% Negative seq. impedance looking into fault point is computed as

Xdiv = X2gen + Xt + XL/Np + X2sys;
X2in = (X2gen + Xt)*(XL/Np + X2sys)/Xdiv;

% Zero seq. impedance looking into fault point is computed as

Xdiv0 = Xt + XL0/Np + X0sys;
X0in = Xt*(XL0/Np + X0sys)/Xdiv0;
Xnc = X2in + X0in;

Prod = X1c*X2c + X2c*Xnc + Xnc*X1c;
Xduring = Prod/Xnc;

R1 = Xbefore/Xduring;
R2 = Xbefore/Xafter;

% Calculate the deltaf, the maximum swing of system
delta_final = pi - asin(sin(delta_init)/R2);
delta_final_deg = rad*delta_final;

% Calculate intermediate term T used in calculating the critical clearing
angle
T = ((delta_final-delta_init)*sin(delta_init) - R1*cos(delta_init) +
R2*cos(delta_final))/(R2 - R1);

% Calculate the critical clearing angle deltac
deltac = acos(T);
deltac_deg = rad*deltac;

% calculate the initial Pmax0
Pmax0=Ef*Esys/Xbefore;

% Calculate the initial input power to the machine
Pmech = Pmax0*sin(delta_init);
```

```
% Calculate Pmax while fault on
% Pmaxon = (Ef*Esys)/Xduring
Pmax = R1*Pmax0;
% Calculate the final Pmax

Pmaxf = R2*Pmax0;

% step size
dt = 0.5/60;
Period=1

Tc1=input('Please input the clearing time in cycles (with 0.5 cylce
increment) = ');
Tc=Tc1/60

% Calculate the Aa and Ad, NOT used for the simulation
deltac_deg=input('please input the clearing angle in degrees =');
deltac=deltac_deg/rad;
% calculate the area of accelerating power Aa
Aa=Pmech*(deltac - delta_init) - Pmax*(-cos(deltac)+cos(delta_init))

% calculate the area of deccelerating power Ad
Ad=Pmaxf*(-cos(delta_final)+cos(deltac))-Pmech*(delta_final - deltac)


% # of steps in 1 second
%nsteps=1+Period*1/dt
nsteps=1+Period/dt;
%nsteps1 = 1 + Tc/dt;
nsteps1=1+2*Tc1;

% The following code solves the swing equation by using the 4th order
% Runge-Kutta method

t(1) = 0.0;
delta(1) = delta_init;
w(1) = 0;

for i = 2:nsteps1
   k1 = dt*w(i-1);
   l1 = dt*(Pmech - Pmax*sin(delta(i-1)))/M;
   k2 = dt*(w(i-1) + l1/2);
   l2 = dt*(Pmech - Pmax*sin(delta(i-1)+ k1/2))/M;
   k3 = dt*(w(i-1) + l2/2);
   l3 = dt*(Pmech - Pmax*sin(delta(i-1)+ k2/2))/M;
   k4 = dt*(w(i-1) + l3);
   l4 = dt*(Pmech - Pmax*sin(delta(i-1)+ k3))/M;
   delta(i) = delta(i-1) + (k1+2*k2+2*k3+k4)/6;
   w(i) = w(i-1) + (l1+2*l2+2*l3+l4)/6;
   t(i) = (i-1)*dt;
end

nsteps2 = nsteps1 + 1

for i = nsteps2:nsteps
   k1 = dt*w(i-1);
```

```
   l1 = dt*(Pmech - Pmaxf*sin(delta(i-1)))/M;
   k2 = dt*(w(i-1) + l1/2);
   l2 = dt*(Pmech - Pmaxf*sin(delta(i-1)+ k1/2))/M;
   k3 = dt*(w(i-1) + l2/2);
   l3 = dt*(Pmech - Pmaxf*sin(delta(i-1)+ k2/2))/M;
   k4 = dt*(w(i-1) + l3);
   l4 = dt*(Pmech - Pmaxf*sin(delta(i-1)+ k3))/M;
   delta(i) = delta(i-1) + (k1+2*k2+2*k3+k4)/6;
   w(i) = w(i-1) + (l1+2*l2+2*l3+l4)/6;
   t(i) = (i-1)*dt;
end


%for i = 2:200
%   a1 = dt*x2(i-1);
%   a2 = dt*(Pmech - Pmax*sin(x1(i -1)))/M;
%   b1 = dt*(x2(i-1) + a2/2);
%   b2 = dt*(Pmech - Pmax*sin(x1(i -1) + a1/2))/M;
%   c1 = dt*(x2(i-1) + b2/2);
%   c2 = dt*(Pmech - Pmax*sin(x1(i -1) + b1/2))/M;
%   d1 = dt*(x2(i-1) + c2);
%   d2 = dt*(Pmech - Pmax*sin(x1(i -1) + c1))/M;
%   x1(i) = x1(i-1)+(a1+2*b1+2*c1+d1)/6;
%   x2(i) = x2(i-1)+(a2+2*b2+2*c2+d2)/6;
%   t(i) = (i - 1)*dt;
%end

for k = 1:i
   deg(k) = rad*delta(k);
end

plot(t,deg,'b');
xlabel('Time(Seconds)')
ylabel('Delta(Degrees)')
title('Swing Equation Solution')
grid on

% figure
% plot(t,x2(:),'b');
% xlabel('Time(Seconds)')
% ylabel('Speed, rad/sec')
% title('Swing Equation Solution')
% grid

% Note: x1 is is degrees and x2 is in radians/sec

 results(:,1) = t(:);
 results(:,2) = deg(:); %x1 in degrees
 %results(:,3) = x2(:);
 results;
```