PDHonline Course E251 (8 PDH)

Short Circuit Current Calculations Using Symmetrical Components

Instructor: Ralph Fehr, Ph.D., P.E.

2012

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone & Fax: 703-988-0088
www.PDHonline.org
www.PDHcenter.com

Short Circuit Current Calculations Using Symmetrical Components

Ralph Fehr, Ph.D., P.E.

Course Content

Introduction

In 1918, Charles Fortescue presented a paper to the American Institute of Electrical Engineers in Atlantic City describing how a system of *n* unbalanced but related phasors can be represented by *n* systems of balanced phasors. Using this principle, any unbalanced three-phase system can be represented by three balanced *sequence networks*.

The theory of symmetrical components and the synthesis of sequence networks for three-phase power systems are instrumental for solving most unbalanced problems such as asymmetrical short-circuit and open-circuit faults. Symmetrical components and sequence networks are also vital for understanding the unbalanced operating conditions of an otherwise balanced system, and the behavior and influence of harmonic voltages and currents.

Unfortunately, the theory of symmetrical components is often learned as a set of abstract algebraic equations, into which known values are substituted and, hopefully, out of which the correct answer emerges. Sequence networks are often synthesized using a *building block approach*, where documented sequence impedance models of various power system elements are connected together much like building blocks to form the sequence network. This method often leads to errors, since topological errors in connecting the *blocks* are easy to make, and topological errors in the sequence network will often produce inaccurate results. But even if the networks are properly constructed, the engineer often lacks the insight and level of understanding to thoroughly comprehend the behavior of the system.

The novel approach for understanding symmetrical components and synthesizing sequence networks presented in this course enlightens the engineer to the reasons behind the behaviors observed on an unbalanced three-phase power system. It is this enlightenment that allows the engineer to fully understand the behavior of the three-phase system under unbalanced conditions. As an additional benefit to applying these approaches, commonly-made errors in unbalanced system calculations will be significantly decreased if not totally eliminated.

Many power system calculations involve analysis of a balanced three-phase system. When this is the case, only one phase needs to be analyzed. The symmetry of the problem allows the behavior of the other two phases to be determined based on the calculated behavior of the first phase. This single-phase equivalent approach is taken to simplify the calculation process.

But when the conditions to be analyzed result in an unbalanced system of voltage and current phasors, the single-phase equivalent approach cannot be directly applied. Such an example is determining the system response to an unbalanced short-circuit fault, such as a line-to-ground fault. The option of analyzing the unbalanced system as a three-phase problem is not an appealing one, since the resulting mathematics would be cumbersome and very difficult to solve.

Using a single-phase approach would be possible if the unbalanced phasors could be resolved into balanced components. Charles Fortescue's theory of symmetrical components shows us that resolving an unbalanced set of voltage or current phasors into a set of balanced components is always possible. Before developing the symmetrical components of an unbalanced set of three-phase phasors, let us look at a more straightforward example of resolving a vector into components.

Physical Example of Vector Components

A basic problem in statics involves calculating the reaction at the attachment point of a cantilevered beam when subjected to a vertical loading, as shown in Fig. 1.
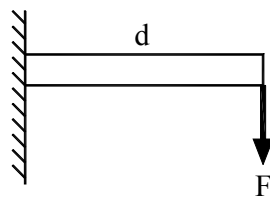
FIGURE 1
Vertical Loading on Cantilevered Beam

The downward force F tries to produce a clockwise rotation of the beam about the attachment point with the wall. The rotational force is called a *moment*. The wall will, hopefully, produce a moment equal in magnitude and opposite in direction to the moment produced by force F. The net moment will then be zero, and the beam will remain stationary.
The moment produced by force F is the product of the force magnitude and the perpendicular distance from the force to the attachment point, or $M = F \times d$.

A slight modification to this problem leads to a complication. Fig. 2 shows another force F applied to the same beam, but this time, F is not directed downward, but at an angle θ with respect to the vertical.
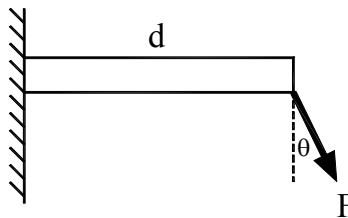
FIGURE 2
Loading Applied to Cantilevered Beam at Angle θ

Now, the resulting moment at the attachment point is not $F \times d$, since the vector from the attachment point to the point of application of the force, which represents d, and the force F are not orthogonal, or perpendicular. In order to calculate the moment, we can resolve the force F into two components: one perpendicular to the d vector (the component we need to calculate the moment), and one parallel to the d vector (which produces no moment). The resolution of F into these components is shown in Fig. 3.
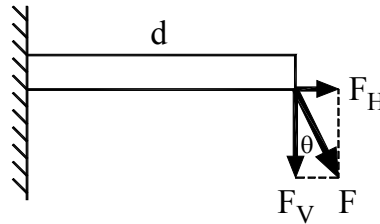
FIGURE 3
Force F Resolved into Appropriate Components

Now, the moment can be calculated as $F_V \times d$. Note that F could have been resolved into components other than F_V and F_H, but F_V and F_H met the criteria required by the moment equation (F_V is perpendicular to d, thus determining the moment, and F_H is parallel to d, which results in no moment).

Trigonometry can be used to express F_V and F_H in terms of F and θ:

$$F_V = F \cos \theta \qquad\qquad \text{Eq. 1}$$

$$F_H = F \sin \theta \qquad\qquad \text{Eq. 2}$$

Equations 1 and 2 allow transformation from the original parameters F and θ into a component environment to facilitate calculation of the moment. A constraint which must be enforced is that the vector sum of F_V and F_H equals the original force F.

$$F = F_V + F_H \qquad\qquad \text{Eq. 3}$$

The angle θ can also be determined if the components F_V and F_H are known.

$$\theta = \tan^{-1}\left(\frac{F_H}{F_V}\right) \qquad\qquad \text{Eq. 4}$$

Equations 3 and 4 allow transformation from the component environment back to the original parameters of F and θ.

So, beginning in the F and θ environment, we can convert to the component environment of F_V and F_H to facilitate calculation of the moment. This same concept can be applied to developing a system of components to facilitate the calculation of unbalanced voltages and currents on a normally-balanced three-phase system. Not all three-phase systems are balanced by design. If the location of the derived neutral point does not coincide with the balanced neutral point, the system is inherently unbalanced. Note that if the three-phase system is unbalanced by design, such as a four-wire delta system, this method of applying symmetrical components cannot be used.
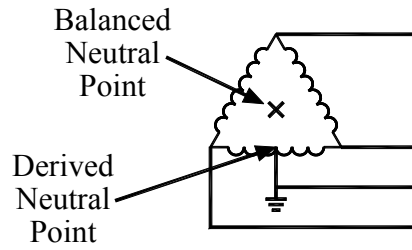
FIGURE 4
Inherently Unbalanced System

Application of Symmetrical Components to a Three-Phase Power System

Now we will apply the same methodology as we did to resolve force F on the beam into suitable components to a normally-balanced three-phase system which is operating in an unbalanced mode. We can consider either voltage or current phasors, since the same methodology applies to both. Since current (the flow of charges) is more easily envisioned than voltage (the difference of two electric potentials), we will use current for this example. Consider the unbalanced set of current phasors shown in Fig. 5.
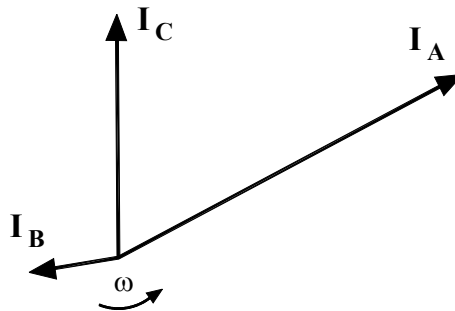

FIGURE 5
Unbalanced Current Phasors

The phasors are rotating *counterclockwise* at the radian fundamental frequency of the system. For a 60 hertz system, this would be $\omega = 2\,\pi\,(60) \approx 377$ radians per second. All phasors rotate counterclockwise – *always*. This is a fact that must be remembered at all times to fully comprehend the concept of symmetrical components.

Since the current phasors are *unbalanced*, meaning that each may have a different magnitude and different angular separation from the other two phasors, we cannot analyze the system taking a single-phase equivalent approach. But by resolving the unbalanced phasors into a suitable set of

components, we could then perform a single-phase equivalent analysis of the system, greatly simplifying the analysis process.

Recalling the previous beam example, the necessary requirements for the components devised to represent the force F were 1) one component must be perpendicular to the distance vector d, and 2) the other component must be parallel to the distance vector d. The constraint was that the vector sum of the two components equals the original force F. Since the three-phase system of current phasors has more degrees of freedom than the beam example, it turns out that we need not two but three sets of components to represent the unbalanced phasors. The necessary requirements for the components of the unbalanced currents are 1) the magnitudes of each of the phasors of a given set of components are equal, and 2) the angular separation between any two phasors in a given set of components is equal. These requirements placed on the components give us the name *symmetrical components*.

Before determining the symmetrical components of the unbalanced phasors, we need to understand the concept of *phase sequencing*. Quite often, phase sequencing is referred to as *phase rotation*, but this terminology is very misleading and is technically incorrect. In fact, *all* phasors rotate counterclockwise – *always*. While the direction of *rotation* never changes, the *sequencing* of the phasors may change.

Referring to Fig. 6, the observer is looking directly at the phase A phasor. The next phasor to come around and point at the observer is the one from phase B. Finally, the phase C phasor will rotate by the observer. This defines an A-B-C, or *positive sequence*, set of components.
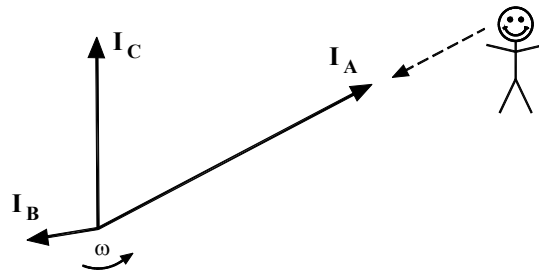

FIGURE 6
Phase Sequencing – A-B-C, or Positive Sequence

In contrast, Fig. 7 shows an example of an A-C-B, or *negative sequence*, set of components.
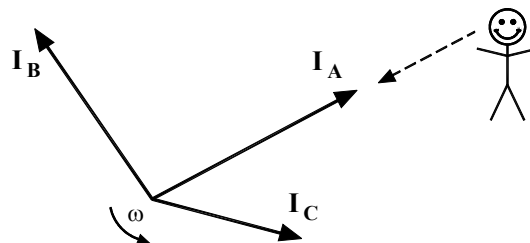

FIGURE 7

Phase Sequencing – A-C-B, or Negative Sequence

Notice that in both Fig. 6 and 7, the phasors are *rotating* in the same (counterclockwise) direction. It is the rotational order, or *sequencing*, of the phasors that differs.

Now we can define the three sets of balanced components to represent the unbalanced set of current phasors shown in Fig. 5. The first set will be a balanced set of phasors having the same phase sequencing as the unbalanced currents. We will call this set the *positive sequence components*, and denote the positive sequence values with the subscript 1.
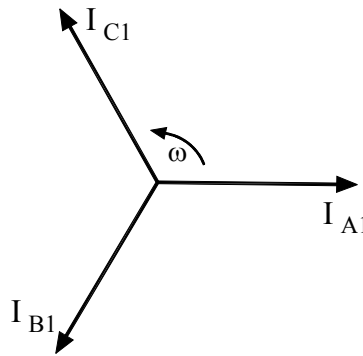
FIGURE 8
Positive Sequence Components

The second set of components will be a balanced set of phasors having A-C-B phase sequencing. We will call this set the *negative sequence components*, and will denote these phasors with the subscript 2.
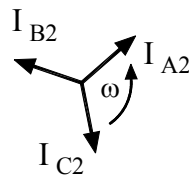
FIGURE 9
Negative Sequence Components

Although the magnitudes of the positive sequence and negative sequence phasors differ, each set contains three phasors of equal magnitude. Also each phasor in both the positive sequence and negative sequence set of components is separated from the other two phasors by equal angular displacements (120°). This fulfills the requirement that each set of components be balanced, or symmetric.

The third set of components must also be balanced. We will choose a set of components with equal magnitudes, and *no* angular displacement between the phases. Note that an angular separation of zero also fulfills the definition of *balanced*, since the same angular displacement exists between any two of the three phasors. This set of components is the *zero sequence*, and we use the subscript 0 to denote them.
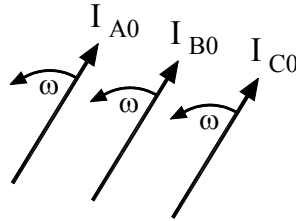

FIGURE 10
Zero Sequence Components

Studying the sequence components shown in Figs. 8 through 10, it is apparent that we can exploit the symmetry of the systems to simplify the nomenclature. We can define an operator **a** such that multiplying any phasor by **a** simply rotates the original phasor by 120°. Thinking in terms of polar coordinates, it becomes obvious that the **a** operator must have a magnitude of 1, or multiplying a phasor by **a** would rescale the phasor. To achieve the 120° rotation, the angle of the **a** operator must be 120°, since angles are additive when multiplying numbers in polar form. Therefore,

$$\mathbf{a} \equiv 1 \; \underline{/120°} \qquad\qquad \text{Eq. 5}$$

The **a** operator can also be expressed in rectangular form as

$$\mathbf{a} = \frac{-1}{2} + \mathbf{j}\,\frac{\sqrt{3}}{2} \qquad\qquad \text{Eq. 6}$$

where $\mathbf{j} \equiv \sqrt{-1}$. Just as **a** can be thought of as a 120° rotator, **j** can be viewed as a 90° rotator. This is an important insight, since it makes the problem less algebraic and more visual and intuitive.

The following powers of the **j** and **a** operators are helpful in visualizing how multiplying a phasor by a power of one of these operators manipulates the original phasor. It should be remembered that squaring an operator repeats its rotation twice, and cubing an operator repeats its rotation three times.

$$\mathbf{j}^2 = 1 \; \underline{/180°} = -1 \qquad\qquad \text{Eq. 7}$$

$$\mathbf{j}^3 = 1 \; \underline{/270°} = -\mathbf{j} \qquad\qquad \text{Eq. 8}$$

$$\mathbf{j}^4 = 1 \; \underline{/360°} = 1 \qquad\qquad \text{Eq. 9}$$

$$\mathbf{a}^2 = 1 \underline{/240°} \qquad \text{Eq. 10}$$

$$\mathbf{a}^3 = 1 \underline{/360°} = 1 \qquad \text{Eq. 11}$$

Using the **a** operator, we can eliminate the double subscript notation used in Figs. 8 through 10 by expressing each phasor in terms of the phase A phasor. This process brings us to a single-phase equivalent of the original system – the goal we were attempting to attain.

Relabeling Figs. 8 through 10 in terms of the phase A quantities by dropping the A subscript and also incorporating the **a** operator has been done in Figs. 11 through 13.
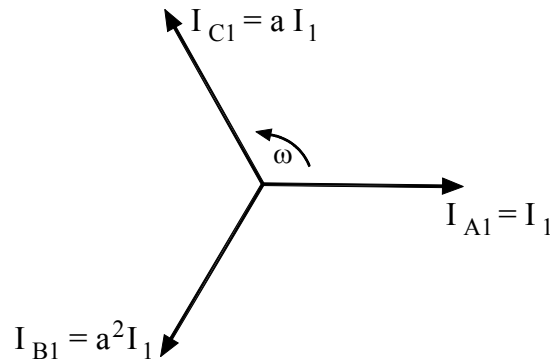
FIGURE 11
Positive Sequence Components
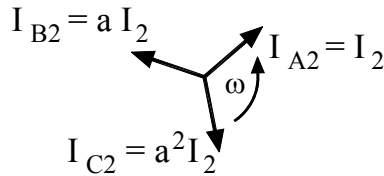Expressed in Terms of Phase A Quantities

FIGURE 12
Negative Sequence Components
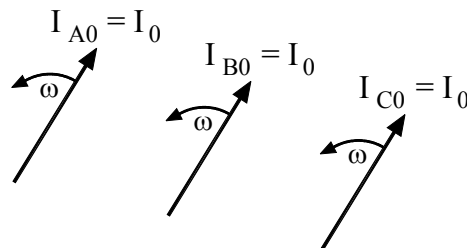Expressed in Terms of Phase A Quantities

FIGURE 13

Zero Sequence Components
Expressed in Terms of Phase A Quantities

Of course, the symmetrical components must satisfy the constraint that their vector sum equals the original set of unbalanced phasors. This is shown graphically in Fig. 14.
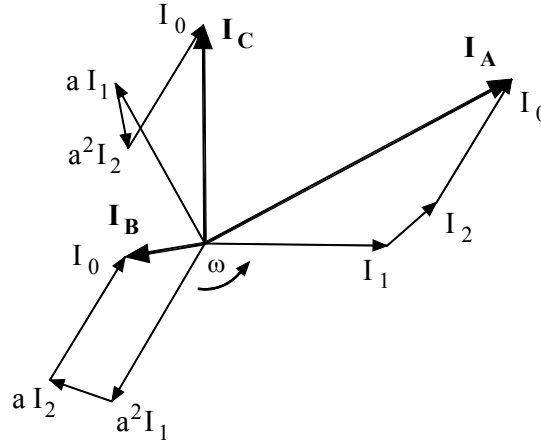

FIGURE 14
Sum of Sequence Components

Now that a suitable set of components to represent the unbalanced currents has been developed, a set of transformation equations similar to Eqs. 1 through 4 must be established.

Expressing the unbalanced currents as the sum of their components provides three of the necessary six transformation equations.

$$I_A = I_0 + I_1 + I_2 \qquad\qquad \text{Eq. 12}$$

$$I_B = I_0 + \mathbf{a}^2\, I_1 + \mathbf{a}\, I_2 \qquad\qquad \text{Eq. 13}$$

$$I_C = I_0 + \mathbf{a}\, I_1 + \mathbf{a}^2\, I_2 \qquad\qquad \text{Eq. 14}$$

The remaining three equations are obtained by solving Eqs. 12 through 14 for I_0, I_1, and I_2. This is cumbersome using algebra. The matrix methods offered by linear algebra greatly simply the process of obtaining solutions for I_0, I_1, and I_2.

The first step is to rewrite Eqs. 12 through 14 as a single matrix equation.

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \qquad\qquad \text{Eq. 15}$$

Multiplying both sides of Eq. 15 by the inverse of the square coefficient matrix gives us the solution for I_0, I_1, and I_2.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} =$$

Eq. 16

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Simplifying and bringing the I_0, I_1, and I_2 vector to the left side of the equal sign,

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

Eq. 17

Inverting the square coefficient matrix gives

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

Eq. 18

Writing 18 as three algebraic equations gives us the last three necessary transformation equations.

$$I_0 = \frac{1}{3}(I_A + I_B + I_C)$$

Eq. 19

$$I_1 = \frac{1}{3}(I_A + \mathbf{a}\, I_B + \mathbf{a}^2\, I_C)$$

Eq. 20

$$I_2 = \frac{1}{3}(I_A + \mathbf{a}^2\, I_B + \mathbf{a}\, I_C)$$

Eq. 21

Using Eqs. 12 through 14 and Eqs. 19 through 21 as transformation equations, we can apply the theory of symmetrical components to unbalanced conditions on otherwise balanced three-phase power systems. But before we do, we must understand the electrical characteristics of the sequence currents so we can understand how they behave in the three-phase system. This knowledge is essential for the proper synthesis of the sequence networks.

Electrical Characteristics of the Sequence Currents

Fig. 15 shows a wye-connected source supplying an unspecified load. The load is required so a closed path exists for current to flow.
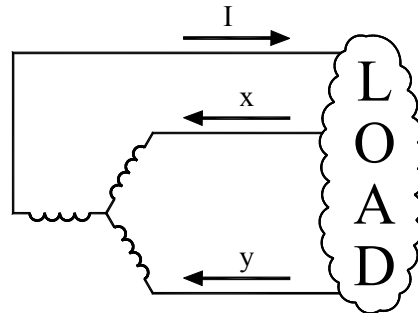

FIGURE 15
Wye-Connected Source Serving Unspecified Load

If the top wire is carrying a current I from the source to the load, we know the only way that current can flow is if there is a return path back to the source. When there is an angular displacement between the line currents, the middle and bottom wires serve as the return path for the source current flowing in the top wire. Writing a node equation using the cloud representing the load as the node, we see that I = x + y. This relationship can be verified graphically by drawing the three line currents in the time domain. At a specific time t = T, the instantaneous values of the currents in the three wires sum to zero, thus honoring Kirchhoff's Current Law.
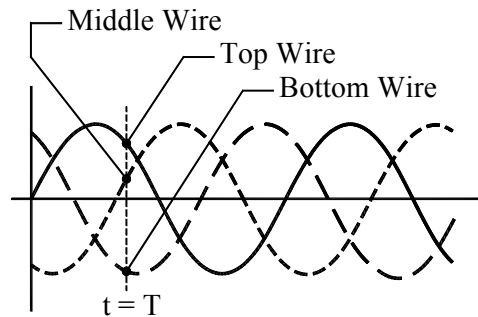

FIGURE 16
Line Currents in Time Domain

But the ability to return the current from the source to the load in the top wire (I) on the bottom two wires requires a phase difference, in this case 120°, between the line currents. In the case of the positive and negative sequence currents, there is a 120° angular displacement between the top, middle, and bottom wire currents. So for the positive and negative sequence, *a current*

supplied by one phase conductor is returned to the source by the other two. This relation is always true in three-phase circuits.

The zero sequence current, however, behaves differently. There is no angular displacement between the phases, so whatever instantaneous current flows on the top wire also must flow on the middle and bottom wires.

Fig. 17 shows a total of 3 I_0 delivered from the source to the load. The only way this current can flow is if it can return to the source. *The zero sequence current is supplied to the load on the phase conductors, but it cannot return to the source on the phase conductors.*
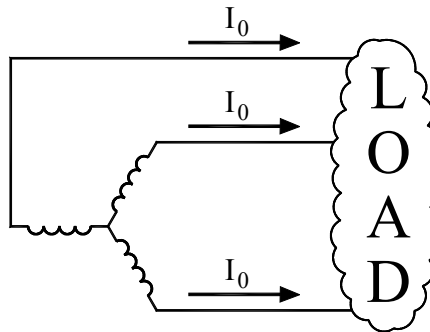

FIGURE 17
Zero-Sequence Current

A fourth conductor must be present to serve as the return path. This fourth conductor is the neutral. The neutral returns the zero-sequence current supplied by each phase conductor, or 3 I_0. *If a fourth conductor (return path) does not exist, zero- sequence current will not flow.* This is always the case with three-phase circuits.
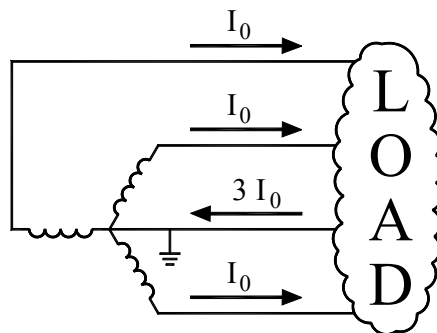

FIGURE 18
Zero-Sequence Current With Return Path

Fig. 18 shows the complete circuit path of the zero-sequence current, being supplied on the phase conductors and being returned on the neutral. Many engineers say that a "ground connection" is required for zero-sequence current to flow. This is really not the case, because a connection to earth is not a requirement for zero-sequence current to flow. It would flow just fine in an

ungrounded (isolated) neutral, but it is very unusual to not ground the neutral for safety reasons. In this situation, the terms *neutral* and *ground* are often [incorrectly] used interchangeably. This paper will make the assumption that the neutral conductor is always grounded, but will refer to the return conductor for zero-sequence current as the *neutral* conductor. Keep in mind that with a multiple-point grounded neutral, as is common on utility systems, the neutral conductor is electrically in parallel with the earth, so the zero-sequence current is returned by both the neutral conductor and the earth.

The circuit depicted in Fig. 18 becomes interesting when trying to apply a single-phase equivalent approach. The "single-phase" circuit is highlighted in Fig. 19.
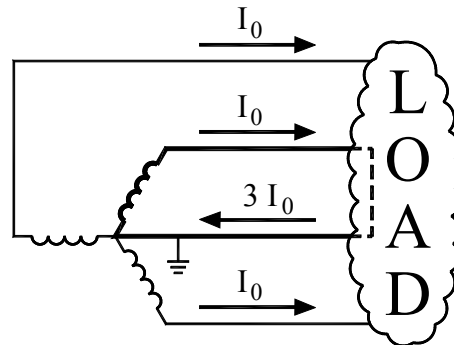

FIGURE 19
Single-Phase Equivalent for Zero-Sequence Current

Note that since the neutral conductor returns not only the zero-sequence current from the phase we are considering as our single-phase equivalent, but also the zero-sequence current from the other two phases. This causes a problem when trying to analyze the single-phase equivalent, since the current supplied (I_0) and the current returned ($3 I_0$) are different. This problem must be fixed, and can be quite simply with a little algebra. For the single-phase equivalent to be valid, the correct voltage drop must be calculated for the neutral return path. If the series reactance in the return path is X_N, the voltage drop for the neutral return path is found using Ohm's Law.

$$V = (3 I_0) \times X_N \qquad\qquad \text{Eq. 22}$$

Forcing the current in the neutral return path of the single-phase equivalent circuit to equal the current supplied by the single phase (I_0), the coefficient 3 must be removed from the current I_0. Simply discarding this coefficient would change the calculated voltage drop for the neutral return path, thus invalidating the single-phase equivalent circuit. But the calculated voltage drop remains correct if the coefficient is simply grouped with the other term (X_N). This is shown in Eq. 23.

$$V = I_0 \times (3 X_N) \qquad\qquad \text{Eq. 23}$$

This subtle algebraic change has a significant physical interpretation. Any impedance in the neutral return path is subjected to three times the zero-sequence current as is flowing in each of the phase conductors; therefore, to provide the proper voltage drop, any impedance in the neutral portion of the circuit must be *tripled* when modeling the circuit as sequence networks. And since

zero-sequence current is the only current component that can flow in the neutral, this condition applies only to the zero-sequence network.

Sequence Networks

Now that the need for modeling unbalanced currents as symmetrical components is understood, the concept of *sequence networks* must be introduced. When a current I flows through an impedance Z, the current should be interpreted as the sum of three balanced components. For phase A,

$$I_A = I_0 + I_1 + I_2 \qquad\qquad \text{Eq. 24}$$

Each component of current can experience a different effective value of impedance. This rather abstract concept must be accepted, although the underlying reasons are not easily understood. Although far from a perfect analogy, one might consider a current containing several harmonic components. Each harmonic component experiences a different resistance value when flowing through a wire. This is due to the fact that AC resistance is a function of frequency. The sequence currents I_0, I_1, and I_2 are all at the system fundamental frequency, so the analogy is not perfect, but like the harmonic currents, the symmetrical components can each experience a different impedance value in a given portion of a system. Ohm's Law can therefore be stated for each sequence component:

$$V_0 = I_0 \times Z_0 \qquad\qquad \text{Eq. 25}$$

$$V_1 = I_1 \times Z_1 \qquad\qquad \text{Eq. 26}$$

$$V_2 = I_2 \times Z_2 \qquad\qquad \text{Eq. 27}$$

Since each component of current experiences a potentially different impedance, three different impedance networks must be developed for any system to be analyzed. Since most studies of unbalanced systems involve short-circuit fault calculations, it is common to neglect the resistive portion of the impedance, since its effect on the short circuit current magnitude is very small. For that reason, we will proceed to develop a positive-, a negative-, and a zero-sequence reactance network. Consider the one-line diagram shown in Fig. 20.
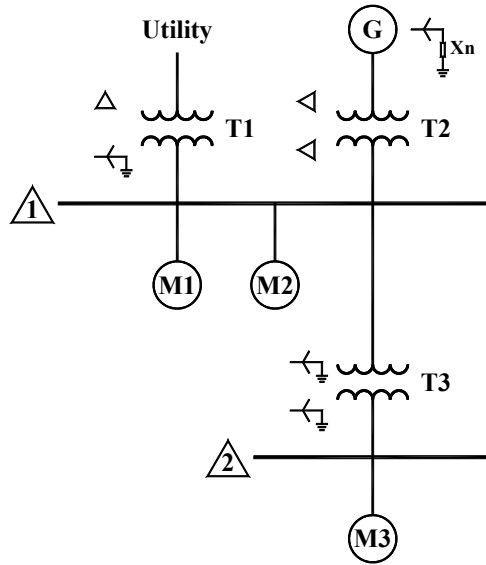
FIGURE 20
One-Line Diagram

The positive sequence reactance network is developed directly from the one-line diagram of the system. First, a Positive-Sequence Reference Bus is drawn. By convention, this bus is drawn at the top of the diagram. Although merely a convention, being consistent with this practice will facilitate both the proper network topology and the correct interconnection of the networks when the fault calculation is done. After the Positive-Sequence Reference Bus is drawn, all sources and loads capable of storing energy (fault current contributors) on the one-line diagram are connected to it. This typically means utility system interconnections, generators, and motors.

The source impedances are modeled in series with an EMF source representing the pre-fault voltage at that point in the system. Since this voltage is generally not known unless a powerflow calculation is performed, it is often assumed as 1.0 per unit, and is assigned the reference angle of zero degrees. The first stage of the positive-sequence network construction is shown in Fig. 21.
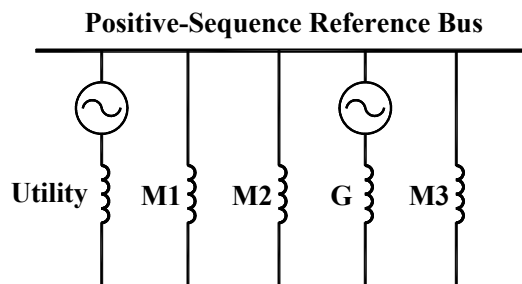

FIGURE 21
First Stage of Positive-Sequence Network Development

Next, the other components from the one-line diagram are modeled as reactances. Transformers T1 and T2 are drawn, and the location of Bus 1 is established. This can be seen in Fig. 22.
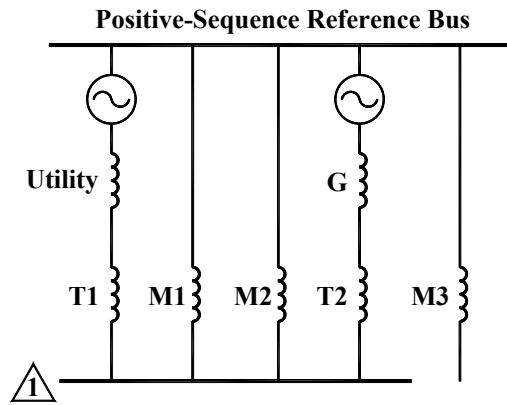

FIGURE 22
Next Stage of Positive-Sequence Network Development

Finally, Transformer T3 and the location of Bus 2 are established, completing the positive-sequence network as shown in Fig. 23.
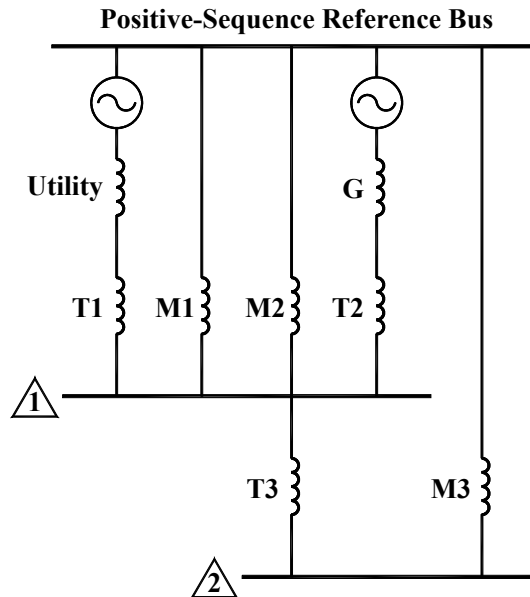

FIGURE 23
Positive-Sequence Network

Note that each numeric reactance value is the positive-sequence reactance for that component.

The negative-sequence network can be developed directly from the positive-sequence network. The following three steps, when applied to the positive-sequence network, will yield the negative-sequence network:

1. Remove (short-circuit) the EMF sources

2. Relabel the reference bus as the "Negative-Sequence Reference Bus"

3. Change the numeric values of the reactances from the positive-sequence values to the negative-sequence values

In the actual system, only positive-sequence voltages are generated. Therefore, all voltage sources will appear in the positive-sequence network only. Note that the reactances behind the generated voltages still appear in the negative-sequence network as negative-sequence reactances.

Most power system components are *bilateral*, meaning that their behavior when current flows through them in one direction is the same as when the direction of current flow reverses. Rotating machines are an exception. Due to the direction of the rotor rotation, the behavior across the air gap changes with the direction of the current flow. This means that, in general, rotating machines will have different negative-sequence reactance values than positive-sequence values. The numeric difference increases as the machine speed decreases and as the pole faces become more salient.

The negative sequence network for the one-line diagram shown in Fig. 20 is shown in Fig. 24.
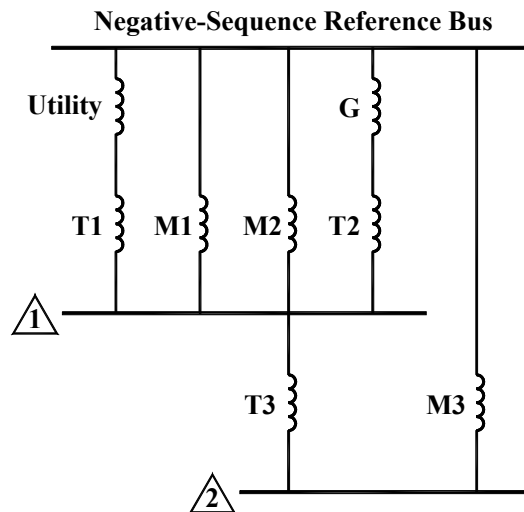

FIGURE 24
Negative-Sequence Network

The zero-sequence network can be developed from the negative-sequence network, but the modification steps are a bit different:
1. Relabel the reference bus as the "Zero-Sequence Reference Bus"

2. Change the numeric values of the reactances from the negative-sequence values to the zero-sequence values

3. Add *three times* the grounding impedance to the numeric reactance value of any machine that is grounded through an impedance

4. Adjust the topology of the network to force proper zero-sequence current behavior

The fourth step is a very important one, and will be addressed in detail. But first, a few comments about steps 2 and 3. The negative- and zero-sequence reactance values are substantially different for most components. A notable exception is the two-winding power transformer, where all three sequence reactances equal the leakage reactance of the transformer. Transmission and distribution lines have zero-sequence reactances that are higher than their positive- and negative-sequence reactances. This is due to the impedance of the earth current return path, and is explained by Carson's Equations. Rotating machines, on the other hand, have zero-sequence reactances that are much lower than their positive- and negative-sequence reactances, due to the large magnitude of the zero-sequence flux across the air gap.

Step three states the need to increase the numeric reactance value of any machine that is grounded through an impedance by three times the value of the grounding impedance. The grounding impedance must be included in the zero-sequence network, and is not included in the other two sequence networks, because only zero-sequence current can flow to ground. The other two sequence currents can flow only in the phase conductors. The reason for tripling the grounding impedance is shown by Eq. 23.

The fourth step is to adjust the topology of the zero-sequence network to force the current flowing in that network to behave like zero-sequence current. The Electrical Characteristics of the Sequence Currents section of this chapter explains that zero-sequence can only flow in the parts of a circuit that have a fourth conductor to serve as a return path. This means that delta and ungrounded wye portions of the system will not allow zero-sequence current to flow. The network topology must be altered to reflect this fact.

Connection	Alteration
Grounded Wye	None
Wye	Open Circuit
Delta	Open Circuit AND Short Circuit to Reference Bus

TABLE 1
Zero-Sequence Network Alteration Rules

These alterations are best understood by example. Before attempting an example, it will be helpful to analyze the zero-sequence current behavior of the delta-wye transformer.

Begin analyzing the delta-wye transformer shown in Fig. 25 with the wye circuit. In order for zero-sequence current to flow to the load on the phase conductors, the total zero-sequence current furnished ($3 I_0$) must return on the neutral.
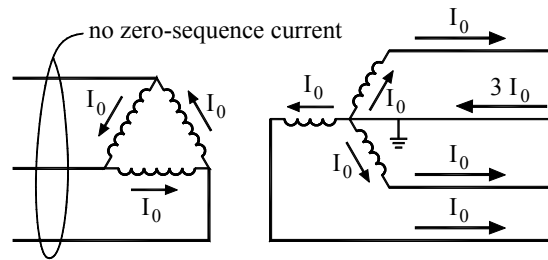

FIGURE 25
Delta-Wye Transformer Zero-Sequence Current Behavior

The per-unit zero-sequence current flowing in each of the wye-connected transformer windings must also flow in the corresponding delta-connected windings. Writing the node equation at each corner of the delta makes it apparent that no zero sequence current can flow out of the delta onto the lines. The zero-sequence current flows in the wye-connected windings, and circulates in the delta-connected windings.

Note that if the circulating current in the delta is in the form of a third harmonic current, the resulting temperature rise due to the higher frequency (and skin effect) may be problematic. Third harmonic currents behave like zero-sequence currents.

In the zero-sequence reactance diagram, the zero-sequence current must be blocked from exiting the delta onto the lines of the three-wire circuit. This is accomplished by introducing an open circuit on the delta side of the transformer. But that open circuit would also prevent zero-sequence current from flowing through the transformer impedance (windings), and we can see in Fig. 25 that this is incorrect. So a short circuit back to the reference bus allows the zero-sequence current to flow from the wye circuit, through the transformer reactance, and to the reference bus, while the open circuit prevents the zero-sequence current from flowing out of the transformer to the delta circuit. This is what Table 4-1 shows as the required topology alteration for a delta connection (Open Circuit AND Short Circuit to Reference Bus). Fig. 26 shows the zero-sequence circuit model for a delta-grounded wye transformer.

Zero-Sequence
Reference Bus

No I_0 →   No I_0 → I_0

$\triangle$ I_0 $\triangle$
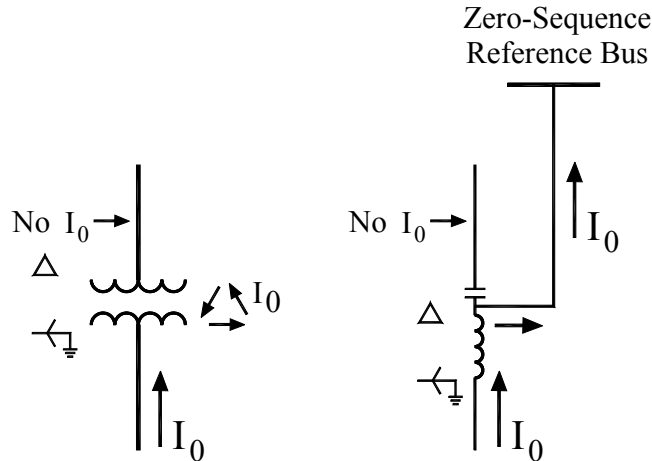
I_0 I_0

I_0 I_0

FIGURE 26
Delta-Wye Transformer Zero-Sequence Circuit Model

Using the alteration rules summarized in Table 1, the topology of each machine (transformer, generator, and motor) reactance can be altered to allow proper zero-sequence behavior. Note that the connection types of the motors shown in Fig. 20 are not specified. This is not a problem, because motors are virtually always wired as a three-wire connection (either delta or wye – not grounded wye). According to Table 1, both the delta and wye connections involve an open circuit, and this open circuit effectively removes the motor from the zero-sequence network.

Begin developing the zero-sequence network from the negative-sequence network by relabeling the reference bus and changing the numeric reactance values from the negative-sequence values to the zero-sequence values. Next, transfer the phasing symbols (deltas, wyes, and grounded wyes) from the one-line diagram to the zero-sequence reactance diagram, as shown in Fig. 27.

Zero-Sequence Reference Bus

Utility   **G**   **Xn**
                     + 3 Xn

$\triangle$
T1   **M1**   **M2**   **T2**

△1

T3   **M3**

△2
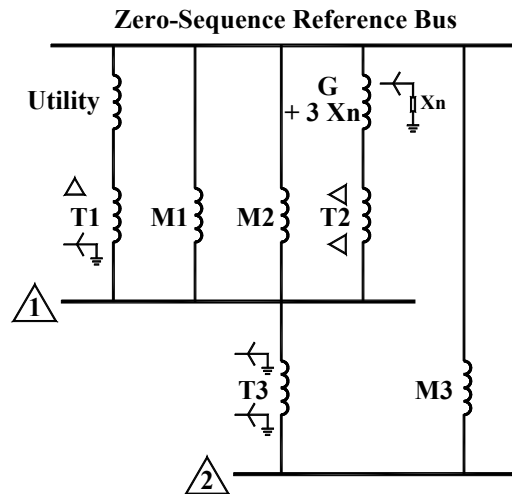
FIGURE 27
First Step of Zero-Sequence Reactance Network Development

At this point, any impedance grounded devices, such as the generator, must have their zero-sequence reactance increased by three times the grounding impedance. Fig. 27 shows the impedance of the generator increased from its original value of G to G + 3 Xn.

Finally, the topology alteration rules of Table 1 can be applied. Any grounded-wye circuit is left unaltered, since zero-sequence current can flow in a four-wire circuit. Ungrounded wye devices are open-circuited, because the lack of return path will prevent zero-sequence current from flowing in a three-wire circuit. And delta-connected components are altered to include both an open circuit (to prevent zero-sequence current from flowing on the three-wire circuit) and a short circuit to the reference bus (to simulate the circulating path provided by the delta-connected windings). The resulting zero-sequence network is shown in Fig. 28.
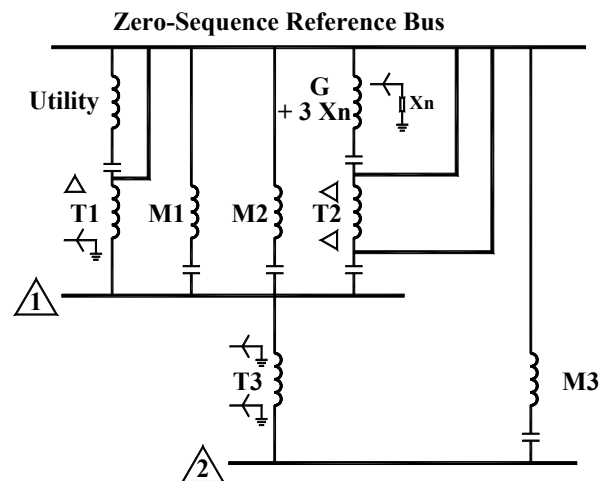
FIGURE 28
Zero-Sequence Network

The method of developing the zero-sequence network by modifying the topology of the negative sequence network, using the rules summarized in Table 1, is novel. Most texts suggest using a *building block approach*, constructing the zero-sequence network from scratch by assembling the zero-sequence impedance models (building blocks) for each system component. The zero-sequence impedance models for many common power system elements are shown in Table 2. While using the information in Table 2 to verify the correctness of a zero-sequence network, **the author strongly discourages use of the *building block approach***, as this method is very prone to errors – even if care is taken while building the network. One subtle misinterpretation of the Table 2 data will result in an incorrect zero-sequence network. On the other hand, a zero-sequence network built by modifying the negative-sequence network can be checked easily for errors as it is being developed, essentially assuring an accurate zero-sequence network when the process is complete.
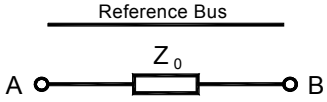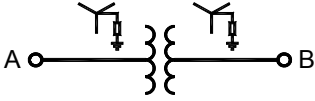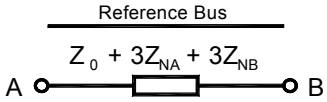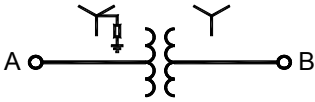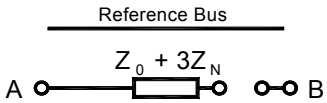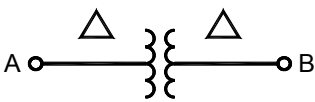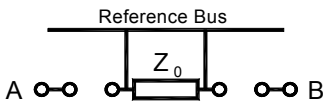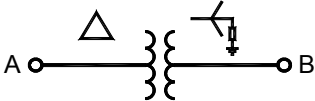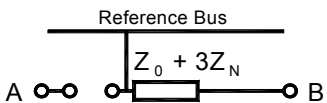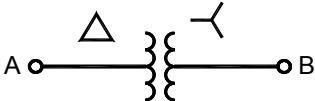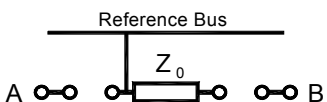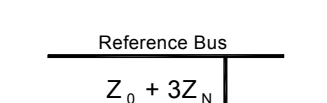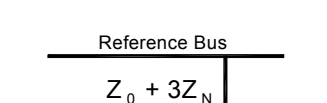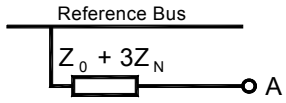
Component	One-Line Symbol	Zero-Sequence Impedance Model

Overhead Line or Cable

Two Winding Transformers

Grounding Transformer

Rotating Machines

TABLE 2
Zero Sequence Impedance Models

Review of Per-Unit System

Due primarily to the abundance of transformers in power systems, many power system problems can be tedious to solve using electrical units. This is because the transformer turns ratio changes the electrical quantities of voltage, current, and impedance differently. If the transformer turns ratio is *n*, as one moves from the high-voltage side of the transformer to the low-voltage side, the voltage changes by $1/n$, the current changes by *n*, and the impedance changes by $1/n^2$. These different factors can make even simple calculations rather complicated.

The complications introduced by the transformer turns ratios can be avoided by applying the *per-unit system*. The per-unit system uses dimensionless quantities instead of electrical units (volts, amps, ohms, watts, etc.). The per-unit system, when properly applied, also changes all transforme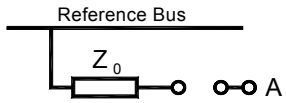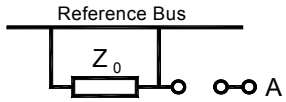r turns ratios to 1. This way, as one moves from the high-voltage side of the transformer to the low-voltage side, the voltage, current, and impedance are all unaffected.

The per-unit system relies on the establishment of four *base quantities*. The required base quantities are: *base power*, *base voltage*, *base current*, and *base impedance*. The base quantities are selected by the person doing the problem. The numeric values of these base quantities are arbitrary. This is because the per-unit system is a mathematical transformation. The problem to be solved is transformed into the per-unit system, solved, and then transformed back to electrical quantities. Since the transformation back to electrical quantities is the inverse of the transformation into the per-unit system, the base quantities which define the transformations can assume any numeric value.

Of the four base quantities, two are mathematically independent. The other two are then defined by the first two. For example, if voltage and current are assumed to be independent, power can be thought of as the product of voltage and current, and impedance as the quotient of voltage and current. The typical way to apply the per-unit system is to arbitrarily assign the power and voltage bases, then using the mathematical relationships between the electrical quantities to determine the current and impedance bases.

Typically, the base power (kVA or MVA base) is selected arbitrarily, often as 10 or 100 MVA. The power base is constant through the entire system. The base voltage (kV base) is frequently assigned as the nominal operating voltage at a given point in the system. At every voltage transformation, the base voltage is adjusted by the transformer turns ratio. Therefore, many different base voltages may exist throughout the system. The proper selection of voltage bases throughout the system effectively makes the transformer turns ratios equal to one, thus removing the complications introduced by the transformer turns ratios.

After the power and voltage bases are chosen, the other two base quantities can be calculated from the established bases by using the formulas

$$\text{Base Current} = \frac{\text{Base kVA}_{3\Phi}}{\sqrt{3} \times \text{Base kV}_{L-L}} \qquad \text{Eq. 28}$$

and

$$\text{Base Impedance} = \frac{\left(\text{Base kV}_{L-L}\right)^2}{\text{Base MVA}_{3\Phi}},\qquad\qquad\text{Eq. 29}$$

where the subscripts 3Φ and L–L respectively denote three-phase power and line-to-line voltage.

Actual electrical quantities are converted to dimensionless per-unit quantities using the formula

$$\text{Per} - \text{Unit Quantity} = \frac{\text{Actual Quantity}}{\text{Base Quantity}}.\qquad\qquad\text{Eq. 30}$$

Often, a per-unit quantity must be converted from a particular base to a new base. Toward that end, we use the relationship

$$\text{Per} - \text{Unit Quantity}_{New} =$$

Eq. 31

$$\text{Per} - \text{Unit Quantity}_{Old} \times \left(\frac{\text{kV Base}_{Old}}{\text{kV Base}_{New}}\right)^2 \times \left(\frac{\text{kVA Base}_{New}}{\text{kVA Base}_{Old}}\right).$$

Examples

1. A generator has an impedance of 2.65 ohms. What is its impedance in per-unit, using bases of 500 MVA and 22 kV?

    Solution:

    Calculate the base impedance using Eq. 29.

    $$\text{Base Impedance} = \frac{\left(\text{Base kV}_{L-L}\right)^2}{\text{Base MVA}_{3\Phi}} = \frac{22^2}{500} = 0.968\,\Omega$$

    Calculate per-unit impedance using Eq. 30.

    $$\text{Per} - \text{Unit Impedance} = \frac{\text{Actual Impedance}}{\text{Base Impedance}} = \frac{2.65\ \Omega}{0.968\ \Omega} = 2.738\ \text{p.u.}$$

2. A transformer has an impedance of 6.25% on 12 MVA and 44 kV bases. What is its percent impedance on 100 MVA and 46 kV bases?
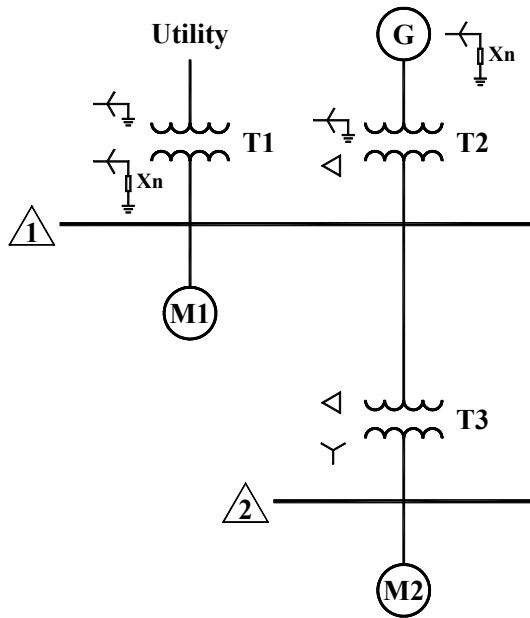
    Solution:

    Convert to new per-unit bases using Eq. 31.

$$\text{Per} - \text{Unit Quantity}_{New} =$$

$$\text{Per} - \text{Unit Quantity}_{Old} \times \left(\frac{\text{kV Base}_{Old}}{\text{kV Base}_{New}}\right)^2 \times \left(\frac{\text{kVA Base}_{New}}{\text{kVA Base}_{Old}}\right) =$$

$$6.25\% \times \left(\frac{44 \text{ kV}}{46 \text{ kV}}\right)^2 \times \left(\frac{100 \text{ MVA}}{12 \text{ MVA}}\right) = 47.65\%$$

3.   Consider the power system shown below:

Rated Voltages:

Utility Source: 12.47 kV

Generator: 460 V

Transformers
T1: 13.2 kV / 4.16 kV
T2: 460 V / 4 kV
T3: 4.16 kV / 480 V

Motors
M1: 4000 V
M2: 460 V

Let the base voltage at Bus 1 be 4.16 kV. Find the base voltage at

a.   the utility connection point
b.   the generator terminals
c.   Bus 2.

Solution:

a.   Starting with the given base voltage at Bus 1 and applying the turns ratio of transformer T1,

$$V_{base\,(Utility)} = 4.16\,\text{kV} \left(\frac{13.2\,\text{kV}}{4.16\,\text{kV}}\right) = 13.2 \text{ kV}$$

b.   Starting with the given base voltage at Bus 1 and applying the turns ratio of transformer T2,

$$V_{base\,(Generator)} = 4.16\,\text{kV}\left(\frac{460\,\text{V}}{4000\,\text{V}}\right) = 478.4\ \text{V}$$

c.   Starting with the given base voltage at Bus 1 and applying the turns ratio of transformer T3,

$$V_{base\,(Bus\,2)} = 4.16\,\text{kV}\left(\frac{480\,\text{V}}{4160\,\text{V}}\right) = 480\ \text{V}$$

Notice that the base voltage is independent of rated voltage, nominal voltage, actual voltage, or any other voltage quantity. It is simply a number to use in a mathematical transformation – nothing more. Since the same numeric value is used when performing the inverse transformation (per-unit back to electrical units), its value is arbitrary. It is essential, however, that after a base voltage is arbitrarily assigned at one bus, that the other voltage bases are determined using transformer turns ratios, as in this example.

Short Circuit Faults

A short circuit fault occurs when one phase is electrically connected, either solidly or through impedance, to another phase or phases and/or ground. The *three-phase fault* is the only symmetrical short circuit fault. The other three types of short circuit faults possible on a three-phase system are asymmetric and must be analyzed as unbalanced faults using the method of symmetrical components.

Short circuit currents are calculated by first determining the Thévenin sequence reactances at the location of the fault, then analyzing the appropriate circuit model of the type of fault to be determined. Recall that the Thévenin sequence reactance is the equivalent reactance from the location of the fault back to the reference bus.

To demonstrate how short circuit currents are calculated, fault current magnitudes will be calculated at Bus 1 of a hypothetical system. To facilitate the fault calculations, the Thévenin reactance of the positive, negative, and zero sequence networks now will be given for Bus 1 of this system. The Thévenin reactance of the positive sequence network at Bus 1 is

$$X_{1\,(Bus\,1)} = j\,0.0140 \text{ per-unit} \qquad\qquad \text{Eq. 32}$$

The Thévenin reactance of the negative sequence network at Bus 1 is

$$X_{2\,(Bus\,1)} = j\,0.0145 \text{ per-unit} \qquad\qquad \text{Eq. 33}$$

The Thévenin reactance of the zero sequence network at Bus 1 is

$$X_{0\,(Bus\,1)} = j\,0.0126 \text{ per-unit} \qquad\qquad \text{Eq. 34}$$

Three-Phase Fault

A three-phase fault is a balanced fault; therefore, the negative- and zero-sequence currents must be zero. The three-phase fault is modeled by shorting the positive-sequence network. Any fault impedance (Z_F) is modeled in series with the positive-sequence network.
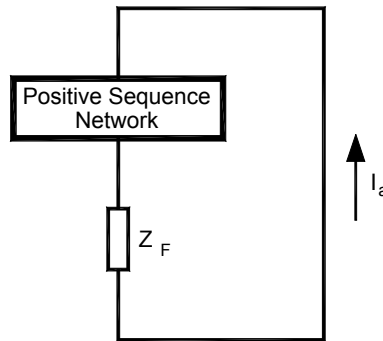

FIGURE 29
Three-Phase Fault Circuit Model

The resulting current that flows is the phase A current due to the short circuit fault. Phase B and c currents can be found by applying principles of symmetry.

$$I_b = I_a \times 1 \underline{/240°} \qquad \qquad \text{Eq. 35}$$

$$I_c = I_a \times 1 \underline{/120°} \qquad \qquad \text{Eq. 36}$$

Using the positive-sequence reactance given in Eq. 32 and assuming a pre-fault voltage of 1.0 $\underline{/0°}$ per unit, the fault current I_a is

$$I_a = \frac{1 \underline{/0°}}{0.0140 \underline{/90°}} = 71.43 \underline{/-90°} \text{ p.u.} \qquad \qquad \text{Eq. 37}$$

Applying Eq. 35 and 36, the phase B and C currents can be found.

$$I_b = 71.43 \underline{/150°} \qquad \qquad \text{Eq. 38}$$

$$I_c = 71.43 \underline{/30°} \qquad \qquad \text{Eq. 39}$$

Of course, these per-unit currents must be multiplied by the base current at the fault location to give a fault current in kA.

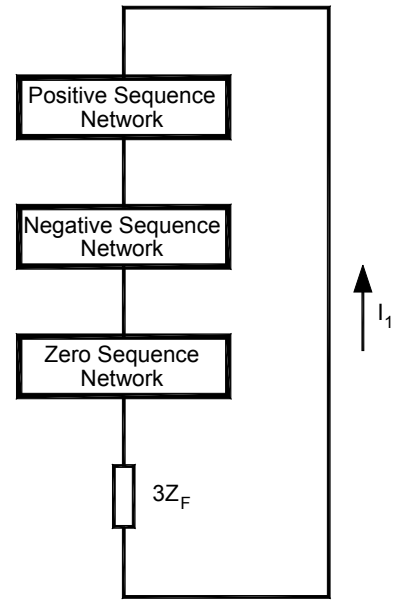## Line-to-Ground Fault

A line-to-ground fault is modeled by connecting the positive, negative, and zero sequence networks in series. Any fault impedance (Z_F) is tripled and put in series. The tripling is necessary because the fault impedance is subjected to the zero-sequence current of all three phases, not just the one phase being analyzed in the single-phase equivalent. Since all sequence

currents are in series, the positive, negative, and zero sequence currents for a line-to-ground fault all must be equal. The current flowing in the series circuit can be designated to represent the phase *a* positive sequence current. The total phase *a* current for a line-to-ground fault equals three times the phase *a* positive sequence current.

$$I_{a\,(3\varnothing\;fault)} = 3\,I_1 \qquad\qquad\qquad \text{Eq. 40}$$

The line-to-ground fault circuit model is shown in Fig. 30.

FIGURE 30
Line-to-Ground Fault Circuit Model

To calculate the fault current magnitude for a line-to-ground fault at Bus 1, the line-to-ground fault circuit model is constructed, as shown in Fig. 31. The current that flows is the sequence current. Since the three sequence networks are in series, the three sequence currents must be equal. This is always the case for a line-to-ground fault. The current in Fig. 31 is simply labeled I_1. Note that the prefault voltage is again assumed to be 1.0 per unit, and that the fault impedance has been neglected to give the worst-case fault current magnitude.
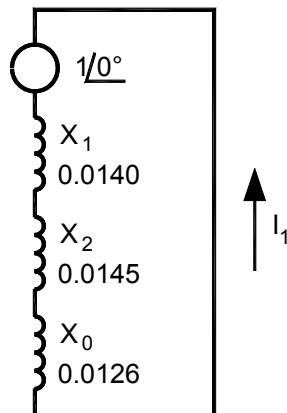

FIGURE 31
Line-to-Ground Fault at Bus 1

Next, the three series reactances are combined to a single reactance of 0.0411 per-unit. The phase *a* positive sequence current is calculated by dividing this reactance into the prefault voltage of 1 $\underline{/\,0^\circ}$ per unit.

$$I_1 = \frac{1\underline{/0^o}}{0.0411\underline{/90^o}} = 24.33\underline{/-90^o} \qquad \text{Eq. 41}$$

The total phase *a* current for a line-to-ground fault is three times the positive sequence current as stated in Eq. 40.

$$I_a = 3\,(24.33) = 72.99 \text{ per-unit} \qquad \text{Eq. 42}$$

Since phases *b* and *c* are unfaulted, no fault current flows in them and ($I_b = I_c = 0$). All fault current flows in phase *a* and returns via ground.

Alternatively, since it is known that for a line-to-ground fault, $I_1 = I_2 = I_0$, Eq. 12 through 14 can be used to find I_a, I_b, and I_c.

Double Line-to-Ground Fault

A double line-to-ground fault is modeled by connecting the positive, negative, and zero sequence networks in parallel. Any fault impedance is tripled and put in series with the zero-sequence network. The current flowing in each parallel branch represents the sequence current of that branch. The double line-to-ground fault circuit model is shown in Fig. 32.
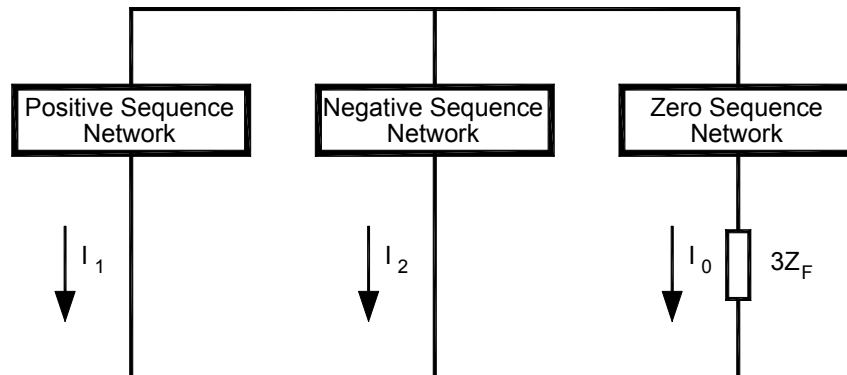

FIGURE 32
Double Line-to-Ground Fault Circuit Model

To calculate the fault current magnitude for a double line-to-ground fault at Bus 1, the double line-to-ground fault circuit model is constructed, as shown in Fig. 33.
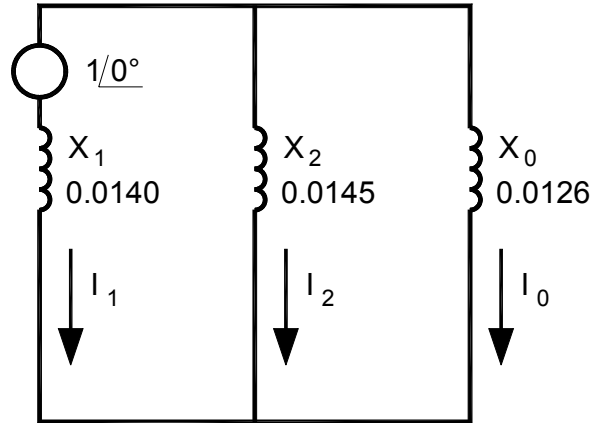
FIGURE 33
Double Line-to-Ground Fault at Bus 1

Basic circuit analysis techniques are used to determine the sequence currents. The X_0 and X_2 reactances can be combined in parallel, then added in series with X_1 to determine current I_1.

$$I_1 = \frac{E}{\dfrac{X_0\,X_2}{X_0 + X_2} + X_1} = \frac{1\underline{/0^o}}{0.0207\underline{/90^o}} = 48.2121\underline{/-90^o} \qquad \text{Eq. 43}$$

The negative and zero sequence currents then can be determined by applying the current divider equation to I_1, which divides between the negative and zero sequence branches.

$$I_2 = -I_1\left(\frac{X_0}{X_0 + X_2}\right) = 48.2121\underline{/90^o}\ (0.4649)$$

Eq. 44

$$= 22.4160\underline{/90^o}$$

$$I_0 = -I_1\left(\frac{X_2}{X_0 + X_2}\right) = 48.2121\underline{/90^o}\ (0.5351)$$

Eq. 45

$$= 25.7961\underline{/90^o}$$

Once the sequence currents have been determined, the phase currents can be calculated by applying Eq. 15.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} 25.7961\underline{/90^o} \\ 48.2121\underline{/-90^o} \\ 22.4160\underline{/90^o} \end{bmatrix} \qquad \text{Eq. 46}$$

Rewriting as scalar equations,

$$I_a = I_0 + I_1 + I_2$$

$$= 25.7961 \,\underline{/\,90°} + 48.2121 \,\underline{/\,{-}90°} + 22.4160 \,\underline{/\,90°}$$

$$= 0 \qquad\qquad \text{Eq. 47}$$

$$I_b = I_0 + \boldsymbol{a}^2\, I_1 + \boldsymbol{a}\, I_2$$

$$= 25.7961 \,\underline{/\,90°} + 48.2121 \,\underline{/\,150°} + 22.4160 \,\underline{/\,210°}$$

$$= 72.3774 \,\underline{/\,147.68°} \qquad\qquad \text{Eq. 48}$$

$$I_c = I_0 + \boldsymbol{a}\, I_1 + \boldsymbol{a}^2\, I_2$$

$$= 25.7961 \,\underline{/\,90°} + 48.2121 \,\underline{/\,30°} + 22.4160 \,\underline{/\,330°}$$

$$= 72.3774 \,\underline{/\,32.32°} \qquad\qquad \text{Eq. 49}$$

Note that the magnitudes of I_b and I_c are equal, and that their angles are supplementary (147.68° + 32.32° = 180°). This is always the case with a double line-to-ground fault.

Line-to-Line Fault

A line-to-line fault is modeled by connecting the positive and negative sequence reactances in parallel. Since there is no connection to ground, zero sequence current cannot flow, so the zero sequence network is open circuited and does not appear in the circuit model. Any fault impedance is put between the two networks. Because of the parallel connection of the positive and negative sequence networks, the positive and negative sequence currents are equal but opposite in sign. The line-to-line fault circuit model is shown in Fig. 34.
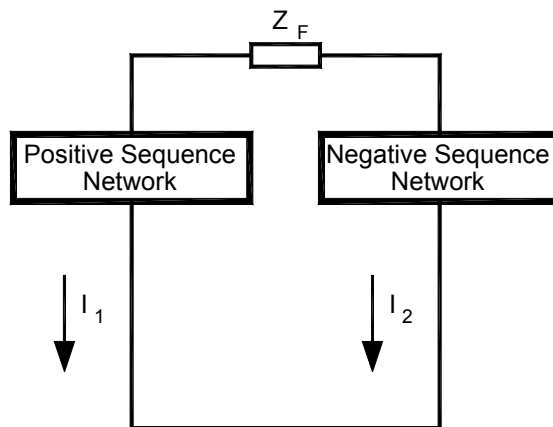

FIGURE 34
Line-to-Line Fault Circuit Model

The fault current magnitude for a line-to-line fault at Bus 1 with no fault impedance can be determined by constructing the circuit model shown in Fig. 35.
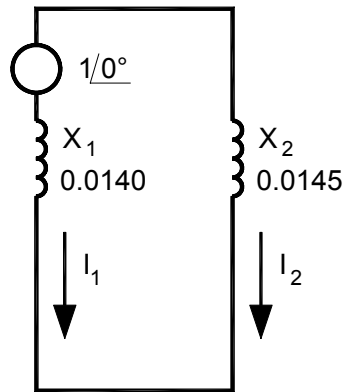
FIGURE 35
Line-to-Line Fault at Bus 1

Adding the two reactances and dividing that sum into the positive sequence voltage E determines the positive sequence current I_1.

$$I_1 = \frac{E}{X_1 + X_2} = \frac{1\underline{/0^o}}{0.0285\underline{/90^o}} = 35.0877\underline{/-90^o} \qquad \text{Eq. 50}$$

The negative sequence current is simply 180° out of phase with I_1.

$$I_2 = -I_1 = 35.0877\underline{/90°} \qquad \text{Eq. 51}$$

The phase currents can be found by using Eq. 12 through 14. Since there is no zero sequence current, $I_0 = 0$.

$$I_a = I_1 + I_2 = 35.0877\underline{/-90°} + 35.0877\underline{/90°} = 0 \qquad \text{Eq. 52}$$

$$I_b = a^2 I_1 + a I_2 = 35.0877\underline{/150°} + 35.0877\underline{/210°} = 60.7737\underline{/180°} \qquad \text{Eq. 53}$$

$$I_c = a I_1 + a^2 I_2 = 35.0877\underline{/30°} + 35.0877\underline{/330°} = 60.7737\underline{/0°} \qquad \text{Eq. 54}$$

These results are also consistent with logic. Since phase *a* is unfaulted, all fault current flows in phases *b* and *c*, and since all the fault current supplied by phase *b* must return on phase *c*, their equal magnitudes and opposite polarities are a necessity.

Summary

Symmetrical components are used to resolve unbalanced electrical systems, typically systems subjected to unbalanced short circuit faults, into balanced systems. The balanced systems are called the *positive*, *negative*, and *zero sequence* networks. The voltage across and the current through the positive sequence network are called the *positive sequence voltage* and *positive sequence current*, respectively. Negative and zero sequence quantities are named similarly.

The positive sequence quantities relate to the generated voltages and currents. The negative sequence quantities relate to the voltages and currents resulting from load imbalance, and the zero sequence quantities show the effects of ground on the system.

Determining the positive sequence network is done directly from the system *impedance* or *one-line diagram*. The negative sequence network has the same topology as the positive sequence network, without the EMF sources. The zero sequence network can only allow current to flow where a ground source is present, so open circuits and short circuits to the reference bus exist in the zero sequence network due to machine connections (delta, wye, or grounded wye).

Because of the widespread use of transformers in power systems, power calculations are more readily done using the *per-unit system*, which transforms electrical quantities to dimensionless quantities. *Base quantities* are assigned for power and voltage, then current and impedance bases are calculated. After the calculation is performed in the per-unit system, an inverse transformation restores the electrical units.

Balanced (three-phase) faults are calculated by analyzing the positive-sequence network. Unbalanced short circuit faults (line-to-ground, double line-to-ground, and line-to-line) are analyzed by appropriately connecting the sequence networks and simplifying the resulting circuit.