PDHonline Course E331 (3 PDH)

Fundamentals of PID Control

Instructor: Anthony K. Ho, PE

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PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

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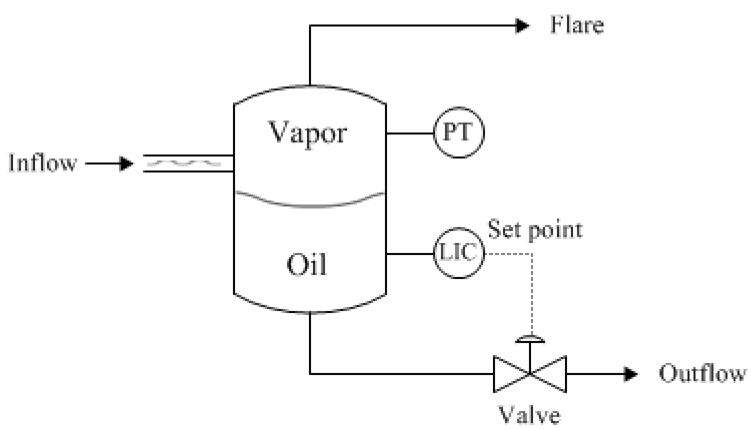
1. History and Background of PID Control

The original technology for industrial proportional, integral, and derivative (PID) control was pneumatic, hydraulic, or mechanical and the controller usually had a simple interface for manual tuning. The first theoretical analysis of a PID controller dates back to 1922 when the Russian American engineer Nicolas Minorsky developed an automatic ship steering system for the U.S. Navy based on observations of the steersmen's use of current error, past error, and rate of change to keep the ship on course. Controllers with electrical systems were developed after World War II.

PID control is used to control and maintain processes. It can be used to control physical variables such as temperature, pressure, flow rate, and tank level. The technique is widely used in today's manufacturing industry to achieve accurate process control under different process conditions. PID is simply an equation that the controller uses to evaluate the controlled variables. A process variable (PV) temperature, for example, is measured, and a feedback signal is sent to the controller. The controller then compares the feedback signal to the set point (SP) and generates an error value. The value is examined with one or more of the three proportional, integral, and derivative methodologies. As a result, the controller issues the necessary commands or alters the control variable (CV) to correct the error (E). These procedures form an iterative process. Below is a common control loop application.



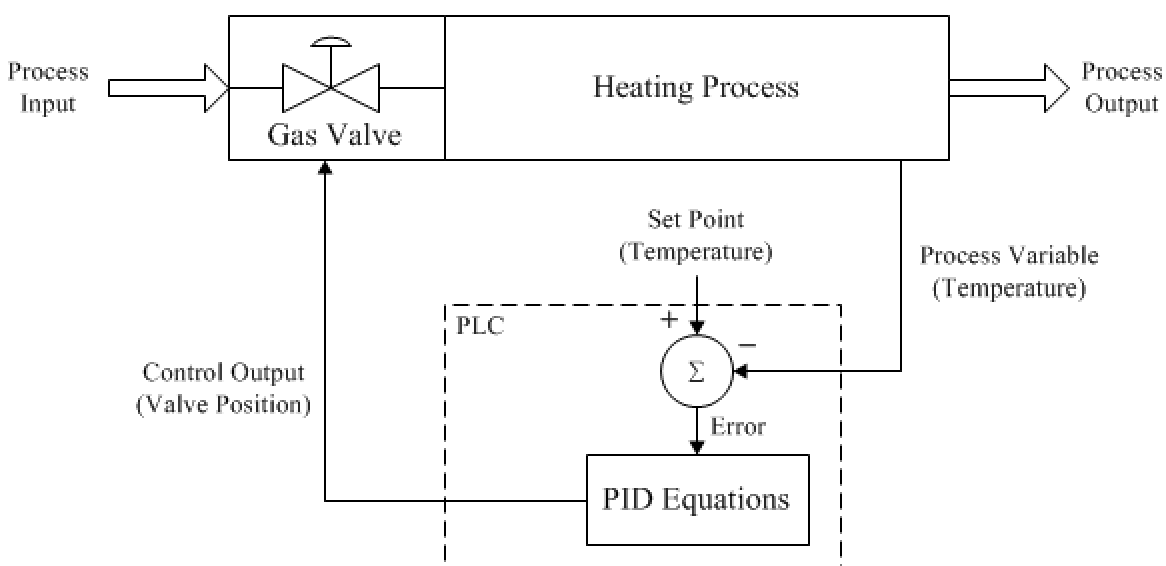
PT	Pressure Transmitter
LIC	Level Indicator Controller

In this example, oil is flowing into the tank in a non-constant rate. The oil level in the tank is the process variable, which is measured, and this information is fed back to the controller. An operator enters a set point for a desired level. The controller compares the current level with the set point and generates a value that is examined with a PID method. The controller then adjusts the valve position to correct the error.

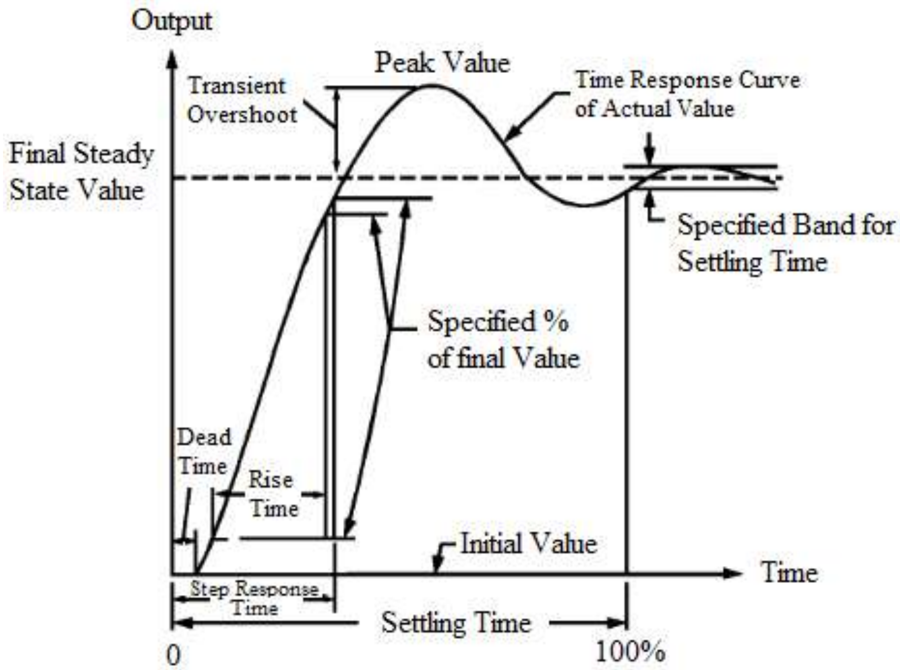
2. Theory of PID Control

PID controllers typically use control loop feedback in industrial and control systems applications. The controller first computes a value of error as the difference between a measured process variable and a preferred set point. It then tries to minimize the error by increasing or decreasing the control inputs or outputs in the process so that the process variable moves closer to the set point. This method is most useful when a mathematical model of the process or control is too complicated or unknown for the system. To increase performance, such as by increasing the system's responsiveness, PID parameters must be adjusted according to the specific application.

The block diagram below shows an example of a heating process that is controlled by a PID controller (programmable logic controller or PLC in this case). The temperature of the furnace is controlled by adjusting the gas valve. The operator sets the desired temperature as the set point. The furnace's temperature is measured, and this information is sent back to the controller. The feedback is compared to the set point, and an error value is calculated. The PID equations then determine the suitable valve position to correct the error. In fact, this is an example of a PID feedback control loop.



The following figure shows a typical step response curve after a controller responds to a set point change. The curve rises from 10% to 90% of final steady state value within a period known as the rise time. The curve rises from 0% to 63.2% of peak value within a period known as the step response time. The rise time is equal to the step response time minus the dead time.



One of the advantages of PID is that for many processes there are straightforward correlations between the process responses and the use and tuning of the three terms (*P*, *I*, and *D*) by the controller. Designing a PID system involves two steps. First, the engineer must choose the structure of the PID controller, for example P only, P and I, or all three terms P, I, and D. Second, to tune the controller, the engineer must choose numerical values for the PID parameters.

These three parameters for the PID algorithm are the proportional, integral, and derivative constants. The proportional constant determines the reaction based on the current error; the integral constant determines the reaction according to the total of recent errors; and the derivative constant determines the reaction using the rate at which the errors have been changing. These three actions are then used to adjust the process through control elements such as the position of a valve. In simple terms, *P* depends on the current error, *I* depends on the sum of past errors, and *D* predicts future errors based on the current rate of change of errors.

2.1. Proportional Control

The proportional element of PID examines the magnitude of the error, and the PID control reacts proportionally. A large error receives a large response. For example, if there is a large temperature error, the fuel valve would be opened a lot, while a small error would receive a small response. In mathematical terms, the proportional term (P_{out}) is expressed as:

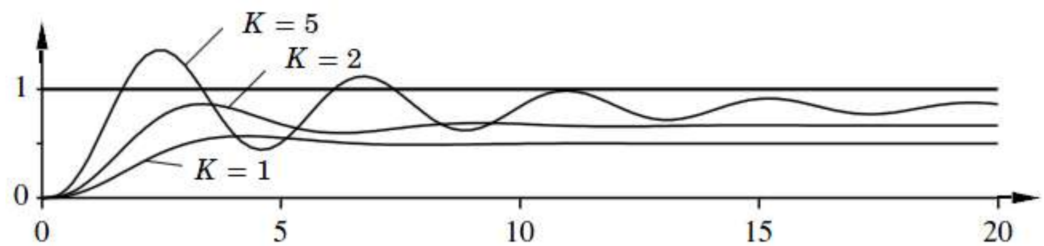
$$P_{out} = K_p\, e$$

where

P_{out}: Proportional portion of controller output
K_p: Proportional gain
e: Error signal, e = set point – process variable

e = SP – PV here represents a reverse acting loop. When e = PV – SP, it refers to a direct-acting loop. In a direct-acting loop, the process variable is greater than the set point; therefore, the appropriate controller action is to increase the output. A typical example of a direct-acting system is one that controls temperature using cooling water. In a reverse-acting loop, the process variable is less than the set point; therefore, the controller's action is to decrease the output. An example would be a system that controls temperature using steam.

The following figure illustrates a proportional control and shows that there is always a steady state error in proportional control. The error will decrease with increasing proportional gain, but the tendency toward oscillation will also increase.



Let's look at a furnace as an example. Assume that between 1500°F and 2000°F, the system response is proportional such that it opens the fuel valve in proportion to the error. When the temperature is below 1500°F, the valve is open 100%. When it is above 2000°F, the valve is closed or 0% open. The proportional band (also known as the reciprocal of gain) in this case is assumed to be 500°F. However, the proportional band is usually expressed in percentage. The percentage is calculated by dividing the proportional band in the engineering unit (°F) by the full controller range and multiplying by 100. The full controller range in this case is 2000°F – 70°F = 1930°F (Assume 70°F is the room temperature.) Therefore, the proportional band in percentage is equal to (500°F / 1930°F) × 100 or 26%. The engineer can adjust the proportional band and make the system more or less responsive to an error. The rule of thumb is that a smaller the proportional band (or large gain) results in a large output or faster response to a given input error, and a larger band (or small gain) results in a less responsive controller. However, one must be aware that an excessively large proportional gain will lead to process instability and oscillation.

One may see that there are issues with proportional control only. One of these issues is that proportional control cannot compensate for very small errors. (These errors are also known as offset.) Another issue is that it cannot adjust its output based on the rate of change in the measured variable. For example, if you are driving on the highway at 55 miles per hour and the car in front of you slows down, you may respond by applying 50% brake pressure. This is a proportional response, because the car in front is slowing down but not coming to a stop. However, if the car slows down even more, you will apply more brake pressure, since the car in front's rate of stopping is greater. In other words, we apply more brake pressure as we see the distance between our cars getting smaller, because we naturally respond to the rate of change of error. However, proportional control systems do not do this; they only respond to the magnitude of the error, not to its rate of change.

2.2. Integral Control

To address the first issue with the proportional control, integral control attempts to correct a small error (offset). The integral examines the error over time and increases the importance of even a small error over time. The integral is equal to error multiplied by the time the error has persisted. A small error at time zero has zero importance. A small error at time 10 has an importance of 10 times error. As such, the integral increases the response of the system to a given error over time until it is corrected. The integral can also be adjusted, and the adjustment is called the reset rate. The reset rate is a time factor. The shorter the reset rate, the quicker the correction of an error. However, too short a reset rate can cause erratic performance. In hardware-based systems, the adjustment can be done by a potentiometer that changes the time constant of an RC circuit. Most of today's applications use a software-based control, such as a PLC module, through which the engineer changes the parameter of the reset rate. The mathematical expression of an integral-only controller (I_{out}) is:

$$I_{out} = \frac{1}{T_i}\int e\,dt = K_i \int e\,dt$$

where

I_{out}: Integral portion of controller output
T_i: Integral time, or reset time
K_i: Integral gain
e: Error signal, e = set point – process variable

2.3. Derivative Control

The derivative part of the control output attempts to look at the rate of change in the error signal. The derivative will cause a greater system response to a rapid rate of change than to a small rate of change. In other words, if a system's error continues to rise, the controller must not be responding with sufficient correction. The derivative senses this rate of change in the error and provides a greater response. The derivative is adjusted as a time factor and therefore is also called rate time. It is essential that too much derivative should not be applied or it can cause overshoot or erratic control. In mathematical terms, the derivative term (D_{out}) is expressed as:

$$D_{out} = T_d \frac{d}{dt} e = K_d \frac{d}{dt} e$$

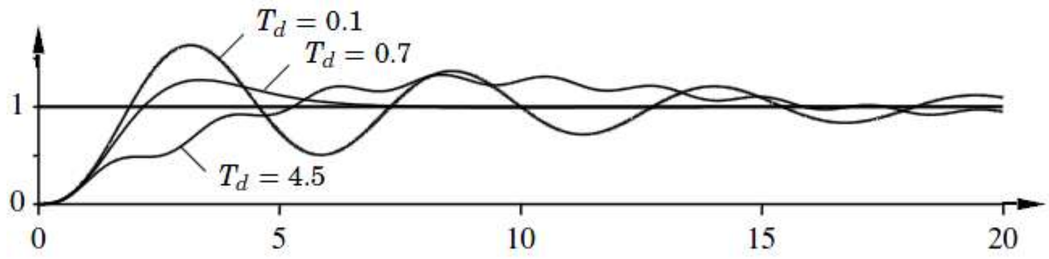
where

D_{out}: Derivative portion of controller output
T_d: Derivative time
K_d: Derivative gain
e: Error signal, e = set point – process variable

To summarize the tasks of all three controls, proportional control causes an input signal to change as a direct ratio of the error signal variation. It responds immediately to the current tracking error, but it cannot achieve the desired set point accuracy without an unacceptably large gain. Thus, the proportional term usually needs the other terms. Integral control causes the output signal to change as a function of the integral of the error signal over time duration. The integral term yields zero steady state error in tracking a constant set point. It also rejects constant disturbances. Derivative action reduces transient errors and causes an output signal to change as a function of the rate of change of the error signal. The contributions of the three terms will yield the control output, or the control variable:

$$\text{Control Variable} = P_{out} + I_{out} + D_{out}$$

The following figure illustrates a closed-loop system with proportional, integral, and derivative control. (*Closed-loop system will be defined later*.)



In practice, most PID controllers can be run in two modes: manual or automatic. In manual mode, the controller output is manipulated directly by the operator, typically by pushing buttons that increase or decrease the controller output. A controller may also operate in combination with other controllers, such as in a cascade or ratio connection, or with nonlinear elements, such as multipliers and selectors. In automatic mode, the PID parameters can be adjusted during operation. When there are changes of modes and parameters, it is important to avoid switching transients.

Since the PID controller is a dynamic system, it is necessary to make sure the state of the system is correct when switching the controller between manual and automatic mode. When the system is in automatic mode, the PID algorithm produces a control variable that may be different from the manually generated control variable. Therefore, it is crucial for the operator to make sure the two control variables match at the time of switching. This is called *bumpless transfer*. The first scenario is switching from automatic to manual. A bumpless transfer will force the manual output to track the output of the controller. If no disturbances occur after switching, the PV will not change. The second scenario is switching from manual to automatic. If the PV is not equal to the SP, the switch will result in an immediate "bump" to the controller output due to proportional action. To avoid the immediate bump, the operator can do one of the following: 1) adjust the SP to the current PV and then switch to automatic and slowly change the SP; 2) slowly adjust the controller output until the PV equals the current SP and then switch to automatic; or 3) use SP tracking while in manual. This would force the current SP to track the PV while in manual so that no proportional bump occurs while switching to automatic.

Below is the pseudo-code of a simple software loop that implements a feedback PID algorithm.

Constants:

Gain	System gain
ResetRate	Reset rate
Derivative	Derivative

Variables:

Input	Process input
InputDev	Process input plus derivative
InputPrev	Process input from previous pass, used in derivative calculation
Error	Difference between input and set point
Setpt	Set point
OutputTemp	Temporary value of output
Output	Output of PID algorithm
Feedback	Result of time delay in positive feedback loop (Sometimes this is called the manual reset or bias)
Mode	Value is "AUTO" if loop is in automatic
Action	Value is "DIRECT" if loop is direct acting

```
IF Mode = "AUTO" THEN
        InputDev = Input + (Input – InputPrev) * Derivative * 60     // Derivative
        InputPrev = Input
        Error = Setpt – InputDev          // Error based on reverse action
        IF Action = "DIRECT" THEN
                Error = 0 – Error         // Change sign of error for direct action
        ENDIF
        OutputTemp = Error * Gain + Feedback
        // Limit output to 0 – 100%
        IF OutPutTemp > 100 THEN OutPutTemp = 100
        IF OutPutTemp < 0 THEN OutPutTemp = 0
        Output = OutputTemp               //  The final output of the controller.
        Feedback = Feedback + (Output – Feedback) * ResetRate / 60
ELSEIF Mode = "MANUAL" THEN
        InputPrev = Input                 // Stay ready for bumpless switch to Auto
        Feedback = Output
ENDIF
```

Feedback control corrects any cumulative measurement errors or other errors in the system. In contrast, feedforward control applies to a system in which a balance between supply and demand is achieved by measuring both demand potential and demand load and using this information to govern supply. Feedforward control applies to a process that must be completely understood and requires no special instrumentation. Feedforward is often used to control processes with transportation and measurement lags. For example, a feedforward value representing "cold water poured into a warm mix" could boost the output value faster than waiting for the process variable to change as a result of the mixing. Another example is that the lag between input and output in a distillation column may vary from a few minutes to many minutes, depending on the size of the column and other variables. When long lags are inherent to this process, feedback control becomes difficult, because it cannot make any required corrections until the error is sensed. Feedforward can be used to reduce the lag and improve control results, because an exact balance between supply and demand potential is achieved before the process is influenced.

3. Implementation of PID Control

As suggested earlier, to implement a PID control, the engineer must first choose the structure of the PID controller and then must choose numerical values for the PID coefficients to tune the controller.

3.1. Choosing the Structure of a PID Controller

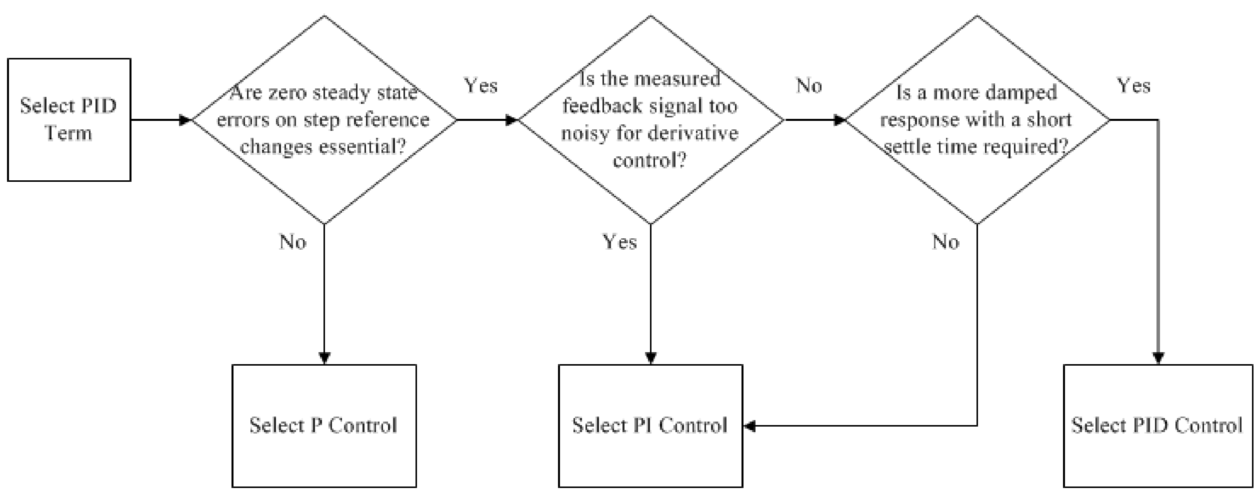
To choose the structure of a controller, refer to the following table of tuning effects of the PID controller terms. This is a reference table that shows how each term of the controller can be selected to accomplish a particular closed-loop system effect of specification.

	Reference Tracking Tuning		**Disturbance Rejection Tuning**	
	Step Reference		**Constant Load Disturbance**	
	Transient	**Steady State**	**Transient**	**Steady State**
P	Increasing $K_p > 0$ speeds up the response.	Increasing $K_p > 0$ reduces but does not eliminate steady state offset.	Increasing $K_p > 0$ speeds up the response.	Increasing $K_p > 0$ reduces but does not eliminate steady state offset.

I	Introducing integral action K_i > 0 gives a wide range of response types.	Introducing integral action K_i > 0 eliminates offset in the reference response.	Introducing integral action K_i > 0 gives a wide range of response types.	Introducing integral action K_i > 0 eliminates steady state offsets.
D	Derivative action K_d > 0 gives a wide range of responses and can be used to tune response damping.	Derivative action has no effect on steady state offset.	Derivative action K_d > 0 gives a wide range of responses and can be used to tune response damping.	Derivative action has no effect on steady state offset.

Let's consider this example: if it is necessary to remove steady state offsets from a closed-loop output response, the table indicates that a D-term will not do this and a P-term will reduce the offset if the proportional gain K_p is increased, but an I-term will eliminate the offset completely. Therefore, an integral term would be chosen for inclusion in the controller structure. Another example occurs when the engineer has elected to use integral action to eliminate constant steady state process disturbance offset errors but now wishes to speed up the closed-loop system response. The table shows that increasing the proportional gain K_p will have just this effect, so both proportional (*P*) action and integral (*I*) action would be selected, giving a PI controller solution.

The examples above show how the PID controller structure is selected to match the desired closed-loop performance. This selection process can be simplified using the following PID term selection flow diagram.



3.2. Tuning the PID Controller

The second part of setting up a PID controller is to tune or choose numerical values for the PID parameters. PID controllers are tuned in terms of their P, I, and D terms. Tuning the control gains can result in the following improvements of responses:

- Proportional gain *(K_p)*
  Larger proportional gain typically means faster response, because the larger the error, the larger the proportional term compensation. However, an excessively large proportional gain may result in process instability and oscillation.

- Integral gain *(K_i)*
  Larger integral gain implies that steady state errors are eliminated faster. However, the trade-off may be a larger overshoot, because any negative error integrated during transient response must be integrated away by positive error before steady state can be reached.

- Derivative gain *(K_d)*
  Larger derivative gain decreases overshoot but slows down transient response and may lead to instability due to signal noise amplification in the differentiation of the error.

The following table lists some common tuning methods and their advantages and disadvantages. The choice of method will mostly depend on whether or not the loop can be taken offline for tuning and the response time of the system. If the system can be taken offline, the best tuning method often involves subjecting the system to a step change in input, measuring the output as a function of time, and using this response to determine the control parameters. Manual tuning methods can be quite inefficient, especially if the loops have response times of longer than a minute.

Tuning Method	**Advantages**	**Disadvantages**
Manual tuning	• No math required • Online method	• Requires experienced personnel
Ziegler-Nichols	• Proven method • Online method	• Process upset • Some trial and error • Very aggressive tuning

Software tools	• Consistent tuning • Either online or offline tuning • May include actuator and sensors analysis	• Involves some cost and training
Cohen-Coon	• Good process models	• Requires some math • Offline method only • Only good for first-order process

Many manufacturing and industrial process companies have in-house guidelines for tuning PID controllers in particular process plant units. Therefore, it is often possible to provide rules and empirical formulas for PID controller tuning procedures. Some of these guidelines base their procedures on the routines of the commonly used Ziegler-Nichols methods. The two Ziegler-Nichols methods use an online process experiment followed by a set of rules to calculate the numerical values of the PID parameters. Numerous improvements and extensions of the associated rules have been achieved since their introduction.

Ziegler-Nichols is a type of continuous cycling method for controller tuning. The term *continuous cycling* refers to a continuous oscillation with constant amplitude and is based on trial-and-error procedure. The following are the steps to implement the method:

1. Allow the process to reach steady state as much as possible, turn off the integral mode (or set time to zero), and then turn off the derivative mode.

2. Assign a small value to proportional-only controller gain K (e.g., 0.5), and place the controller in automatic mode.

3. Make a small set point change so that the control variable moves away from the set point.

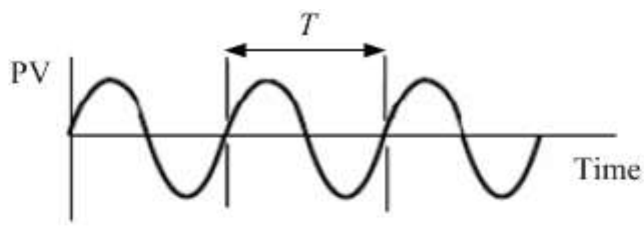
4. Increase the gain slightly.

5. Repeat steps 3 and 4 until continuous oscillation is achieved. This is known as the ultimate gain.

6. Calculate the PID controller settings using the Ziegler-Nichols tuning relations in the following table.

7. Evaluate the Ziegler-Nichols controller settings by introducing a small set point change and observing the closed-loop response. Fine tune the settings if necessary.

	P Control	**PI Control**	**PID Control**
K_p	0.5 K	0.45 K	0.6 K
K_i		$\frac{1.2}{T}$	$\frac{2}{T}$
K_d			$\frac{T}{8}$



Note: K is the ultimate gain (the gain at which the oscillations continue with constant amplitude), and T is the time or ultimate period in minutes measured in the gain calibration above (the time it takes the process variable to complete one full oscillation while the system is at steady state).

By tuning the three parameters in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The controller's response can be described in terms of the controller's responsiveness to error, the degree to which the controller overshoots the set point, and the degree of system oscillation. Note that the use of the PID algorithm for control does not guarantee optimal control of the system or system stability. Some applications may require using only one or two modes to provide the appropriate system control. A PID controller will be called a P, I, PI, or PD controller in the absence of respective control actions. If the PID controller parameters are improperly chosen, the controlled process input can become unstable. Instability is often caused by excess gain, particularly in the presence of significant time delay. In most applications, stability of response is required, and the process must not oscillate for any combination of process conditions.

4. Open-Loop vs. Closed-Loop Control

An open-loop controller, also called a non-feedback controller, computes its input into a system using only the current state and its model of the system. The controller does not receive a feedback signal from the process and it only has a set-point and a fixed output signal. The output signal does not change, regardless of the system conditions and disturbances. Consequently, an open-loop system cannot engage in machine learning or correct any error that it produces. It also may not compensate for disturbances in the system.

For example, an irrigation sprinkler system programmed to turn on at fixed times is an example of an open-loop system if it does not measure soil moisture as a form of feedback. Even when it is raining, the sprinkler system would still activate on schedule.

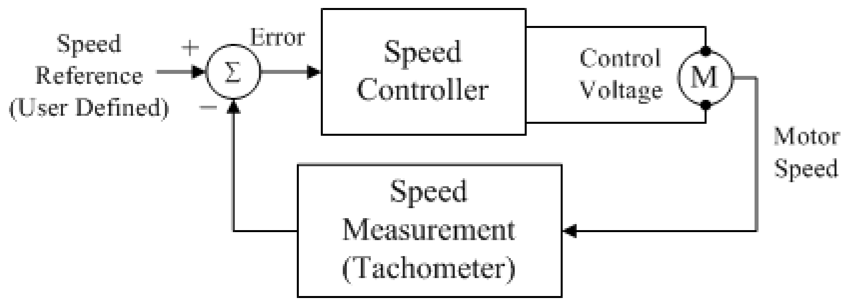
An open-loop controller is often used in simple processes because of its simplicity and low cost, particularly in systems where feedback is not critical. A typical example would be a conventional dryer, for which the length of machine drying time is entirely based on the judgment and estimation of the human operator. To obtain a more accurate or more adaptive control, it is necessary to feed the output of the system back to the controller. This type of system is called a closed-loop system.

In a closed-loop system, also called a feedback system, the controller has a feedback signal from the process. The controller has a set point, a feedback input signal, and a varying output signal. The output signal increases or decreases proportionally to the error of the set point compared to the input signal. The input signal varies proportionally to the system disturbances and the gain of the measurement sensor.

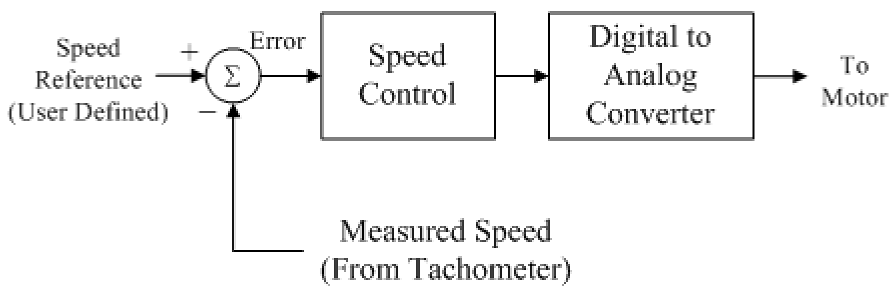
An example of a closed-loop system would be an automobile’s cruise control. When the car goes up a hill, the car will power up to maintain the set point speed set by the driver; the steeper the hill, the more power will be applied. The increase in slope is a system disturbance, but there can be more than one disturbance on a system. A stronger head wind would add to the error of an increasing slope, requiring the car use even more power to keep up with the set point.

5. Example: PID Controller for DC Motor

In this example, the desired target speed of the motor is set by the user. This value is fed into the speed controller to change the motor speed, and the loop is closed by a tachometer. The controller constantly adjusts the value of the DC voltage applied to the motor to maintain the desired speed. The control loop is shown in the following figure:



The speed controller contains two components: speed control and a digital-to-analog converter. The speed control component compares the desired speed with the measured speed and generates a digital value proportional to the DC value to be applied to the motor. The digital-to-analog converter is implemented using a signal generator and a low pass filter to smooth out the signal.



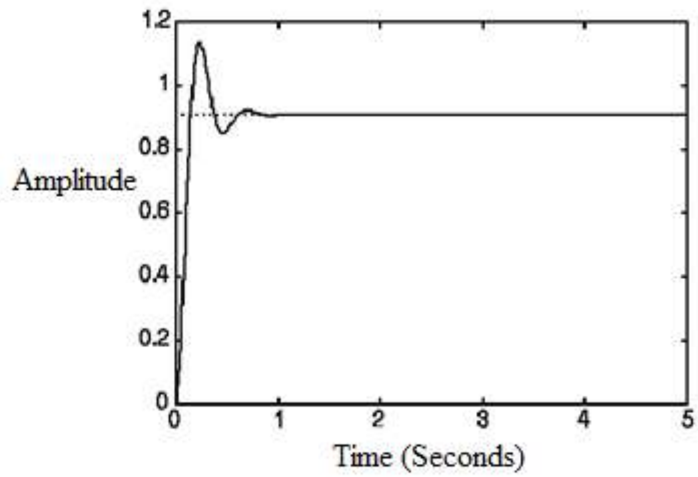
The transfer function for the DC motor's speed is expressed in Laplace form as:

$$\frac{\theta}{V} = \frac{K}{(J+b)(L+R)+K^2}$$

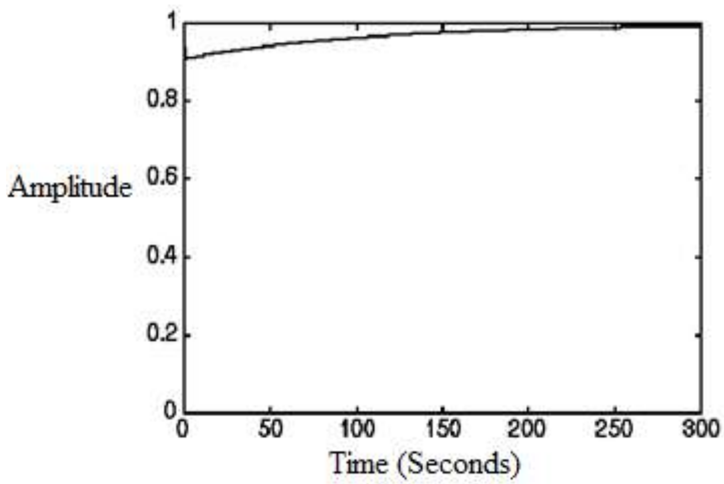
where

K:	Electromotive force control = 0.01 Nm/Amp
R:	Electrical resistance = 1 Ω
L:	Electrical inductance = 0.5H
J:	Moment of inertia of rotor = 0.01 kg m^2/s^2
b:	Damping ratio of mechanical system = 0.1 Nms
V:	Source voltage
θ:	Rotating speed

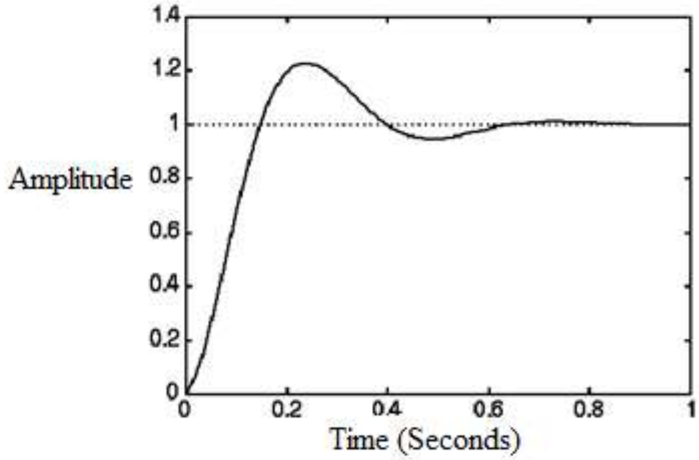
Therefore, the rotating speed of the motor is directly proportional to the input voltage. First, let's try a proportional-only control with a gain (K_p) of 100 and step input of 1 rad/sec. A step response is received as follows:



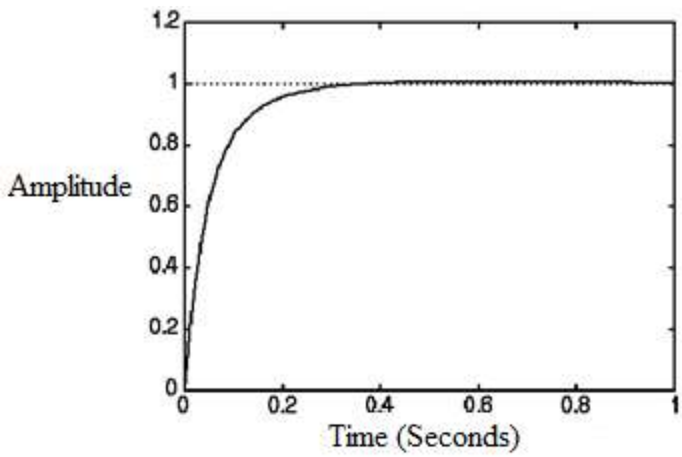
From the figure above, we see that both the steady state error and overshoot are too large. Since adding an integral term will eliminate the steady state error and a derivative term will reduce the overshoot, we try a PID controller with small $K_i = 1$ and $K_d = 1$. After that, the step response curve looks like this:



However, now the settling time is too long. So we increase K_i to 200 to reduce the settling time. After that, the step response curve looks like the following:



Now we see that the response is much faster than before; however, the large K_i has worsened the transient response (big overshoot). So we increase K_d to 10 to reduce the overshoot. After that, the step response curve looks like:

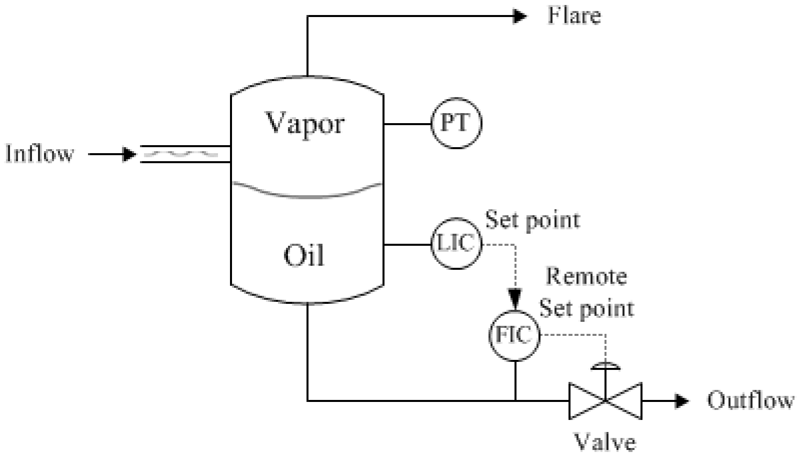


We can now use a PID controller with the following parameters to adjust the value of the DC voltage applied to the motor to maintain the desired speed.

$$K_p = 100$$
$$K_i = 200$$
$$K_d = 10$$

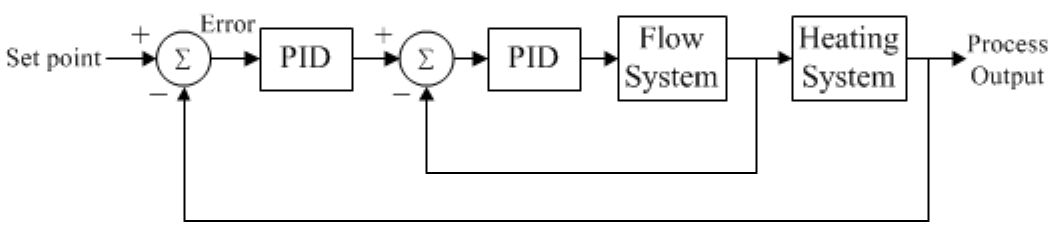
6. Cascade Control

PID control can be improved by changing the PID parameters, improving measurement, such as sampling rate, precision, and accuracy, and using low-pass filtering. Another proven method is cascading multiple PID controllers, which can be used to yield better dynamic performance. Using a previous example, a cascade control would look like this:



PT	Pressure Transmitter
LIC	Level Indicator Controller
FIC	Flow Indicator Controller

In a cascade control, there are two PID controllers, with one controlling the set point of the other. A PID controller acts as an outer loop (primary) controller, which controls the primary physical parameter, such as temperature. The other controller then acts as an inner loop (secondary) controller, which reads the output of the outer loop controller as its set point, usually controlling a more rapid changing parameter, such as flow rate. The inner loop must have a response time that is faster than that of the outer loop. Since the outer loop controller provides the set point for the inner loop, the inner loop must be faster than the outer loop so that the set point will not change too fast for the inner loop to catch; otherwise, stability problems may result. Below is a block diagram of a cascade control loop.



Cascade control is especially advantageous when the main disturbance is in the inner loop. This is because with the cascade configuration, the correction of the inner disturbance occurs as soon as the secondary sensor detects the upset. There are three main advantages gained by the use of cascade control:

1. Disturbances affecting the secondary variable can be corrected by the secondary controller before their influence is felt by the primary variable.

2. Closing the control loop around the secondary part of the process reduces the phase lag seen by the primary controller, resulting in increased speed of the response. Because of the increased speed of response, the sensitivity of the primary process variable to process upsets is also reduced.

3. The use of the secondary loop can reduce the effect of control valve sticking or actuator nonlinearity.

7. Troubleshooting PID Control

If a control loop is not performing satisfactorily, troubleshooting is necessary to identify the source of the problem. In many process applications, a control loop that once operated satisfactorily can become either unstable or excessively sluggish for a variety of reasons, including:

- Changing process conditions, usually changes in throughput rate
- A sticking control valve stem
- A plugged line in a pressure or differential pressure transmitter
- Fouled heat exchangers, particularly reboilers for distillation columns
- Cavitating pumps (usually caused by a suction pressure that is too low)

The starting point for troubleshooting is to obtain enough background information to define the problem clearly. The following questions must be answered:

- What are the control objectives?
- What is the process being controlled?
- What are the control variables?
- Are closed-loop response data available?
- Is the controller in manual or automatic mode? Is it reverse or direct acting?
- If the process is cycling, what is the cycling frequency?

- What PID controller algorithm is used? What are the PID controller parameters?
- Is the process open-loop stable?
- What additional documentation is available, such as control loop summary sheets, piping and instrumentation diagrams (P&ID), etc.?