PDHonline Course E344 (6 PDH)

# Calculating and Measuring Power in Three Phase Circuits 

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## TABLE OF CONTENTS

Section - Description ..... Page

1. Introduction ..... 5
2. Calculating Power in Single Phase Circuits ..... 6
2A. Single Phase Voltage and Single Phase Current ..... 6
2B. Single Phase Power ..... 8
2C. Kilowatts, Lumens, BTU's and Horsepower ..... 13
3. Calculating Power in Balanced Three Phase Circuits ..... 14
3A. General ..... 14
3B. Calculating Power in Balanced Three Phase Wye Circuits ..... 18
3B1. General ..... 19
3B2. Calculating Power in Balanced Three Phase Wye Circuits - Resistive Loads ..... 20
3B2a. Using Phase Parameters ..... 20
3B2b. Using Line Parameters ..... 20
3B3. Calculating Power in Balanced Three Phase Wye Circuits - Inductive or Capacitive Loads ..... 21
3B3a. Using Phase Parameters ..... 21
3B3b. Using Line Parameters ..... 21
3C. Calculating Power in Balanced Delta Three Phase Circuits ..... 21
3C1. General ..... 21
3C2. Calculating Power in Balanced Three Phase Delta Circuits - Resistive Loads ..... 22
3C2a. Using Phase Parameters ..... 22
3C2b. Using Line Parameters ..... 22
3C3. Calculating Power in Balanced Three Phase Delta Circuits - Inductive or Capacitive ..... 23
3C3a. Using Phase Parameters ..... 23
3C3b. Using Line Parameters ..... 23
4. Calculating Power in Unbalanced Three Phase Circuits ..... 23
4A. General ..... 23
4B. Calculating Power in Unbalanced Three Phase Wye Circuits ..... 23
4B1. General ..... 23
4B2. Calculating Power in Unbalanced Three Phase Wye Circuits - Resistive Loads ..... 24
4B2a. Using Phase Parameters ..... 24
4B2b. Using Line Parameters ..... 25
4B3. Calculating Power in Unbalanced Three Phase Wye Circuits - Inductive or Capacitive Loads 26
4B3a. Using Phase Parameters ..... 26
4B3b. Using Line Parameters ..... 27
4C. Calculating Power in Unbalanced Three Phase Delta Circuits ..... 29
4 C 1 . General ..... 29
4C2. Calculating Power in Unbalanced Three Phase Delta Circuits - Resistive Loads ..... 29
4C2a. Using Phase Parameters ..... 29
4C2b. Using Line Parameters ..... 30
4C3. Calculating Power in Unbalanced Three Phase Delta Circuits - Inductive or Capacitive ..... 30
4C3a. Using Phase Parameters ..... 30
4C3b. Using Line Parameters ..... 31
4D. Calculating Power in a Three Phase Circuit with Mixed Wye and Delta Loads ..... 35
4D1. General ..... 35
4D2. Using Phase Data ..... 36
4D3. Using Line Data ..... 36
5. Measuring Power in a Single Phase Circuit ..... 36
5A. General ..... 36
6. Measuring Power in Three Phase Three Wire Circuits ..... 37
6A. General ..... 38
6B. Using One Single Phase Wattmeter to Measure Power in a Three Phase Three Wire Circuit ..... 40
6E. Using a Three Phase Wattmeter to Measure Power in a Three Wire Three Phase Circuit ..... 46
7. Measuring Power in a Three Phase Four Wire Circuit ..... 48
7A. General ..... 48
7B. Using One Single Phase Wattmeter to Measure Power in a Three Phase Four Wire Circuit ..... 48
7C. Using Three Single Phase Wattmeters to Measure Power in a Three Phase Four Wire Circuit ..... 49
7D. Using a Three Phase Wattmeter to Measure Power in a Three Phase Four Wire Circuit ..... 50
8. Power Analyzers and Power Quality Meters ..... 51
9. Summary of Symbols and Equations __ 53

9A. Symbols $\longrightarrow 53$
9B. Equations_54
10. References_ 58

Appendix A-Demonstration that Equations 105 is the equivalent of Equation $106 \ldots{ }_{60}$ Appendix B

- Table of Computed Values of Instantaneous Power for Example 1 _ 62

Appendix C-Demonstration that Total Power of an Unbalanced Delta Circuit is
Equivalent to that of an Assumed Wye Circuit
Appendix D - Proof of Two Wattmeter and Three Wattmeter Methods.__ 70
Appendix E - Table of Typical Three Phase Power Values vs. $\omega \boldsymbol{t}$ for One Cycle___ 79
Appendix F - Equations for Calculating Currents $\quad \mathbf{8 2}$

## Calculating and Measuring Power in Three Phase Circuits

## 1. Introduction

The generation and transmission of electricity is commonly accomplished by means of three phase circuits. Although electrical service to residential buildings in the USA are exclusively by single phase circuits, electrical service to commercial and industrial users are usually by means of three phase circuits. The distribution of electrical power within these buildings is mostly accomplished by means of three phase circuits. And, all larger motors are almost exclusively of the three phase type. In short, three phase circuits are the most practical means of dealing with larger amounts of electrical power.

When planning a new three phase circuit, it very often becomes necessary for an engineer to determine the power those circuits will require. Likewise, the need may arise to determine the power consumption of an existing installation. Sometimes, these values may be established by calculation. On the other hand, it sometimes becomes necessary to take measurements in order to determine the power that is being used by an existing installation. In the case of an existing installation, calculations and measurements go hand in hand. If measurements are to be made, a person must first know what measurements are to be made and, secondly, how are field measurements converted to actual power values. A number of conditions have a bearing on the subject. There are delta connected loads and wye connected loads; there are three phase three wire circuits and three phase four wire circuits. And, of course, there are balanced loads and unbalanced loads. This spectrum of possible scenarios can be confusing to a person who may not regularly deal with three phase power.

This course treats the subject of three phase power in detail and in a manner that a reader, well experienced in three phase circuits or otherwise, will find
easy to follow. The course considers all of the possible types of three phase circuits - balanced, unbalanced, three wire, four wire, wye and delta circuits. The course is arranged so that power values may be determined from calculations or measurements and from either phase parameters or line parameters.

Numerous diagrams and examples are used in the course to explain the concepts of the course in simple and unambiguously terms. In the spirit of presenting subject matter in easily understood concepts the course avoids the use of complex variables and polar notation. Rather the more simple method of vector algebra is used.

## 2. Calculating Power in Single Phase Circuits

To better understand three phase power, a person would be well advised to first review and understand the principles applicable to single phase power. After all, a three phase circuit is essentially a combination of three separate single phase circuits which happen to have peaks and valleys separated by a period of time. Following is a brief review of the principles involved in single phase power.

## 2A. Single Phase Voltage and Single Phase Current

In general, the instantaneous voltage in a single phase AC electrical circuit with respect to time can be expressed by the relationship,
$v^{i}=\left(V_{\mathrm{PK}}\right) \sin \omega t \ldots$ Equation 101
where,
$\boldsymbol{v}^{i}=$ instantaneous value of voltage (volts)
$\nu_{\mathrm{PK}}=$ peak value of instantaneous voltage $\nu^{i}$ (volts)
$\omega=2 \pi \mathrm{f}$ (radians)
$\mathrm{f}=$ frequency (hz)
$t=$ time (sec)

Similarly, instantaneous current can be expressed as, $i^{i}=i_{\mathrm{PK}} \sin \left(\omega t+\theta_{\mathrm{SP}}\right) \ldots$ Equation 102
where,
$i^{i}=$ instantaneous value of current (amps)
$i_{\mathrm{PK}}=$ peak value of instantaneous current $i^{i}$ (amps)
$\theta_{\mathrm{SP}}=$ angle of lead or angle of lag (radians) of current with respect to voltage in a single phase circuit. (The subscript "SP" designates "single phase.")
for a lagging power factor, $\theta_{\mathrm{SP}}<0$
for a leading power factor, $\theta_{\mathrm{SP}}>0$

The trace of the voltage and current in a typical electrical circuit with a lagging power factor is shown in Fig. 1. In a circuit that is said to have a lagging current it is the current that lags voltage.

The trace of the


$\mathrm{V}(t)=\mathrm{V} \sin (\omega t) \ldots$ Equation 103
$\mathrm{I}(t)=\mathrm{I} \sin \left(\omega t+\theta_{\mathrm{SP}}\right) \ldots$ Equation 104
where,
$\mathrm{V}=\mathrm{rms}$ voltage
$\mathrm{I}=\mathrm{rms}$ current
(Note: Equations 101 and 102 are accurate expressions for instantaneous voltage and instantaneous current with respect to time. Equation 101 can be used to compute the precise value of voltage at a selected value of time $(t)$. Likewise, Equation 102 will compute the precise value of current at a selected value of time $(t)$. On the other hand Equations 103 and 104, while convenient for calculating power and other electrical properties of a circuit, do not provide precise values of voltage and current at selected values of time. Equations 103 and 104 are not used in this course for calculating instantaneous electrical properties.)

The commonly recognized relationships between instantaneous values of rms voltage and rms current are,
$\mathrm{V}=(1 / \sqrt{2}) \nu_{\mathrm{PK}}^{i}$, and
$\mathrm{I}=(1 / \sqrt{2}) i_{\mathrm{PK}}^{l}$

## 2B. Single Phase Power

Power is defined as the rate of flow of energy with time. In the MKS (Meter-Kilogram-Second) system of units, which today is more commonly called the SI (Système International) system, the unit of the flow of electrical energy is called the watt. One unit of energy in the SI system is the joule. One joule flowing for one second is one watt. In other words, 1 watt-second $=1$ joule. Of course, the more commonly used measure of electrical energy is the kilowatt-hour which would be the energy equivalent of one kilowatt flowing for one hour. The watt has been the standard unit for the measure of electrical power and it appears the use will continue well into the foreseeable future. However, some of the other units of measure associated with the use of electrical power are changing. Common uses for electrical power have been
for lighting, to drive electrical motors and for heating. All of these usages are being subject to changes in the units used to describe the particular end use. Instantaneous power in a single phase circuit is given by the expression, $\mathrm{P}^{i}=\boldsymbol{v}^{i} \cdot i^{i}$ (watts)
From Equation 101,
$v^{i}=\left(\mathcal{V}_{\mathrm{PK}}\right) \sin \omega t$
and from Equation 102,
$i^{i}=i_{\mathrm{PK}} \sin \left(\omega t+\theta_{\mathrm{SP}}\right)$
Therefore,
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right) \sin \omega t\right]\left[i_{\mathrm{PK}} \sin \left(\omega t+\theta_{\mathrm{SP}}\right)\right]$, or
$\mathrm{P}^{i}=\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)(\sin \omega t)\left[\sin \left(\omega t+\theta_{\mathrm{SP}}\right)\right]$
Since, $\sin \left(\omega t+\theta_{\mathrm{SP}}\right)=\sin \omega t \cos \theta_{\mathrm{SP}}+\cos \omega t \sin \theta_{\mathrm{SP}}$
$\mathrm{P}^{i}=\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)(\sin \omega t)\left[\sin \omega t \cos \theta_{\mathrm{SP}}+\cos \omega t \sin \theta_{\mathrm{SP}}\right]$
$\mathrm{P}^{i}=\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)+\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$

## ... Equation 105

The equation for instantaneous power $\left(\mathrm{P}^{i}\right)$ is often presented in a different form. One common version is:
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta_{\mathrm{SP}}-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \left(2 \omega t+\theta_{\mathrm{SP}}\right) \ldots$ Equation 106
(Reference \#1) As demonstrated in Appendix A, Equations 105 and 106 are equivalent equations although each is stated in a different form.

## Example 1

It will be informative to plot the computed values of Equation 105 for one complete cycle ( $\omega t=0^{\circ}$ to $\omega t=360^{\circ}$ ) of the variable $\omega t$ for a typical single phase application. The plot provides a pictorial view of the instantaneous value of power as a function of time. In the way of illustration, consider a case having the following parameters:
$\mathrm{V}_{\mathrm{rms}}=480 \mathrm{VAC}$, single phase
$V_{\mathrm{PK}}=480 \sqrt{2}=678.82 \mathrm{VAC}$
$\mathrm{I}_{\mathrm{rms}}=10 \mathrm{amps}$
$i_{\mathrm{PK}}=10 \sqrt{2}=14.14 \mathrm{amps}$

Power Factor $=0.70$, lagging \& $\cos \theta=0.70$, and $\theta=-45.57^{\circ}$
Equation 105 states,
$\mathrm{P}^{i}=\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)+\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$
Let,
$\mathrm{A}=\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta \sin ^{2} \omega t$, and
$\mathrm{B}=\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \sin \theta \sin \omega t \cos \omega t$
$\mathrm{P}^{i}=\mathrm{A}+\mathrm{B}$
$A=(480 \sqrt{2})(10 \sqrt{2})(0.7) \sin ^{2} \omega t$
$\mathrm{A}=(6720.00)$ $\sin ^{2} \omega t$, and $\mathrm{B}=(480 \sqrt{2})(10$ $\sqrt{2})\left(\sin -45.57^{\circ}\right)$ $\sin \omega t \cos \omega t$ $\mathrm{B}=(9600)(-$ .7141) $\sin \omega t \cos$ $\omega t$

The computed values of Function A and Function B in this

example are contained in Appendix B. A scaled plot of the values of Function A is shown in Fig. 3. A scaled plot of the values of Function B is shown in Fig. 4 and a plot of Function A plus Function B is shown in Fig. 5.

It is interesting to note that in Fig. 5, which describes power flow in a typical single phase circuit with a power factor of 0.70 , power flow has two large peaks in the positive direction and two smaller valleys in the

negative direction. This power flow will be compared below to power flow in a three phase circuit which, as demonstrated below, is drastically different.

According to Equation 105, the instantaneous power in a single phase circuit is: $\mathrm{P}^{i}=\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)+\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$

Generally speaking the average power is of the most interest. To determine the average power a single cycle of $\omega t$ is considered, i.e. from the value of $\omega t=0$ to $\omega t=360^{\circ}$. To determine average power, the net power over a single cycle is determined and then divided by the time of one period.

Let $\mathrm{T}=$ the time for a single cycle (from $\omega t=0^{\circ}$ to $\omega t=360^{\circ}$ ). The net power in the period is determined from integration of the equation for instantaneous power (Eq. 101) with respect to $t$. Let,
$\mathrm{P}^{i}=\mathrm{A}(t)+\mathrm{B}(t)$, whereby
$\mathrm{A}(t)=\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta \sin ^{2} \omega t$, and
$\mathrm{B}(t)=\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \sin \theta \sin \omega t \cos \omega t$
$\int_{\mathrm{A}}(t)=\int\left(\nu_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta \sin ^{2} \omega t d t=\left(\nu_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta \int_{\sin ^{2} \omega t} \omega t$
$\int_{\mathrm{A}}(t)=\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta[t / 2-(\sin 2 \omega t) / 4 \omega]$, evaluated from $t=0$ to $t=\mathrm{T}$.

The integral is evaluated from $t=0$ to $t=\mathrm{T}$, (with value " T " occurring at $\omega t=360^{\circ}$ )
$\int \mathrm{A}(t)=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta\right]\{[\mathrm{T} / 2-(\sin 2 \omega T) / 4 \omega]-[0 / 2-(\sin 0) / 4 \omega]\}$
$\int_{\mathrm{A}}(t)=\left[\left(\nu_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta\right](\mathrm{T} / 2)$
The average value of $\int_{\mathrm{A}}(t)$ throughout the period is:
$\mathrm{P}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta\right][\mathrm{T} / 2] / \mathrm{T}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta\right] / 2$
It will be apparent that the contribution of $\mathrm{B}(t)$ to the value of the integral of $\mathrm{P}^{i}$ throughout the period $t=0$ to $t=\mathrm{T}$ is $\mathrm{B}(t)=0$. This is the case since half of the function is positive during the period under consideration and half is negative, the net result being that the total contribution throughout the period is zero.

Voltages and currents are commonly used in rms (root mean squared) values. By definition,
$\mathrm{V}=v_{\mathrm{PK}} / \sqrt{2}$, and
$\mathrm{I}=i_{\mathrm{PK}} / \sqrt{2}$
And,
$\mathrm{P}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) \cos \theta\right] / 2$
Substituting rms values for peak values of voltage and current gives:
$\mathrm{P}=\mathrm{VI} \cos \theta \ldots$ Equation 107
Where,
$\mathrm{P}=$ power (watts)
$\mathrm{V}=$ potential (rms voltage)
$\mathrm{I}=$ current (rms amperage)
$\cos \theta=$ power factor
(The valid range of $\theta$ for a single phase circuit is from $-90^{\circ}+90^{\circ}$, and the power factor will always be between 0 and +1.0.)
Equation 107 is the commonly recognized equation for calculating power in a single phase circuit.

## 2C. Kilowatts, Lumens, BTU's and Horsepower

It is fair to say that a lage usage of three phase electrical power can be attributed to lighting, electric motors and heating.
Electrical lighting, and especially incandescent lighting, has long been defined by the amount of power to activate the light, i.e. by the wattage. This practice is changing as more efficient light sources have become available. More and more the various types of lights are being defined by the amount of light produced rather by the amount of electrical power the light requires. One unit that seems to be gaining in acceptance is the 'lumens', which is a metric unit. The incandescent light which remains the most common means of providing illumination, at around 17 lumens per watt, is one of the least efficient means of converting electrical energy into light. There are of course other popular units of measure for defining the amount of light a device produces. In any event the unit of the electrical power to activate a light will most certainly remain the "watt."

When describing heat, the BTU (British Thermal Unit) has long been used in the United States and in Commonwealth Countries. In the United States, the BTU will probably continue is use for the foreseeable future although overseas the BTU is being replaced by the "calorie" as well as other units. Nevertheless, electric resistance heating is mostly classified by its kilowatt rating.
Another big change occurring in the United States is in the way electrical motors are defined. Traditionally the size of a motor has been defined by "horsepower." Because of globalization and increased international trade, the term "horsepower" is being replaced by the "watt." In electrical units, 1 horsepower $=746$ watts. For example, a 1 kilowatt motor with an efficiency of $74.6 \%$ would require 1 kilowatt ( 1,000 watts) of electrical energy to operate and would produce 1 kilowatt X 0.746 kilowatts of output power (brake power), or 746 watts of output which would be the equivalent to 1
horsepower of output. A 1 kilowatt motor with an efficiency of $80 \%$ would produce 0.80 X 1 kilowatt of output power or $(800 / 746)=1.07 \mathrm{HP}$.

## 3. Calculating Power in Balanced Three Phase Circuits

## 3A. General

By definition, a three phase circuit consists of three separate circuits. A typical trace of the three voltages with respect to time would
 be similar to that represented in Fig. 6. The phases are commonly identified as Phase A, Phase B and Phase C and the common sequence is A-B-C. Each of the phases is $120^{\circ}$ from one another in a time plot.

There are two basic types of three phase circuits: delta and wye. A typical delta circuit is shown in Fig. 7 and a typical wye circuit is shown in Fig. 8. In a delta circuit, phase voltage equals line voltage but phase current is not equal to line current. In a wye circuit, line current equals phase current but phase voltage is not equal to line voltage. So, in order to perform correct power calculations, one must make the

distinction between line parameters and phase parameters and then apply the appropriate calculation.

A wye circuit could be the three-wire or four-wire type. If a wye circuit is the four wire type, a fourth conductor is
 extended from the wye neutral point to ground. A three wire wye circuit is absent a conductor extending from the neutral point to ground.

To calculate the power of a three phase circuit, specific parameters must be known. Voltage must be known. Also, currents and the leads or lags of currents with respect to voltages must be known. Unless care is taken to use the appropriate equation, a correct answer cannot be expected.

When computing three phase power, whether for a wye circuit or a delta circuit, it is particularly important that a person clearly understands the significance of current lead and current lag as well as the bearing that lead/lag has on the determination of power. Power factor in a single phase circuit is a clearly understood condition. More specifically, power factor in a single phase circuit is the cosine of the angle between current and voltage. A leading current with a lead angle of, say, $30^{\circ}$ has a power factor equal to the cosine of $30^{\circ}$ or 0.866 . A lagging current of $30^{\circ}$ would likewise have a power factor of 0.866 . In three phase circuits, the matter of power factor is somewhat more complicated.

A motor with a nameplate that states the motor power factor is giving the power factor of the phases. In other words, a motor power factor is the cosine
of the angle between the phase current and the phase voltage whether the motor is a wye wound motor or a delta wound motor. The most common type of three phase motor is the induction motor which always has a lagging power factor. Equation 12-1 in Appendix 12 gives the relationship between phase lead/lag and line lead/lag for balanced wye circuits and balanced delta circuits. Equation 12-1 can be helpful when a person needs to determine line currents and the associated lead/lag of the line currents in order to perform power calculations.

With a wye circuit, line currents and line voltages of an existing installation can generally be read with ease, but phase voltages for a three wire circuit may be difficult to read. In a delta circuit line voltages, which are the same as phase voltages, can be conveniently read but phase currents may be difficult to read. For example, to measure the phase currents of a delta wound motor would require removing the terminal box cover and moving existing connections. Particularly with high voltage motors, an effort of this type may be especially hazardous and undesirable.

By definition a balanced circuit has line voltages, line currents, phase voltages, phase currents and the lead/lag of the currents that are all identical. According to Equation 107, the power of a single phase circuit is described by the relationship,
$\mathrm{P}=\mathrm{VI} \cos \theta$, where the parameters are the single phase parameters. It follows then that the power for a balanced three phase circuit is, $\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}} \ldots$ (Reference 1 )
where,
$\mathrm{P}=$ power (watts)
$\mathrm{V}_{\mathrm{P}}=$ phase voltage (rms)
$\mathrm{I}_{\mathrm{P}}=$ phase current (rms)
$\cos \theta_{\mathrm{P}}=$ power factor of phase
$\theta_{\mathrm{P}}=$ angle of lead or angle of lag (radians or degrees) (phase current with respect to phase voltage) (The subscript ' P ' designates 'phase'.)
for a lagging power factor $\theta_{\mathrm{P}}<0$
for a leading power factor, $\theta_{\mathrm{P}}>0$

As in a single phase circuit, instantaneous power of each of the phases of a three phase is given by Equation 105.

## Example 2

In above Example 1, the values of instantaneous power are plotted for one complete cycle in a typical single phase application. As will be demonstrated, a plot of three phase power throughout the same period has a very different appearance. Consider a case having the following parameters which are typically characteristic of a three phase motor:
$\mathrm{V}_{\mathrm{rms}}=480 \mathrm{VAC}$, three phase
$\mathrm{V}_{\mathrm{PK}}=480 \sqrt{2} \mathrm{VAC}$
$\mathrm{I}_{\mathrm{rms}}=10 \mathrm{amps}$ (phase current)
$\mathrm{I}_{\mathrm{PK}}=10 \sqrt{2} \mathrm{amps}$
Power Factor $=0.7$, lagging
Therefore,
$\cos \theta=0.70$, and
$\theta=-45.57^{\circ}$
Equation 105 states that for each of the three phases, $\mathrm{P}^{l}=\mathrm{V}_{\mathrm{PK}} \mathrm{I}_{\mathrm{PK}} \cos \theta$ $\sin ^{2} \omega t+\mathrm{V}_{\mathrm{PK}} \mathrm{I}_{\mathrm{PK}} \sin$ $\theta \sin \omega t \cos \omega t \ldots$ Equation 105

It may be seen that the power factor is the same as that assumed in Example

1. In this later example the user is a
 three phase motor. A summary of the power computations of instantaneous power vs. $\omega t$ for Example 2 are tabulated in Appendix E. A plot of the values of power for the assumed parameters for one complete cycle is shown in Fig. 9. It is significant to note that the value of total instantaneous power throughout
the cycle is constant. In other words, the sum of the powers of the three phases is calculated to be exactly a constant value, in this instance $10,080.00$ watts. The computations of Example 1 and Example 2 demonstrate one of the significant differences between a single phase motor and a three phase motor. The power of the single phase motor in a single cycle has two sharp peaks and two sharp valleys per cycle whereas the power of the three phase motor is exactly constant for all values of $\omega t$.
As mentioned, the nameplate on a three phase motor will bear the power factor which is applicable to the phase currents and phase voltages. The nameplate will also state the full load amperage, as 'FLA', which is the magnitude of the line current. This practice is seemingly inconsistent, but nonetheless it has become standard practice.

It is usually a good idea to establish the sequence of a circuit under consideration. Knowing the phase sequence is especially important when dealing with three phase motors. For this purpose a phase sequence meter can come in handy.


## 3B. Calculating Power in Balanced Three Phase Wye Circuits

## 3B1. General

In a balanced three phase wye circuit the phase currents of all three phases are of the same magnitude as the line currents, and all three currents are at the same lead/lag angle. The phase voltages are at a magnitude that is a fixed ratio of the line voltages.

In the typical three phase wye circuit as represented in Fig. 8 the line current in Conductor A is the current of Phase A-D. The line current in Conductor B is the current of Phase B-D, and the line current in Conductor C is the current of Phase B-D. For a wye circuit the magnitude of the phase voltages are a fixed ratio of the line voltages and are given by the expression,
$V_{L}=(\sqrt{3}) V_{P} \ldots$ Equation 109
or,
$\mathrm{V}_{\mathrm{P}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}}$
where,
$\mathrm{V}_{\mathrm{L}}=$ line voltage (rms)
$\mathrm{V}_{\mathrm{P}}=$ phase voltage (rms)
The lead/lag of the line currents in a wye circuit are related to the lead/lag of the phase currents by the general expression,
$\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ}$ (Reference: Equation 12-1 in Appendix F.)
Where,
$\theta_{\mathrm{P}}=$ lead/lag of phase current with respect to phase voltage
$\theta_{\mathrm{L}}=$ lead/lag of line current with respect to line voltage
More specifically,

$$
\begin{aligned}
\theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}} & =\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ} \\
\theta_{\mathrm{P}-\mathrm{B} / \mathrm{BD}} & =\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ} \\
\theta_{\mathrm{P}-\mathrm{C} / \mathrm{CD}} & =\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}
\end{aligned}
$$

## 3B2. Calculating Power in Balanced Three Phase Wye Circuits Resistive Loads

## 3B2a. Using Phase Parameters

According to above Equation 108, $\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
In a resistive circuit the power factor is unity, or $\cos \theta_{\mathrm{P}}=1.0$.
Thus,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \ldots$ Equation 3B2a

## 3B2b. Using Line Parameters

From Equation 109,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
From Equation 108,
$V_{L}=(\sqrt{3}) V_{P}$, or
$\mathrm{V}_{\mathrm{P}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}}$
In a balanced wye circuit the magnitude of the line current is equal to the magnitude of the phase current (although the lead/lag of the current with respect to phase voltage is different from the lead/lag of the current with respect to line voltage).
$\mathrm{I}_{\mathrm{P}}=\mathrm{I}_{\mathrm{L}}$
For a balanced or unbalanced wye circuit the current lead/lag are related by the expression:
$\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ} \quad \ldots$ Equation 12
Since $\cos \theta_{\mathrm{P}}=1.0$ for resistive loads, $\theta_{\mathrm{P}}=0^{\circ}$
$\theta_{\mathrm{P}}=0^{\circ}=\theta_{\mathrm{L}}-30^{\circ}$
$\theta_{\mathrm{L}}=+30^{\circ}$
Thus,
The net power for all three phases becomes,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}=3\left\{[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}}\right\} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)$
$\mathrm{P}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(30^{\circ}-30^{\circ}\right)$
$=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(0^{\circ}\right)=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}}(1)$
$\mathrm{P}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \ldots$ Equation 3B2b

## 3B3. Calculating Power in Balanced Three Phase Wye Circuits Inductive or Capacitive Loads

## 3B3a. Using Phase Parameters

From Equation 109,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}} \quad \ldots \quad$ Equation 3B3a

## 3B3b. Using Line Parameters

From Equation 109, $\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
From Equation 12, $\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ}$
$\mathrm{V}_{\mathrm{P}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}}$
In a balanced wye circuit the magnitude of the line current is equal to the magnitude of the phase current.
$\mathrm{I}_{\mathrm{P}}=\mathrm{I}_{\mathrm{L}}$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
From Equation 12

$$
\begin{aligned}
& \theta_{\mathrm{P}}=\theta_{\mathrm{L}}+30^{\circ} \\
& \mathrm{P}=3[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right) \\
& \mathrm{P}=(\sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right) \quad \ldots \text { Equation 3B3b }
\end{aligned}
$$

Note: Equation 3B3b can be used for any balanced three phase load whether the user is wye, delta or a mix. The reasons for this condition are explained below.

## 3C. Calculating Power in Balanced Delta Three Phase Circuits

## 3C1. General

In a balanced three phase delta circuit the phase voltages are the same as the line voltages and the line currents are a fixed ratio of the phase currents.

In the representation of a typical delta circuit in Fig. 7 line voltage $A-B$ is the voltage of Phase A-B. Line voltage B-C is the voltage of Phase B-C, and Line voltage $B-C$ is the voltage of Phase B-C.

Since the phase voltages are of the same magnitude as the line voltages,
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{P}}$
Phase currents are related to line currents by the expression,
$I_{L}=(\sqrt{3}) I_{P} \ldots$ Reference 2
As with balanced wye circuits the lead/lag of the line currents in a balanced delta circuit are related to the lead/lag of the phase currents by the expression, $\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ} \quad$ (Reference: Equation 12-1 of Appendix F.)

$$
\begin{aligned}
& \theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{B} / \mathrm{BD}}=\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{C} / \mathrm{CD}}=\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}
\end{aligned}
$$

## 3C2. Calculating Power in Balanced Three Phase Delta Circuits Resistive Loads

## 3C2a. Using Phase Parameters

From Equation 109
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
Since the power factor in a resistive load is unity,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \ldots$ Equation 3C2a

## 3C2b.Using Line Parameters

$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{P}}$
$\mathrm{I}_{\mathrm{L}}=(\sqrt{3}) \mathrm{I}_{\mathrm{P}, \text { or }} \mathrm{I}_{\mathrm{P}}=[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}$
$\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ}$
$\theta_{\mathrm{P}}=0^{\circ}=\theta_{\mathrm{L}}-30^{\circ}$
$\theta_{\mathrm{L}}=+30^{\circ}$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}=3 \mathrm{~V}_{\mathrm{L}}\left\{[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}\right\} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{L}}\left\{[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}\right\} \cos \left(+30^{\circ}-30^{\circ}\right)=\mathrm{P}=3 \mathrm{~V}_{\mathrm{L}}\left\{[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}\right\} \cos \left(0^{\circ}\right)$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{L}}\left\{[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}\right\}=(\sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}}$
$\mathrm{P}=(\sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \quad \ldots$ Equation 3C2b

## 3C3. Calculating Power in Balanced Three Phase Delta Circuits Inductive or Capacitive

3C3a. Using Phase Parameters
From Equation 109,
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}} \quad \ldots$ Equation 3C3a
3C3b. Using Line Parameters
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
$\mathrm{I}_{\mathrm{L}}=(\sqrt{3}) \mathrm{I}_{\mathrm{P}}$
$\mathrm{I}_{\mathrm{P}}=[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}}$
$\mathrm{V}_{\mathrm{P}}=\mathrm{V}_{\mathrm{L}}$
$\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ}$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}=3 \mathrm{~V}_{\mathrm{L}}[1 /(\sqrt{3})] \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)$
$\mathrm{P}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right) \quad \ldots$ Equation 3C3b

## 4. Calculating Power in Unbalanced Three Phase Circuits

## 4A. General

Unlike power computations for an unbalanced wye circuits, the computations of power for an unbalanced delta circuits cannot be performed as easily. This is in large part due to the fact that in a delta circuit line currents are the product of two phase currents. If the delta circuit is a balanced circuit, the computations are relatively straightforward. On the other hand, unbalanced delta circuits require more attention to detail and more time overall.

## 4B. Calculating Power in Unbalanced Three Phase Wye Circuits

## 4B1. General

A viable method to determine the total power of an unbalanced three phase wye circuit is to treat the circuit as a combination of three single phase circuits. The total power then becomes the sum of the three individual power determinations. The computations are made especially easy since phase
current is equal to line current and phase voltage is a fixed ratio of line voltage.

## 4B2. Calculating Power in Unbalanced Three Phase Wye Circuits Resistive Loads

## 4B2a. Using Phase Parameters

For Phase A-D,
$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{A}-\mathrm{D}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}$
For Phase B-D,
$\mathrm{P}_{\mathrm{B}}=\mathrm{V}_{\mathrm{B}-\mathrm{D}} \mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}$
For Phase C-D,
$\mathrm{P}_{\mathrm{C}}=\mathrm{V}_{\mathrm{C}-\mathrm{D}} \mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}$
Where,
$\mathrm{P}_{\mathrm{A}}=$ power attributed to phase A
$\mathrm{P}_{\mathrm{B}}=$ power attributed to phase B
$\mathrm{P}_{\mathrm{C}}=$ power attributed to phase C
$\mathrm{I}_{\mathrm{A}}=$ current in conductor A and phase A-D
$\mathrm{I}_{\mathrm{B}}=$ current in conductor B and phase B-D
$\mathrm{I}_{\mathrm{A}}=$ current in conductor A and phase C-D
$\theta_{\mathrm{P}-\mathrm{AD}}=$ lead/lag of current in phase A with respect to voltage $\mathrm{V}_{\mathrm{A}-\mathrm{D}}$
$\theta_{\mathrm{P}-\mathrm{BD}}=$ lead/lag of current in phase B with respect to voltage $\mathrm{V}_{\mathrm{B}-\mathrm{D}}$
$\theta_{\mathrm{P}-\mathrm{CD}}=$ lead/lag of current in phase C with respect to voltage $\mathrm{V}_{\mathrm{C}-\mathrm{D}}$
For resistive loads, $\cos \theta_{\mathrm{P}-\mathrm{AD}}=\cos \theta_{\mathrm{P}-\mathrm{BD}}=\cos \theta_{\mathrm{P}-\mathrm{CD}}=1.0$
$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{A}-\mathrm{D}} \mathrm{I}_{\mathrm{A}}$
$\mathrm{P}_{\mathrm{B}}=\mathrm{V}_{\mathrm{B}-\mathrm{D}} \mathrm{I}_{\mathrm{B}}$
$\mathrm{P}_{\mathrm{C}}=\mathrm{V}_{\mathrm{C}-\mathrm{D}} \mathrm{I}_{\mathrm{C}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}+\mathrm{P}_{\mathrm{C}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{A}-\mathrm{D}} \mathrm{I}_{\mathrm{A}}+\mathrm{V}_{\mathrm{B}-\mathrm{D}} \mathrm{I}_{\mathrm{B}}+\mathrm{V}_{\mathrm{C}-\mathrm{D}} \mathrm{I}_{\mathrm{C}}$
Since, $V_{A-D}=V_{B-D}=V_{C-D}=V_{P}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right] \quad \ldots$ Eq. 4B2a

## Example 3

Consider a wye circuit the equivalent to that of Fig. 8. Assume the following conditions:
Voltage: 480-3-60
Currents:
$\mathrm{I}_{\mathrm{A}}=7 \mathrm{amp}$
$\mathrm{I}_{\mathrm{B}}=11 \mathrm{amp}$
$\mathrm{I}_{\mathrm{C}}=15 \mathrm{amp}$
Calculate total power.
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right]$
$\mathrm{V}_{\mathrm{P}}=480 / \sqrt{3}=277.128$ volts
According to Equation 4B2a,

$$
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right]=277.128[7+11+15]=9145.228 \text { watts }
$$

## 4B2b. Using Line Parameters

Phase currents equal line currents.
For Phase A-D,
$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{DA}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}$
$\mathrm{I}_{\mathrm{P}-\mathrm{A}}=\mathrm{I}_{\mathrm{L}-\mathrm{A}}=\mathrm{I}_{\mathrm{A}}$
$\mathrm{V}_{\mathrm{P}-\mathrm{AD}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{CA}}$
$\theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}$
$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{DA}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}=\left\{[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{CA}}\right\}\left[\mathrm{I}_{\mathrm{A}}\right] \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)$
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}+\mathrm{P}_{\mathrm{C}}$
The general form of the applicable equation is,
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right.$ $\left.\mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \ldots$ Eq. 4B2b
It is noted that for resistive loads, $\theta_{\mathrm{P}-\mathrm{A}}=0^{\circ}$
$\theta_{\text {P-A }}=\theta_{\text {L-A }}-30^{\circ}=0^{\circ}$
$\theta_{\mathrm{L}-\mathrm{A}}=+30^{\circ}$
$\cos \left(\theta_{\mathrm{L}-\mathrm{A}}-30^{\circ}\right)=\cos \left(+30^{\circ}-30^{\circ}\right)=\cos 0^{\circ}=1.0$
$\mathrm{P}_{\mathrm{DA}}=\left\{[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{CA}}\right\}\left[\mathrm{I}_{\mathrm{A}}\right](1.0)=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}-\mathrm{CA}} \mathrm{I}_{\mathrm{L}-\mathrm{A}}$
For Phase B-D,
$\mathrm{P}_{\mathrm{B}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}-\mathrm{AB}} \mathrm{I}_{\mathrm{L}-\mathrm{B}}$

For Phase C-D,
$\mathrm{P}_{\mathrm{C}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}-\mathrm{CA}} \mathrm{I}_{\mathrm{L}-\mathrm{C}}$
The net power then is,
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}+\mathrm{P}_{\mathrm{C}}$
Since, $\mathrm{V}_{\mathrm{L}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}-\mathrm{AB}}=\mathrm{V}_{\mathrm{L}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}}$
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}-\mathrm{A}}+(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}-\mathrm{B}}+(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}-\mathrm{C}}$
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}}+\mathrm{I}_{\mathrm{L}-\mathrm{B}}+\mathrm{I}_{\mathrm{L}-\mathrm{C}}\right]$

## Example 4

Assume the parameters of Example 3 and calculate total power using line parameters.

Voltage: 480-3-60
Currents:
$\mathrm{I}_{\mathrm{A}}=7 \mathrm{amp}$
$\mathrm{I}_{\mathrm{B}}=11 \mathrm{amp}$
$\mathrm{I}_{\mathrm{C}}=15 \mathrm{amp}$
Calculate total power.
In wye circuits phase currents equal line currents.
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right]$
$\mathrm{V}_{\mathrm{P}}=480 / \sqrt{3}=277.128$ volts
According to Equation 4B2b,

$$
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right]=277.128[7+11+15]=9145.228 \text { watts }
$$

## 4B3. Calculating Power in Unbalanced Three Phase Wye Circuits Inductive or Capacitive Loads

## 4B3a. Using Phase Parameters

$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{A}-\mathrm{D}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}$
$\mathrm{P}_{\mathrm{B}}=\mathrm{V}_{\mathrm{B}-\mathrm{D}} \mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}$
$\mathrm{P}_{\mathrm{C}}=\mathrm{V}_{\mathrm{C}-\mathrm{D}} \mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}$
The net power is,
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}+\mathrm{P}_{\mathrm{C}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{A}-\mathrm{D}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}+\mathrm{V}_{\mathrm{B}-\mathrm{D}} \mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}+\mathrm{V}_{\mathrm{C}-\mathrm{D}} \mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}$
Since, $V_{A-D}=V_{B-D}=V_{C-D}=V_{P}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}+\mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}+\mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}\right] \ldots$ Eq. 4B3a

## Example 5

Assume the following conditions for an unbalanced wye circuit:
Voltage: 480-3-60
Currents:
$\mathrm{I}_{\mathrm{A}}=8 \mathrm{amp}, \mathrm{PF}_{\mathrm{A}}=.60$
$\mathrm{I}_{\mathrm{B}}=12 \mathrm{amp}, \mathrm{PF}_{\mathrm{B}}=.70$
$\mathrm{I}_{\mathrm{C}}=16 \mathrm{amp}, \mathrm{PF}_{\mathrm{C}}=.80$
Calculate total power using phase parameters.
Power factors are descriptive of the lag/lead of phase current with respect
to phase voltage. Therefore,
$\cos \theta_{\mathrm{P}-\mathrm{AD}}=0.60$, lagging
$\cos \theta_{\mathrm{P}-\mathrm{BD}}=0.70$, lagging
$\cos \theta_{\mathrm{P}-\mathrm{CD}}=0.80$, lagging
$\mathrm{V}_{\mathrm{P}}=\mathrm{V}_{\mathrm{L}} / \sqrt{3}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}+\mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}+\mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}\right]$
$\mathrm{P}_{\mathrm{T}}=[480 / \sqrt{3}][(8)(.60)+(12)(.70)+(16)(.80)]$
$\mathrm{P}_{\mathrm{T}}=(277.128)[4.8+8.4+12.8]=(277.128)[26]=7,205.33$ watts

## 4B3b. Using Line Parameters

$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{AD}} \mathrm{I}_{\mathrm{P}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}$
$\mathrm{V}_{\mathrm{A}-\mathrm{D}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{CA}}$
$\mathrm{I}_{\mathrm{P}-\mathrm{A}}=\mathrm{I}_{\mathrm{A}}$
$\theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}$
$\mathrm{P}_{\mathrm{A}}=\mathrm{V}_{\mathrm{AD}} \mathrm{I}_{\mathrm{P}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{CA}} \mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)$
Similarly,
$\mathrm{P}_{\mathrm{B}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{AB}} \mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)$
$\mathrm{P}_{\mathrm{C}}=[1 /(\sqrt{3})] \mathrm{V}_{\mathrm{L}-\mathrm{BC}} \mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)$
Since, $\mathrm{V}_{\mathrm{L}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}-\mathrm{AB}}=\mathrm{V}_{\mathrm{L}-\mathrm{BC}}=\mathrm{V}_{\mathrm{L}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}+\mathrm{P}_{\mathrm{C}}$
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right.$
$\left.\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]$... Eq. 4B3b

## Example 6

Assume the same circuit parameters used in above Example 3 for a wye circuit and calculate total circuit power using line parameters.

Voltage: 480-3-60
Circuit: wye
Currents:
$\mathrm{I}_{\mathrm{A}}=8 \mathrm{amp}, \mathrm{PF}_{\mathrm{A}}=.60$, lagging
$\mathrm{I}_{\mathrm{B}}=12 \mathrm{amp}, \mathrm{PF}_{\mathrm{B}}=.70$, lagging
$\mathrm{I}_{\mathrm{C}}=16 \mathrm{amp}, \mathrm{PF}_{\mathrm{C}}=.80$, lagging
Calculate total power using line values.
Power factors are descriptive of the lag/lead of phase current with respect to phase voltage. Therefore,
$\cos \theta_{\mathrm{P}-\mathrm{AD}}=0.60$
$\theta_{\mathrm{P}-\mathrm{AD}}=-53.130^{\circ}$
$\cos \theta_{\mathrm{P}-\mathrm{BD}}=0.70$
$\theta_{\mathrm{P}-\mathrm{BD}}=-45.572^{\circ}$
$\cos \theta_{\mathrm{P}-\mathrm{CD}}=0.80$
$\theta_{\mathrm{P}-\mathrm{CD}}=-36.869^{\circ}$
According to Equation 4B3b,
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right.$
$\left.\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]$
$\theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}=-53.130^{\circ}$
$\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}=-53.130^{\circ}+30^{\circ}=-23.130^{\circ}$
$\theta_{\mathrm{P}-\mathrm{B} / \mathrm{BD}}=\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}=-45.572^{\circ}$
$\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}=-45.572^{\circ}+30^{\circ}=-15.572^{\circ}$
$\theta_{\mathrm{P}-\mathrm{C} / \mathrm{CD}}=\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}=-36.869^{\circ}$
$\theta_{\mathrm{L}-\mathrm{C} / \mathrm{CD}}=-36.869^{\circ}+30^{\circ}=-6.869^{\circ}$
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right.$ $\left.\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]$
$\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3})(480)\left[(8) \cos \left(-23.130^{\circ}-30^{\circ}\right)+\right.$
(12) $\left.\cos \left(-15.572^{\circ}-30^{\circ}\right)+(16) \cos \left(-6.869^{\circ}-30^{\circ}\right)\right]$

$$
\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3})(480)[(8)(.6)+(12)(.7)+(16)(.8)]
$$

$\mathrm{P}_{\mathrm{T}}=7205.33$ watts
This computed value of power is in agreement with the calculation of total circuit power using phase parameters which confirms the validity of both methods.

## 4C. Calculating Power in Unbalanced Three Phase Delta Circuits

## 4C1. General

Much as with a wye circuit, total power in an unbalanced three phase delta circuit can be determined by treating the circuit as a combination of three single phase circuits. The power of each of the three phases is separately determined and the total of the three becomes the three phase power of the circuit. This method, of course, assumes that the values of the currents and the respective leads/lags are available. The alternate method is to determine circuit power from the properties of the line currents.

## 4C2. Calculating Power in Unbalanced Three Phase Delta Circuits Resistive Loads

## 4C2a. Using Phase Parameters

$\mathrm{P}_{\mathrm{AB}}=\mathrm{V}_{\mathrm{A}-\mathrm{B}} \mathrm{I}_{\mathrm{P}-\mathrm{AB}} \cos \theta_{\mathrm{P}-\mathrm{AB}}, \mathrm{P}_{\mathrm{BC}}=\mathrm{V}_{\mathrm{B}-\mathrm{C}} \mathrm{I}_{\mathrm{P}-\mathrm{BC}} \cos \theta_{\mathrm{P}-\mathrm{BC}} \&$
$\mathrm{P}_{\mathrm{CA}}=\mathrm{V}_{\mathrm{C}-\mathrm{A}} \mathrm{I}_{\mathrm{P}-\mathrm{CA}} \cos \theta_{\mathrm{P}-\mathrm{CA}}$
For a circuit with all loads resistive,
$\cos \theta_{\mathrm{P}-\mathrm{AB}}=1.0, \cos \theta_{\mathrm{P}-\mathrm{BC}}=1.0 \& \cos \theta_{\mathrm{P}-\mathrm{CA}}=1.0$
$\mathrm{P}_{\mathrm{AB}}=\mathrm{V}_{\mathrm{L}-\mathrm{AB}} \mathrm{I}_{\mathrm{P}-\mathrm{AB}}$
$\mathrm{P}_{\mathrm{BC}}=\mathrm{V}_{\mathrm{L}-\mathrm{BC}} \mathrm{I}_{\mathrm{P}-\mathrm{BC}}$
$\mathrm{P}_{\mathrm{CA}}=\mathrm{V}_{\mathrm{L}-\mathrm{CA}} \mathrm{I}_{\mathrm{P}-\mathrm{CA}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{AB}}+\mathrm{P}_{\mathrm{BC}}+\mathrm{P}_{\mathrm{CA}}$, or
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}-\mathrm{AB}} \mathrm{I}_{\mathrm{P}-\mathrm{AB}}+\mathrm{V}_{\mathrm{L}-\mathrm{BC}} \mathrm{I}_{\mathrm{P}-\mathrm{BC}}+\mathrm{V}_{\mathrm{L}-\mathrm{CA}} \mathrm{I}_{\mathrm{P}-\mathrm{CA}}$

Since, $\mathrm{V}_{\mathrm{L}-\mathrm{AB}}=\mathrm{V}_{\mathrm{L}-\mathrm{BC}}=\mathrm{V}_{\mathrm{L}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}}$,
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{P}-\mathrm{AB}}+\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{P}-\mathrm{BC}}+\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{P}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{P}-\mathrm{BC}}+\mathrm{I}_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{P}-\mathrm{BC}}+\mathrm{I}_{\mathrm{P}-\mathrm{CA}}\right] \ldots$ Eq. 4C2a

## 4C2b.Using Line Parameters

Line parameters are identified as,
Voltages: $\mathrm{V}_{\mathrm{A}-\mathrm{B}}, \mathrm{V}_{\mathrm{B}-\mathrm{C} \&} \mathrm{~V}_{\mathrm{C}-\mathrm{A}}$
Currents:
$\mathrm{I}_{\mathrm{A}}-$ Current in conductor A
$\mathrm{I}_{\mathrm{B}}$ - Current in conductor B
$\mathrm{I}_{\mathrm{C}}-$ Current in conductor C
Lead/lag (currents with respect to line voltages):
$\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-$ lead/lag of line current A with respect to line voltage C-A
$\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-$ lead/lag of line current B with respect to line voltage A-B
$\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-$ lead/lag of line current C with respect to line voltage $\mathrm{B}-\mathrm{C}$
The applicable equation for circuit power as a function of line parameters with all resistance loads would be the same as that for inductive or capacitive loads as treated in below Section 4C3b.

$$
\begin{array}{r}
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right. \\
\left.\mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \quad \ldots \text { Eq. 4C2b }
\end{array}
$$

## 4C3. Calculating Power in Unbalanced Three Phase Delta Circuits Inductive or Capacitive

## 4C3a. Using Phase Parameters

$\mathrm{P}_{\mathrm{AB}}=\mathrm{V}_{\mathrm{A}-\mathrm{B}} \mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}, \mathrm{P}_{\mathrm{BC}}=\mathrm{V}_{\mathrm{B}-\mathrm{C}} \mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}} \&$
$\mathrm{P}_{\mathrm{CA}}=\mathrm{V}_{\mathrm{C}-\mathrm{A}} \mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{AB}}+\mathrm{P}_{\mathrm{BC}}+\mathrm{P}_{\mathrm{CA}}$, or
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{A}-\mathrm{B}} \mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}+\mathrm{V}_{\mathrm{B}-\mathrm{C}} \mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}}+\mathrm{V}_{\mathrm{C}-\mathrm{A}} \mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}$
Since, $\mathrm{V}_{\mathrm{A}-\mathrm{B}}=\mathrm{V}_{\mathrm{B}-\mathrm{C}}=\mathrm{V}_{\mathrm{C}-\mathrm{A}}=\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{P}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}+\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}}+\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}}+\mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}\right] \ldots$ Eq. 4C3a

## Example 7

Assume the following conditions for an unbalanced delta circuit:
Line potential: 480-3-60 volts
$\mathrm{I}_{\mathrm{ab}}=5 \mathrm{amps} @ \mathrm{PF}=1.00$
$\mathrm{I}_{\mathrm{bc}}=10 \mathrm{amps} @ \mathrm{PF}=0.90$ lagging
$\mathrm{I}_{\mathrm{ca}}=15 \mathrm{amps} @ \mathrm{PF}=0.80$ leading
Find line currents in conductors A, B and C.
Solution:
$\theta_{\mathrm{P}-\mathrm{AB}}=\cos ^{-1} 1.00=0$
$\theta_{\mathrm{P}-\mathrm{BC}}=\cos ^{-1} 0.90=-25.84193^{\circ}$
$\theta_{\text {P-CA }}=-\cos ^{-1} 0.80=+36.86989^{\circ}$
According to Equation 4C3a,
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}}+\mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{P}_{\mathrm{T}}=(480)[(5)(1)+(10)(.90)+(15)(.80)]$
$\mathrm{P}_{\mathrm{T}}=(480)[5+9+12]=(480)[26]=12480$ watts

## 4C3b.Using Line Parameters

Line parameters are identified as,
Voltages: $\mathrm{V}_{\mathrm{A}-\mathrm{B}}, \mathrm{V}_{\mathrm{B}-\mathrm{C} \&} \mathrm{~V}_{\mathrm{C}-\mathrm{A}}$
Currents:
$\mathrm{I}_{\mathrm{A}}-$ Current in conductor A
$\mathrm{I}_{\mathrm{B}}$ - Current in conductor B
$\mathrm{I}_{\mathrm{C}}$ - Current in conductor C
Lead/lag (currents with respect to line voltages):
$\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-$ lead/lag of line current A with respect to line voltage C-A
$\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-$ lead/lag of line current B with respect to line voltage $\mathrm{A}-\mathrm{B}$
$\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-$ lead/lag of line current C with respect to line voltage $\mathrm{B}-\mathrm{C}$
Total circuit power can be determined by assuming that there is a single user and that user is a wye circuit. This method is treated and corroborated in Appendix C.
$\mathrm{P}_{\mathrm{A}-\mathrm{D}}=\mathrm{V}_{\mathrm{P}-\mathrm{AD}} \mathrm{I}_{\mathrm{P}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}$
$\mathrm{V}_{\mathrm{P}-\mathrm{AD}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{CA}}$
$\mathrm{I}_{\mathrm{P}-\mathrm{A}}=\mathrm{I}_{\mathrm{L}-\mathrm{A}}$

$$
\begin{aligned}
& \theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ} \\
& \mathrm{P}_{\mathrm{A}-\mathrm{D}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{CA}} \mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{B}-\mathrm{D}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{AB}} \mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right) \text {, and } \\
& \mathrm{P}_{\mathrm{C}-\mathrm{D}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{BC}} \mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right) \\
& \mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\mathrm{A}-\mathrm{D}}+\mathrm{P}_{\mathrm{B}-\mathrm{D}}+\mathrm{P}_{\mathrm{C}-\mathrm{D}}, \text { or } \\
& \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{CA}} \mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+ \\
& \quad \mathrm{P}_{\mathrm{B}-\mathrm{D}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{AB}} \mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+ \\
& \quad \mathrm{P}_{\mathrm{C}-\mathrm{D}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{BC}} \mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)
\end{aligned}
$$

$$
\text { Since } \mathrm{V}_{\mathrm{L}-\mathrm{AB}}=\mathrm{V}_{\mathrm{L}-\mathrm{BC}}=\mathrm{V}_{\mathrm{L}-\mathrm{CA}}=\mathrm{V}_{\mathrm{L}}
$$

$$
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right.
$$

$$
\left.\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \quad \ldots \quad \text { Eq. 4C3b }
$$

## Example 8

Assume the same delta phase values that were used in above Example 7.
Compute the associated line parameters and then compute total power using the values of line parameters.
Line potential: 480-3-60 volts
$\mathrm{I}_{\mathrm{ab}}=5 \mathrm{amps} @ \mathrm{PF}=1.00$
$\mathrm{I}_{\mathrm{bc}}=10 \mathrm{amps} @ \mathrm{PF}=0.90$ lagging
$\mathrm{I}_{\mathrm{ca}}=15 \mathrm{amps} @ \mathrm{PF}=0.80$ leading
Find line currents in conductors A, B and C.
Solution:

$$
\begin{aligned}
\theta_{\mathrm{P}-\mathrm{AB}} & =\cos ^{-1} 1.00=0 \\
\theta_{\mathrm{P}-\mathrm{BC}} & =\cos ^{-1} 0.90=-25.84193^{\circ} \\
\theta_{\mathrm{P}-\mathrm{CA}} & =-\cos ^{-1} 0.80=+36.86989^{\circ}
\end{aligned}
$$

Next, the associated line parameters are determined.
From Equation 9-1 and Equation 9-2 (contained in Appendix F),
$\mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}$
$\theta_{\mathrm{L}-\mathrm{A}}=\left(120^{\circ}-\lambda\right)$
where,

From Equation 9-3 and Equation 9-4,

$$
\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2}
$$

$$
\theta_{\mathrm{L}-\mathrm{B}}=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right)
$$

where

$$
\begin{aligned}
\mathrm{X}_{\mathrm{B}} & =\mathrm{X}_{\mathrm{ab}}+\mathrm{X}_{\mathrm{cb}} \\
\mathrm{Y}_{\mathrm{B}} & =\mathrm{Y}_{\mathrm{ab}}+\mathrm{Y}_{\mathrm{cb}} \\
\mathrm{X}_{\mathrm{ab}} & =\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}=(5) \cos 0=5 \\
\mathrm{Y}_{\mathrm{ab}} & =\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}=(5) \sin 0=0 \\
\mathrm{X}_{\mathrm{cb}} & =-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right] \\
& =-(10)(1 / 2)\left[(\sqrt{3}) \sin -25.84193^{\circ}-\cos -25.84193^{\circ}\right] \\
& =-(5)[(\sqrt{3})(-0.4358898)-(.9]=-(5)[(-.754983)-(.9)] \\
& =8.274917 \\
\mathrm{Y}_{\mathrm{cb}} & =\mathrm{I}_{\mathrm{cb}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{BC}}+\sin \theta_{\mathrm{P}-\mathrm{BC}}\right] \\
& =(10)(1 / 2)\left[(\sqrt{3}) \cos -25.84193^{\circ}+\sin -25.84193^{\circ}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}=-(5) \cos 0=-5 \\
& \mathrm{Y}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}=-\mathrm{I}_{\mathrm{ba}}(0)=0 \\
& \mathrm{X}_{\mathrm{ca}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right] \\
& \mathrm{X}_{\mathrm{ca}}=-(15)(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right] \\
& =-(7.5)\left[(\sqrt{3}) \sin \left(36.86989^{\circ}\right)+\cos \left(36.86989^{\circ}\right)\right] \\
& =-(7.5)[(\sqrt{3})(0.6)+(0.8)]=-(7.5)[1.039230+0.8]=-13.794228 \\
& \mathrm{Y}_{\mathrm{ca}}=\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right] \\
& Y_{c a}=I_{c a}(1 / 2)\left[(\sqrt{3}) \cos \left(36.86989^{\circ}\right)-\sin \left(36.86989^{\circ}\right)\right] \\
& =(7.5)[(\sqrt{3})(.80)-(-0.6)]=5.892304 \\
& \mathrm{X}_{\mathrm{A}}=\mathrm{X}_{\mathrm{ba}}+\mathrm{X}_{\mathrm{ca}}=-5+(-13.794228)=-18.794228 \\
& \mathrm{Y}_{\mathrm{A}}=\mathrm{Y}_{\mathrm{ba}}+\mathrm{Y}_{\mathrm{ca}}=0+5.892304=5.892304 \\
& \mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}^{1 / 2}=\left\{(-18.794229)^{2}+(5.892304)^{2}\right\}^{1 / 2} \\
& =19.696250 \mathrm{amps} \\
& \lambda=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{A}} \div \mathrm{I}_{\mathrm{A}}\right)=\sin ^{1}[5.892304 \div 19.696250]=\sin ^{-1} .2991586 \\
& \mathrm{I}_{\mathrm{A}} \text { is in quadrant II } \\
& \lambda=\sin ^{-1} .299158=162.5929^{\circ} \\
& \theta_{\mathrm{L}-\mathrm{A}}=\left(\lambda-120^{\circ}\right)=\left(162.5929^{\circ}-120^{\circ}\right)=42.5929^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
&=(5)[(\sqrt{3})(.9)+(-0.435889)]=(5)[1.55884-0.435889] \\
&=5.61475 \\
& \mathrm{X}_{\mathrm{B}}=\mathrm{X}_{\mathrm{ab}}+\mathrm{X}_{\mathrm{cb}}=5+(8.274917)=13.274917 \\
& \mathrm{Y}_{\mathrm{B}}= \mathrm{Y}_{\mathrm{ab}}+\mathrm{Y}_{\mathrm{cb}}=0+5.61475=5.61475 \\
& \mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2}=\left\{(13.274917)^{2}+(5.61475)^{2}\right\}^{1 / 2} \\
&= 14.413495 \mathrm{amps} \\
& \theta_{\mathrm{L}-\mathrm{B}}=-\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right)=\sin ^{-1}(5.61475 \div 14.41349)=22.9263^{\circ}
\end{aligned}
$$

From Equation 9-5 and Equation 9-6,

$$
\mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2}
$$

where,
$\mathrm{X}_{\mathrm{bc}}=\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$=(10)(1 / 2)\left[(\sqrt{3}) \sin -25.84193^{\circ}-\cos -25.84193^{\circ}\right]$
$=(5)[(\sqrt{3})(-0.435889)-(0.9)]=(5)[-.754983-0.9]=-8.27491$
$\mathrm{Y}_{\mathrm{bc}}=-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{BC}}+\sin \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$=-(10)(1 / 2)\left[(\sqrt{3}) \cos -25.84193^{\circ}+\sin -25.84193^{\circ}\right]$
$=-(5)[(\sqrt{3})(.9)+(-.435889)]=-(5)[1.55884-.435889]=-5.61478$
$\mathrm{X}_{\mathrm{ac}}=\mathrm{I}_{\mathrm{Ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$=(15)(1 / 2)\left[(\sqrt{3}) \sin 36.86989^{\circ}+\cos 36.86989^{\circ}\right]$
$=(7.5)[(\sqrt{3})(.6)+(.8)]=(7.5)[1.039230+.8]=13.794228$
$\mathrm{Y}_{\mathrm{ac}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$=-(15)(1 / 2)\left[(\sqrt{3}) \cos 36.86989^{\circ}-\sin 36.86989^{\circ}\right]$
$=-(7.5)[(\sqrt{3})(.8)-(0.6)]=-(7.5)[1.385640-0.6)]=-5.89230$
$\mathrm{X}_{\mathrm{C}}=\mathrm{X}_{\mathrm{bc}}+\mathrm{X}_{\mathrm{ac}}=-8.27491+13.794228=5.519318$
$\mathrm{Y}_{\mathrm{C}}=\mathrm{Y}_{\mathrm{bc}}+\mathrm{Y}_{\mathrm{ac}}=-5.61478+(-5.89230)=-11.50708$
$\mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2}=\left\{(5.519318)^{2}+(-11.50708)^{2}\right\}^{1 / 2}$
$=12.762278 \mathrm{amps}$
$\theta_{\mathrm{L}-\mathrm{C}}=\left(\varphi-240^{\circ}\right)$
$\mathrm{I}_{\mathrm{C}}$ is in Quadrant IV
$\varphi=\sin ^{-1}\left(Y_{C} \div \mathrm{I}_{\mathrm{C}}\right)=\sin ^{-1}(-11.50708 \div 12.76227)=295.6244^{\circ}$

$$
\theta_{\mathrm{L}-\mathrm{C}}=\left(\varphi-240^{\circ}\right)=295.6244^{\circ}-240^{\circ}=55.6244^{\circ}
$$

Summary of calculated values for the delta circuit:

| Line Current | Lead/lag of line current |
| :---: | :---: |
| $\mathrm{I}_{\mathrm{A}}=19.6962506 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}=+42.5929^{\circ}$ |
| $\mathrm{I}_{\mathrm{B}}=14.413495 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}=+22.9263^{\circ}$ |
| $\mathrm{I}_{\mathrm{C}}=12.762278 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}=+55.6244^{\circ}$ |

According to Equation 4C3b,

$$
\begin{gathered}
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right. \\
\left.\mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \\
\mathrm{P}_{\mathrm{T}}=(480)(1 / \sqrt{3})\left[(19.6962506) \cos \left(42.5929^{\circ}-30^{\circ}\right)+\right. \\
(14.413495) \cos \left(22.9263^{\circ}-30^{\circ}\right)+ \\
(12.762278) \cos \left(55.6244^{\circ}-30^{\circ}\right] \\
\mathrm{P}_{\mathrm{T}}=(277.12812)\left[(19.6962506) \cos \left(12.5929^{\circ}\right)+\right. \\
(14.413495) \cos \left(-7.0737^{\circ}\right)+ \\
\left.(12.762278) \cos \left(25.6244^{\circ}\right)\right] \\
\mathrm{P}_{\mathrm{T}}=(277.12812)[19.22243+14.30378+11.50719] \\
\mathrm{P}_{\mathrm{T}}=(277.12812)[45.0334]=12480.02 \text { watts } \approx 12480 \text { watts }
\end{gathered}
$$

Thus, it is seen that the computation using line parameters yields the same result that was obtained using phase parameters as demonstrated in Example 7.

## 4D. Calculating Power in a Three Phase Circuit with Mixed Wye and Delta Loads

## 4D1. General

It is common to find a three phase feeder that serves a mix of both wye and delta three phase circuits. There could also be single phase loads taken from any two of the phases. If data is available for all of the users then, obviously, the total power consumption is the sum of the power of all of the users. This total could be determined from either phase parameters or line parameters of the users. It can also be determined if adequate line data is available for the common feeder.

## 4D2. Using Phase Data

For wye loads the power can be determined by one of the applicable equations presented above for wye loads using phase data. Similarly the individual loads for delta circuits can be determined with an applicable delta equation. This would also be true of single phase loads. The total power is then the sum of the individual loads.

## 4D3. Using Line Data

Above, the various equations are treated for both balanced and unbalanced circuits including both wye and delta circuits. If the type of load is known then the applicable equation may be selected. What if, on the other hand, the type of load is unknown? Or, what if the load is a mix of wye, delta and single phase? The answer is relatively simple!

It may be noted that of all the equations of power, those that define power from line parameters are identical for both wye and delta circuits. The general form for power as determined from line parameters is that which is defined by Equation 4B3b or 4C3b which are identical:

$$
\begin{array}{r}
\mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\right. \\
\left.\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]
\end{array}
$$

The same general rule is applicable to those above equations that use phase values. Of course, the numerical value of phase potential in a particular circuit will be different from the numerical value of line potential.

## 5. Measuring Power in a Single Phase Circuit

## 5A. General

Power in a single phase circuit is typically measured with a wattmeter. By one means or another, a single phase wattmeter measures current, voltage and the lead or lag of the current with respect to the measured voltage. The wattmeter essentially performs the calculation,
$\mathrm{P}=\mathrm{VI} \cos \theta_{\mathrm{P}}$, where
$\mathrm{V}=$ potential (voltage)
$\mathrm{I}=$ current (amps)
$\theta_{\mathrm{P}}=$ phase lead/lag angle between phase current and phase voltage (degrees or radians) (for lagging current, $\theta_{\mathrm{P}}<0$; for leading current, $\theta_{\mathrm{P}}>0$ ) In a single phase circuit, $\theta_{\mathrm{P}}>-90^{\circ}$ and $\theta_{\mathrm{P}}<+90^{\circ}$


The Amprobe Model ACD-41PQ clamp-on meter provides a convenient means to measure single phase current, power or power factor without the need to disconnect a conductor

## 6. Measuring Power in Three Phase Three Wire Circuits

## 6A. General

A customer of a three phase electrical service will have a permanently installed three phase watt-hour meter that is owned by the utility and provided for billing purposes. Depending on the area and utility policies additional meters may be provided to measure and record power factor and load "demand." The watt-hour meter integrates watts with time to determine total energy consumer by the customer within a specific period of time. The user will then be billed for energy consumed within that period of time. Aside from metering conducted by the utility a customer may have need to conduct independent metering of power for a variety of purposes. Spot metering may be done with portable meters that are used temporarily and then stored when not in use. Or, permanently installed meters may be used for continuous monitoring. A permanently installed meter could typically be the panel mounted type.

Temporary measurements of a three phase three wire circuit may be conducted by any one of several methods. The task can be carried out by means of a single phase watt meters. This option might be selected if, for example, a three phase meter is not conveniently available. Three phase power can be measured with one single phase wattmeter, two single phase wattmeters, three single phase wattmeters, a three phase wattmeter, a power quality meter, a power analyzer or other instruments. The selection of meters and the method would depend on several conditions, including:
a. Types and number of meters on hand.
b. Balanced or unbalanced load
c. Rate of change of load.
d. Frequency at which readings are to be made, and
e. Required accuracy.

If only one single phase wattmeter is on hand, there is only an occasional need for wattage measurements and the loads change slowly, one single phase wattmeter can usually provide adequate results. At the other end of the spectrum where there is a frequent need for measurements and the loads
change rapidly, a three phase meter, a power quality meter or a power analyzer may be the better selection.
When taking readings of power with a power meter it is important to know the phase sequence of the conductors. If the conductors under consideration followed a color coding, that color coding would be the first clue of the phase sequence. However, color coding around the world vary. (Frequently in the USA the sequence 'brown-orange (or violet)-yellow' is followed for three phase circuits 480 VAC conductors. Often black conductors are used and the ends taped with colored tape.) On the other hand, color coded conductors may not accurately reflect the true phase sequence. Conductors at voltages above 480 VAC are not generally color coded. When in doubt, the phase sequence should be checked with a phase sequence meter. Phase sequence meters are especially helpful to confirm that a three phase motor has been wired properly. For example, a large motor may require conductors A-B-C to be wired respectively to terminals T1-T2-T3, T2-T3-T1 or T3-T1-T2 to ensure proper rotation of the motor. If the source sequence is different, as A-C-B, the motor will rotate in reverse direction to the intended rotation. In some cases a motor that is started in a reverse direction may result in significant property damage and, conceivably, injury to personnel. When in doubt the sequence should be confirmed. The various, possible methods of measuring three phase power with single phase meters are treated below.
If a person understands how single phase meters can be used to measure power, it then becomes much easier to understand how a three phase power meter operates. Power can also be determined by any means that permits measuring current, voltage and current lead/lag.
It is pertinent to note that for at least the first 100 years following the use of electrical power, power usage was measured by means of meters that contained coils within the casing of the meter. Power was measured by electromechanical current and voltage coils located within the casing of the meter. Today, power is being measured more and more by digital meters that measure current with external current transformers (CT's). Voltages are brought to the meter and power is computed by digital algorithms.

## 6B. Using One Single Phase Wattmeter to Measure Power in a Three Phase Three Wire Circuit

A possible procedure to measure the total circuit power of a balanced three phase three wire circuit would involve the measurement of the power of only one phase. Since, by definition, all three phases have equal power consumption, the total power consumption is triple the power measurement of any one phase. This procedure would, of
 course, require access to the conductors that carry the phase currents as well as the conductors that deliver voltage potentials to the phases.
A procedure for measuring the phase power of a balanced delta circuit is shown in Fig. 10. The phase potentials of a delta circuit are usually accessible but the conductors carrying the phase currents may not be conveniently accessible. Only one phase needs to be measured since, by definition, all three phases are of equal power. The total power consumption of the circuit is triple the power of one phase.

A procedure for measuring the phase power of one phase of a balanced wye circuit is shown in Fig. 11. However, in many instances the measurements depicted in Fig. 11 may not be physically practical. Generally the current of a wye user may be measured since the phase current is the same as the line current. However, access to the point where the phases tie together (Point "d" in Fig. 11) may not be readily accessible.
Under some circumstances, a single portable wattmeter can be used to obtain adequate readings in an unbalanced three phase three wire circuit. As mentioned above, this arrangement would be practical only if the measured
parameters remain constant throughout the period of time during which the measurements are made. The arrangement of Fig. 12 shows a typical configuration that could be used to determine total power with a single meter. The meter is first connected as shown in Position \#1. The current in Conductor A is measured as well as voltage $\mathrm{C}-\mathrm{A}$. A reading is taken and the meter is then moved to Position \#2. In position \#2 the current in Conductor C is measured as well as voltage C-B. The two readings are added to determine


Fig. 11
Measuring Power of a Wye Circuit


Fig. 12 - Use of Single Wattmeter
total circuit power. Alternate positions for measuring total power would involve the use of different conductors as explained below in Section 6C. The method to use only two wattmeters to measure the total power of a three phase three wire circuit is commonly called the "two wattmeter" which is discussed in more detail below.

## 6C. Using Two Single Phase Wattmeters to Measure Power in a Three Phase Three Wire Circuit

An effective and practical means of measuring power in unbalanced three phase three wire circuits involves the use of only two wattmeters in what is commonly called the "two wattmeter method." Total power is determined at any selected instant by the sum of the readings indicated on the two wattmeters. The wattmeters measure power by measuring line parameters, i.e. line potentials and currents, rather than phase parameters. The two wattmeters are to be located in the three phase
 lines that lead to
the load as depicted in Fig. 13. Only two currents are measured and the voltages are measured as shown in the figure. In Fig. 13 the two wattmeters are designated Meter \#1 and Meter \#2. In one possible configuration, Meter $\# 1$ measures the current in Conductor A as well as voltage C-A. The positive voltage lead of Meter \#1 is connected to Phase A and the negative voltage lead is connected to Phase C. Meter \#2 measures current in Conductor B as well as voltage C-B. The positive voltage lead is connected to Phase B and the negative lead is connected to Phase C. An alternate two meter configurations are shown in Fig. 14 where the current coils are shown in Phase A and Phase C. In fact, the current measurements can be in any two of the three phases: $\mathrm{A} \& \mathrm{~B}, \mathrm{~A} \& \mathrm{C}$ or $\mathrm{B} \& \mathrm{C}$. The positive potential leads are in contact with the respective phase conductor and the negative leads joined together and in
contact with any voltage. Proof of the two wattmeter method for three phase three wire circuits is presented in Appendix D along with proof of the three wattmeter method discussed below.

While an acceptable
 method of measuring total power of a three phase circuit, the two wattmeter method has certain limitation. If the (line) current measured by Meter \#1 (of Fig. 13 or Fig. 14) leads the reference potential of the meter by less than $90^{\circ}$, Meter\#1 will correctly indicate the value of measured power. (A line current that leads potential by $90^{\circ}$ would correspond to a power factor in a balanced circuit of 0.50 .) However, when line current leads the measured potential by more than $90^{\circ}$ problems may ensue depending on the type of meter used. If the wattmeter is of the electromechanical type, i.e. one that uses internal coils to determine power, beyond $90^{\circ}$ there would be no indication of power level. In this case, a reading may be obtained by reversing potential leads and subtracting the reading of Meter \#1 from the reading of Meter\#2 since the reading of Meter \#1 would be a negative value. Some digital type wattmeters would indicate a negative reading in which case there would be no need to reverse leads.

It is important to note that with the two wattmeter method of measuring circuit power it is possible to have some 20 or more possible incorrect connections. Yet there is only one set of connections that will provide correct readings. For this reason, it is important to ensure that all CT (or coil) connections and potential connections are made correctly.

## 6D. Using Three Single Phase Wattmeters to Measure Power in a Three Phase Three Wire Circuit

It is possible, and at times desirable, to measure all three line currents of a three phase three wire circuit along with the three sets of line potentials. This might be the case, for example, if the three line currents are to be monitored in addition
 to total circuit power.

In the case of an unbalanced delta circuit, total power consumption can be determined by measuring the power of each phase. This could be done, typically, with the configuration represented in Fig. 15. The total power consumption of the circuit would be the sum of the readings of the three wattmeters. As indicated in Fig. 15, the current transformers are located in each of the three phases. However, in many instances the arrangement of Fig. 15 is not practical since it is often not convenient to read the currents of the phases of a delta circuit.

In the case of a three phase three wire wye circuit, three single phase wattmeters could be used in some cases to measure total power in the configuration of Fig. 16. The total circuit power is the sum of the phase powers. It is to be noted that if the user is, say, configured in a wye configuration, as would be the case with a wye wound motor, it would generally not be possible to access the "d" point represented in Fig. 16. So, in
practice, there are many three phase four wire use three wattmeters as represented in Fig. 16.

An alternate method of measuring circuit power involves using three separate single phase wattmeters to measure line parameters as represented in Fig. 17. The users could be delta, wye or any mix thereof. The positive potential lead of each of the meters is be in contact with the respective phase in which the CT is located. The negative


Fig. 16
Measuring Power of a Wye Circuit
 leads of the
instruments are joined together. Often the negative leads are connected to ground although a connection to ground is not necessary.

The configuration represented in Fig. 17 is an effective method of measuring the total power of a three phase three wire circuit. If only total circuit power is needed the use of three wattmeters as depicted in Fig. 17 offers no advantage over the two wattmeter method treated in above Section 6C. In fact, the three
wattmeter method requires three current measurements instead of the two of the two wattmeter method. On the other hand, the use of three CT's allows monitoring of the line currents as well. It is pertinent to note that the use of a single three phase wattmeter, rather than the use of three single phase wattmeters, would generally be a more practical means to measure circuit power of a three phase three wire circuit.

At first, it may seem strange to a reader that with the use of three wattmeters as depicted in Fig. 17 all three negative leads of the instruments are joined together. This subject is discussed further in Appendix D where it is explained that, in fact, this is the correct procedure. Regardless of the voltage of the negative leads, the sum of the measurements of the wattmeters is the total circuit power. In addition, if the three phase three wire circuit is derived from a three phase four wire circuit and the negative leads of the three wattmeters are connected to ground potential, the measurements of each wattmeter will indicate the respective power delivered by that phase, i.e. Phase A, Phase B and Phase C.

## 6E. Using a Three Phase Wattmeter to Measure Power in a Three Wire Three Phase Circuit

Above it was demonstrate how single phase wattmeters may be used to measure the power of a three wire three phase circuit. If a person understands the principles involved in applying single phase wattmeters to measure power, it then becomes much easier to understand the principles upon which a three phase wattmeter operates. In fact a three phase wattmeter would almost always be the preferred method to measure three phase power. And, a three phase watt-hour meter would be the preferred means for measuring three phase energy, i.e. watt-hours.

A number of instruments are available for monitoring (and in some cases recording) three phase power. The various instruments for measuring three phase power are available in a variety of forms. There are hand held wattmeters, power analyzers and power quality meters. Some of these
instruments are intended to be readily portable and held by hand. Some are less portable and intended primarily for mounting on a bench. Some are intended for only permanent mounting in a panel.

Three phase power


Fig. 18-3ф3W Wattmeter meters function by, first, making separate, single phase measurements of currents and potentials. Depending on the application and the capability of the instrument, the single phase measurements are then combined by algorithms within the envelope of the instrument to produce a variety of values. The available parameters may include: phase currents, phase voltages, total power, phase powers, VAR, reactive power and apparent power.

If only total circuit power of a balanced three phase three wire circuit is to be determined, a single current measurement will suffice. For unbalanced circuits, a minimum of two current measurements are necessary to establish a value of total circuit power. If each of the three phases are to be monitored for currents as well as power, then three current measurements are required. For this reason, wattmeters are available for use with one, two or three CT's. The possible configurations using a single instrument are depicted in Fig. 18. A summary of the possible configurations, the number of CT's required and the measurements of total power and line currents that can be provided are summarized in Table 6E-1.

The algorithm used within a wattmeter to compute the various parameters will be dependent on the number of CT's used to measure currents. The algorithms
applicable to the various configurations for three phase three wire circuits are treated in above Sections 3 and 4.

Table 6E-1
Power and Current Values Available - Three Wire Circuit

| Type Load | Total Power | Line <br> Currents | Number of <br> CT's Required |
| :--- | :---: | :---: | :---: |
| Balanced | $\bullet$ | $\bullet$ | 1 |
| Unbalanced | $\bullet$ |  | 2 |
| Unbalanced | $\bullet$ | $\bullet$ | 3 |

## 7. Measuring Power in a Three Phase Four Wire Circuit

## 7A. General

One single phase wattmeter can be used to measure total power in a balanced three phase four wire circuit or in an unbalanced three phase four wire circuit if the loads are steady. The two wattmeter method described in Section 6 for the three phase three wire balanced or unbalanced circuits cannot be used to measure power in a three phase four wire unbalanced circuit. For unbalanced three phase four wire circuits a minimum of three single phase meters would be required. On the other hand a three phase meter would generally be the better choice.

## 7B. Using One Single Phase Wattmeter to Measure Power in a Three Phase Four Wire Circuit

One single phase wattmeter can be used to measure the total circuit power of a three phase four wire circuit. In order to obtain an accurate measurement of total power the measured parameters must remain relatively constant throughout the period during which the measurements are made. Otherwise accurate readings may not be possible.

The total circuit power is the sum of the phase powers. It is noted that if the user is, say, configured in a wye configuration, as would be the case with a wye would motor, it would generally not be possible to access the "d" point represented in Fig. 11. So, in practice, there are many applications that could not use three wattmeters as represented in Fig. 11. The more common usage of three wattmeters in a wye circuit would be in the configuration of Fig. 12.

## 7C. Using Three Single Phase Wattmeters to Measure Power in a Three Phase Four Wire Circuit

In above Section 6D it was demonstrated that the power of a three phase three wire circuit may be determined by measuring the power of each of the phases, whether the circuit is a delta circuit or a wye circuit. Of course, the same remains true for a three phase four wire circuit - delta or wye. As with a three phase three wire circuit the power of a three phase four wire circuit may be measured by measuring line parameters. And, actually, that is generally the better way to do it.

Three single phase wattmeters can be used to accurately measure the total power of a three phase four wire circuit. The CT's and potential measurements would typically be made as shown in Fig. 19.


Fig. 19-3 $\$ 4$ W Measurement

## 7D. Using a Three Phase Wattmeter to Measure Power in a Three Phase Four Wire Circuit

If a three phase four wire circuit is balanced, then total power may be determined from a single current measurement. However, most three phase four wire circuits are unbalanced. For this reason three current readings are normally required to measure total power. With a single three phase meter to measure total power the meter would be


Fig. 20-3 34 W Wattmeter connected as represented in Fig. 20.

A summary of the possible configurations, the number of CT's required and the measurements of total power and line currents that can be provided are summarized in Table 7D-1.

| Table 7D-1 |  |  |  |
| :--- | :---: | :---: | :---: |
| Power and Current Values Available - Four Wire Circuit |  |  |  |
| Type Load | Total Power | Line <br> Currents | Number of <br> CT's Required |
| Balanced | $\bullet$ | $\bullet$ | 1 |
| Unbalanced | $\bullet$ | $\bullet$ | 3 |

## 8. Power Analyzers and Power Quality Meters

For many years instruments described as "power analyzers" have been the traditional means of examining the characteristics of a three phase power source. Power analyzers come in a wide variety of shapes, sizes, prices and with a wide variety of features. In general, power analyzers can provide numerous details characteristic of a circuit including parameters as currents, voltages, power factors, power, VARS, VA, frequency, phase sequence and lead/lag angle. Some can show a circuit's phasor diagrams. Information at a power analyzer is generally available on a display on the instrument and some power analyzers can transmit data serially over a cable to remote locations. Many power analyzers can be used to make a print-out of the measured parameters. Although more costly than wattmeters, in many instances the greater cost of a power analyzer is justified by the greater amount of information that it provides. More or less by custom the term "power analyzer" is applicable to either hand held or bench mounted instruments that are portable. Most power analyzers measure currents by means of clamp-on CT's. In general power analyzers are relatively expensive instruments. In recent years digital "power quality" meters have become available for the continuous monitoring of three phase power sources. Many of the new generations of digital power quality meters are intended for permanent mounting in a panel. Unlike portable power analyzers current measurements for the panel mounted power quality meters are made with permanently installed CT's. Many of the newer power quality meters can provide data comparable to that obtained with the traditional power analyzers. And the digital power quality meters are generally much less expensive than the traditional power analyzers.


The GIMA Three Phase Digital Panel Meter by Simpson Electric allows monitoring of a number of 3Ф3W and 3Ф4W electrical properties including voltages, currents, power, VA,

VAR, energy and many other significant parameters

## 9. Summary of Symbols and Equations

Following is a summary of the symbols and equations used in this course. (Note: Equations that are helpful for calculating currents are presented in Appendix F.)

## 9A. Symbols

Following is a summary of the symbols used in this course:
$\Delta=$ symbol for (three phase) delta loads
$\Phi-$ Symbol for "phase" (e.g. $3 \Phi=$ three phase)
$\nu^{i}=$ instantaneous voltage (volts)
$v_{X}^{i}=$ value of instantaneous voltage $v^{i}$ at time " X " (volts)
$\boldsymbol{\nu}_{\mathrm{XY}}=$ instantaneous value of voltage measured from "X" to " Y " (volts)
$\nu_{\mathrm{PK}}=$ peak value of instantaneous voltage $\nu^{i}$ (volts)
$\mathrm{V}(t)=$ voltage expressed as a function of time (rms volts)
$\mathrm{V}(t)_{\mathrm{ab}}=$ voltage A-B expressed as a function of time (rms volts)
$\mathrm{V}=$ (absolute) numerical value of DC voltage or (rms) AC voltage (volts)
$\mathrm{V}_{\mathrm{L}}=$ line voltage (rms volts - used in reference to a three phase source)
$\mathrm{V}_{\mathrm{L}-\mathrm{XY}}=$ line voltage between phase X and phase Y (rms volts - used in
reference to a three phase source)
$\mathrm{V}_{\mathrm{P}}=$ voltage (rms volts - used in reference to a phase of a three phase load)
$\mathrm{V}_{\mathrm{XY}}=\mathrm{AC}$ voltage with positive direction measured from " X " to " Y "
(rms volts or voltage vector)
$i_{i}^{i}=$ instantaneous current (amps)
$i_{X}^{i}=$ value of instantaneous current $i^{i}$ at time " X " (amps)
$i_{\mathrm{PK}}=$ peak value of instantaneous current $i^{i}$ (amps)
$\mathrm{I}=$ (absolute) numerical value of DC current or (rms) AC current (amps)
$\mathrm{I}(t)=$ current expressed as a function of time (rms amps)
$\mathrm{I}(t)_{\mathrm{ab}}=$ current A-B expressed as a function of time (rms amps)
$\mathrm{I}_{\mathrm{L}}=$ line current (rms amps) (used in reference to a three phase source)
$\mathrm{I}_{\mathrm{L}-\mathrm{X}}=$ current in line conductor X (rms amps)
$\mathrm{I}_{\mathrm{P}}=$ phase current (rms amps) (used in reference to a three phase load) $\mathrm{I}_{\mathrm{P}-\mathrm{XY}}=$ current in Phase X-Y (rms amps)
$\mathrm{I}_{\mathrm{XY}}=\mathrm{AC}$ current vector with positive direction measured from " X " to "Y" (rms amps or current vector)
$L=$ general representation of an electrical load (which could be resistive, capacitive, inductive or any combination thereof)
$\mathrm{R}=$ electrical resistance (ohms)
$\mathrm{P}=$ electrical power (watts)
$P_{X Y}=$ electrical power in circuit "X-Y" (watts)
$\mathrm{P}^{l}=$ instantaneous power (watts)
$t=$ time (seconds)
$t_{\mathrm{X}}=$ time at " X " (seconds)
$\mathrm{f}=$ frequency (hz)
$\theta_{\mathrm{SP}}=$ single phase lead/lag angle between current and voltage (degrees)
$\theta_{\mathrm{L}}=$ lead/lag angle between line current and line voltage (degrees or radians)(for leading current, $\theta_{\mathrm{L}}>0$; for lagging current, $\theta_{\mathrm{L}}<0$ )
$\theta_{\mathrm{L}-\mathrm{CA}}$ - lead/lag of line current A with respect to line voltage C-A
$\theta_{\mathrm{L}-\mathrm{AB}}-$ lead/lag of line current B with respect to line voltage A-B
$\theta_{\mathrm{L}-\mathrm{BC}}-$ lead/lag of line current C with respect to line voltage B-C
$\theta_{\mathrm{P}}=$ phase lead/lag angle between phase current and phase voltage (degrees/radians)(for leading current, $\theta_{\mathrm{P}}>0$; for lagging current, $\theta_{\mathrm{P}}<0$ ) $\theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=$ lead $/$ lag of phase current A with respect to line voltage A-D
$\theta_{\mathrm{P}-\mathrm{B} / \mathrm{BD}}=$ lead/lag of phase current B with respect to line voltage B-D
$\theta_{\mathrm{P}-\mathrm{C} / \mathrm{CD}}=$ lead/lag of phase current C with respect to line voltage C-D
$\mathrm{PF}=$ power factor $=\cos \theta_{\mathrm{P}}$ (for balanced delta or balanced wye loads) $\omega=2 \pi \mathrm{f}$ (radians)

## 9B. Equations

Below is a summary of applicable equations used in this course.

Equation 101- Instantaneous voltage expressed as a function of time:
$v^{i}=\left(v_{\mathrm{PK}}\right) \sin \omega t \ldots$ Equation 101

Equation 102 - Instantaneous current expressed as a function of time: $i^{i}=i_{\mathrm{PK}} \sin \left(\omega t+\theta_{\mathrm{SP}}\right) \ldots$ Equation 102
$\qquad$
Equation 103 - RMS voltage as a function of time:
$\mathrm{V}(t)=\mathrm{V} \sin (\omega t) \ldots$ Equation 103

Equation 104 - RMS current as a function of time:
$\mathrm{I}(t)=\mathrm{I} \sin \left(\omega t+\theta_{\mathrm{SP}}\right) \ldots$ Equation 104

Equation 105 - Instantaneous power as a function of time:
$\mathrm{P}^{i}=\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)+\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$

## ... Equation 105

Equation 106 - Alternate expression of instantaneous power as a function of time:
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta_{\mathrm{SP}}-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \left(2 \omega t+\theta_{\mathrm{SP}}\right) \ldots .$. Equation 106

Equation 107 - General expression of power in a single phase AC circuit $\mathrm{P}=\mathrm{VI} \cos \theta_{\mathrm{SP}} \ldots$ Equation 107

Equation 108 - Computation of power in a balanced three phase wye circuit: $\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}} \quad \ldots$ Equation 108

Equation 109 - Relationship between phase voltage and line voltage in a wye circuit:
$V_{L}=(\sqrt{3}) V_{P} \ldots$ Equation 109

## Summary of Three Phase Power Equations

| Three Phase Wye Circuit Power Equations |  |  |
| :---: | :---: | :---: |
| Type Circuit | Equation <br> Number | Phase parameters* |
|  |  | Line parameters |
| Balanced Resistive | 3B2a | $\mathrm{P}_{\mathrm{T}}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}}$ |
|  | 3B2b | $\mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}}$ |
| Balanced Inductive or Capacitive | 3B3a | $\mathrm{P}_{\mathrm{T}}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$ |
|  | 3B3b | $\mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)$ |
| Unbalanced Resistive | 4B2a | $\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right]$ |
|  | 4B2b | $\begin{aligned} & \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\right. \\ & \left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \end{aligned}$ |
| Unbalanced Inductive or Capacitive | 4B3a | $\begin{aligned} & \mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{AD}}+\mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{BD}}+\right. \\ & \left.\mathrm{I}_{\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{CD}}\right] \end{aligned}$ |
|  | 4B3b | $\left[\begin{array}{l} \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\right. \\ \left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \end{array}\right.$ |

${ }_{+}^{*}$ - In a wye circuit, $\mathrm{V}_{\mathrm{P}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}$

- The equation number is the same as the associated paragraph number.

Three Phase Delta Circuit Power Equations

| Type Circuit | Equation <br> Number ${ }^{+}$ | Phase parameters** |
| :---: | :---: | :---: |
|  |  | Line parameters |
| Balanced Resistive | 3C2a | $\mathrm{P}_{\mathrm{T}}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}}$ |
|  | 3C2b | $\mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}}$ |
| Balanced Inductive or Capacitive | 3C3a | $\mathrm{P}_{\mathrm{T}}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$ |
|  | 3C3b | $\mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right)$ |
| Unbalanced Resistive | 4C2a | $\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{P}-\mathrm{BC}}+\mathrm{I}_{\mathrm{P}-\mathrm{CA}}\right]$ |
|  | 4C2b | $\begin{aligned} & \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\right. \\ & \left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \end{aligned}$ |
| Unbalanced Inductive or Capacitive | 4C3a | $\begin{aligned} & \mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{P}}\left[\mathrm{I}_{\mathrm{A}-\mathrm{B}} \cos \theta_{\mathrm{P}-\mathrm{AB}}+\mathrm{I}_{\mathrm{B}-\mathrm{C}} \cos \theta_{\mathrm{P}-\mathrm{BC}}+\right. \\ & \left.\mathrm{I}_{\mathrm{C}-\mathrm{A}} \cos \theta_{\mathrm{P}-\mathrm{CA}}\right] \end{aligned}$ |
|  | 4C3b | $\begin{aligned} & \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\right. \\ & \left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \end{aligned}$ |

$\stackrel{* *}{+}$ - In a delta circuit $\mathrm{V}_{\mathrm{P}}=\mathrm{V}_{\mathrm{L}}$
${ }^{+}$- The equation number is the same as the associated paragraph number.

# Three Phase Delta, Wye or Mixed Circuit Power Equations ---Using Line Parameters--- 

\(\left.$$
\begin{array}{|c|c|}\hline \begin{array}{c}\text { Type } \\
\text { Circuit }\end{array} & \text { Equation } \\
\hline \begin{array}{c}\text { Balanced } \\
\text { Resistive }\end{array} & \mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \\
\hline \begin{array}{c}\text { Balanced } \\
\text { Inductive or } \\
\text { Capacitive }\end{array} & \mathrm{P}_{\mathrm{T}}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \left(\theta_{\mathrm{L}}-30^{\circ}\right) \\
\hline \begin{array}{c}\text { Unbalanced } \\
\text { Resistive }\end{array}
$$ \& \mathrm{P}_{\mathrm{T}}=(1 / \sqrt{3}) \mathrm{V}_{\mathrm{L}}\left[\mathrm{I}_{\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\right. <br>

\left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]\end{array}\right]\)|  |
| :---: |
| Unbalanced <br> Inductive or <br> Capacitive |
| $\left.\mathrm{I}_{\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right]$ |

## 10. References

1. Electrical Engineering Fundamentals, $2^{\text {nd }}$ Ed., V. Del Toro, PrenticeHall, pp 305-314
2. Electrical Fundamentals. $2^{\text {nd }}$ Ed., J.J. DeFrance, Prentice-Hall, pp 604-621
3. Calculating Currents in Balanced and Unbalanced three Phase Circuits, Joseph E. Fleckenstein, PDH Online Course E336,

## Appendix A - Demonstration that Equations 105 is the equivalent of Equation 106

Equation 105 states,
$\mathrm{P}^{i}=\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)+\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$
... Equation 105

A common form of instantaneous power found in texts on the subject of three phase power is:
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos (2 \omega t+\theta) \ldots .$. Equation 106
These two equations are equivalent as demonstrated below.
Reference: Handbook of Mathematical Tables and Formulas, P. 18:

$$
\cos (2 \omega t+\theta)=\cos 2 \omega t \cos \theta-\sin 2 \omega t \sin \theta
$$

Thus, Equation 106 becomes,
$\mathrm{P}_{i}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\cos 2 \omega t \cos \theta-\sin 2 \omega t \sin \theta]$
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\cos 2 \omega t \cos \theta]$
$+\left[\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\sin 2 \omega t \sin \theta]$
Since, $\cos 2 \omega t=1-2 \sin ^{2} \omega t$
$\left.\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right]\left[1-2 \sin ^{2} \omega t\right] \cos \theta\right]$
$+\left[\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\sin 2 \omega t \sin \theta]$
$\mathrm{P}^{i}=\left[\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta$

$$
+\left[\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right]\left[2 \sin ^{2} \omega t \cos \theta\right]
$$

$+\left[\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\sin 2 \omega t \sin \theta]$
$\mathrm{P}^{i}=\left[\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\right] \cos \theta \sin ^{2} \omega t+\left[\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][\sin 2 \omega t \sin \theta]$
Reference: Handbook of Mathematical Tables and Formulas, P. 18: $\sin 2 \omega t=2 \sin \omega t \cos \omega t$
$\mathrm{P}_{i}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\right] \cos \theta \sin ^{2} \omega t+\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right][(2 \sin \omega t \cos \omega t) \sin \theta]$
$\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\right] \cos \theta \sin ^{2} \omega t+\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\right][(\sin \theta)(\sin \omega t)(\cos \omega t)]$,
Q.E.D.
(Note: In this course, instantaneous current is described by the equation $i^{i}=i_{\mathrm{PK}} \sin \left(\omega t+\theta_{\mathrm{SP}}\right)$. If current leads voltage then $\theta_{\mathrm{L}}>0$ and if current lags
voltage $\theta_{\mathrm{L}}<0$. However, many texts on the subject describe instantaneous current as $i_{\mathrm{PK}} \sin \left(\omega t-\theta_{\mathrm{SP}}\right)$ in which case $\theta_{\mathrm{L}}<0$ for a leading current and $\theta_{\mathrm{L}}>0$ for a lagging current. If instantaneous current is assumed to be described by the expression $i_{\mathrm{PK}} \sin \left(\omega t-\theta_{\mathrm{SP}}\right)$ then the corresponding equivalent equations to Equations 105 and 106 become, respectively: $\mathrm{P}_{i}^{t}=\left(v_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\cos \theta_{\mathrm{SP}}\right)\left(\sin ^{2} \omega t\right)-\left(V_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right)\left(\sin \theta_{\mathrm{SP}}\right)(\sin \omega t)(\cos \omega t)$, and $\mathrm{P}^{i}=\left[\left(\mathcal{V}_{\mathrm{PK}}\right)\left(i_{\mathrm{PK}}\right) / 2\right] \cos \theta-\left[(\mathcal{V F K})\left(i_{\mathrm{PK}}\right) / 2\right] \cos (2 \omega t-\theta)$

## Appendix B - Table of Computed Values of Instantaneous Power for Example 1

$$
\begin{aligned}
& \mathrm{P}^{t^{*}}=\mathrm{A}+\mathrm{B} \\
& \mathrm{~A}=(6720) \sin ^{2} \omega t ; \mathrm{B}=-(6900)\left[\sin \left(\cos ^{-1}\right)\right] \sin \omega t \cos \omega t
\end{aligned}
$$

| $\boldsymbol{\omega t}$ | Function <br> $\mathbf{A}$ | Function <br> $\mathbf{B}$ | $\mathbf{P}^{\boldsymbol{i}}=\mathbf{A +} \mathbf{B}$ |
| :--- | :--- | :--- | :--- |
| $0^{\circ}$ | 0 | 0 | 0 |
| $5^{\circ}$ | 51.04 | -595.24 | -544.20 |
| $10^{\circ}$ | 202.63 | -1172.40 | -969.77 |
| $15^{\circ}$ | 450.15 | -1713.94 | -1263.78 |
| $20^{\circ}$ | 786.09 | -2203.40 | -1417.31 |
| $25^{\circ}$ | 1200.23 | -2625.91 | -1425.67 |
| $30^{\circ}$ | 1680.00 | -2968.63 | -1288.63 |
| $35^{\circ}$ | 2210.81 | -3221.15 | -1010.34 |
| $40^{\circ}$ | 2776.54 | -3375.80 | -599.26 |
| $45^{\circ}$ | 3360.00 | -3427.88 | -67.88 |
| $50^{\circ}$ | 3943.45 | -3375.80 | 567.64 |
| $55^{\circ}$ | 4509.18 | -3221.15 | 1288.02 |
| $60^{\circ}$ | 5040.00 | -2968.63 | 2071.36 |
| $65^{\circ}$ | 5519.76 | -2625.81 | 2893.85 |
| $70^{\circ}$ | 5933.90 | -2203.40 | 3730.50 |
| $75^{\circ}$ | 6269.84 | -1713.94 | 4555.90 |
| $80^{\circ}$ | 6517.36 | -1172.40 | 5344.96 |
| $85^{\circ}$ | 6668.95 | -595.24 | 6073.70 |
| $90^{\circ}$ | 6720.00 | 0 | 6720.00 |
| $95^{\circ}$ | 6668.95 | 595.24 | 7264.20 |
| $100^{\circ}$ | 6517.36 | 1172.40 | 7689.77 |
| $105^{\circ}$ | 6269.84 | 1713.94 | 7983.78 |
| $110^{\circ}$ | 5933.90 | 2203.40 | 8137.31 |
| $115^{\circ}$ | 5519.76 | 2625.81 | 8145.67 |
| $120^{\circ}$ | 5040.00 | 2968.63 | 8008.63 |
| $125^{\circ}$ | 4509.18 | 3221.15 | 7730.34 |
| $130^{\circ}$ | 3943.45 | 3375.80 | 7319.26 |
| $135^{\circ}$ | 3360.00 | 3427.88 | 6787.88 |
| $140^{\circ}$ | 2776.54 | 3375.80 | 6152.35 |
| $145^{\circ}$ | 2210.81 | 3221.15 | 5431.97 |
| $150^{\circ}$ | 1680.00 | 2968.63 | 4648.63 |
| $155^{\circ}$ | 1200.23 | 2625.91 | 3826.14 |
| $160^{\circ}$ | 786.09 | 2203.40 | 2989.49 |
|  |  |  |  |


| $165^{\circ}$ | 450.15 | 1713.94 | 2164.09 |
| :--- | :--- | :--- | :--- |
| $170^{\circ}$ | 202.63 | 1172.40 | 1375.03 |
| $175^{\circ}$ | 51.04 | 595.24 | 646.29 |
| $180^{\circ}$ | 0 | 0 | 0 |
| $185^{\circ}$ | 51.04 | -595.24 | -544.20 |
| $190^{\circ}$ | 202.63 | -1172.40 | -969.77 |
| $195^{\circ}$ | 450.15 | -1713.94 | -1263.78 |
| $200^{\circ}$ | 786.09 | -2203.40 | -1417.31 |
| $205^{\circ}$ | 1200.23 | -2625.91 | -1425.67 |
| $210^{\circ}$ | 1680.00 | -2968.63 | -1288.63 |
| $215^{\circ}$ | 2210.81 | -3221.15 | -1010.34 |
| $220^{\circ}$ | 2776.54 | -3375.80 | -599.26 |
| $225^{\circ}$ | 3360.00 | -3427.88 | -67.88 |
| $230^{\circ}$ | 3943.45 | -3375.80 | 567.64 |
| $235^{\circ}$ | 4509.18 | -3221.15 | 1288.02 |
| $240^{\circ}$ | 5040.00 | -2968.63 | 2071.36 |
| $245^{\circ}$ | 5519.76 | -2625.81 | 2893.85 |
| $250^{\circ}$ | 5933.90 | -2203.40 | 3730.50 |
| $255^{\circ}$ | 6269.84 | -1713.94 | 4555.90 |
| $260^{\circ}$ | 6517.36 | -1172.40 | 5344.96 |
| $265^{\circ}$ | 6668.95 | -595.24 | 6073.70 |
| $270^{\circ}$ | 6720.00 | 0 | 6720.00 |
| $275^{\circ}$ | 6668.95 | 595.24 | 7264.20 |
| $280^{\circ}$ | 6517.36 | 1172.40 | 7689.77 |
| $285^{\circ}$ | 6269.84 | 1713.94 | 7983.78 |
| $290^{\circ}$ | 5933.90 | 2203.40 | 8137.31 |
| $295^{\circ}$ | 5519.76 | 2625.81 | 8145.67 |
| $300^{\circ}$ | 5040.00 | 2968.63 | 8008.63 |
| $305^{\circ}$ | 4509.18 | 3221.15 | 7730.34 |
| $310^{\circ}$ | 3943.45 | 3375.80 | 7319.26 |
| $315^{\circ}$ | 3360.00 | 3427.88 | 6787.88 |
| $320^{\circ}$ | 2776.54 | 3375.80 | 6152.35 |
| $325^{\circ}$ | 2210.81 | 3221.15 | 5431.97 |
| $330^{\circ}$ | 1680.00 | 2968.63 | 4648.63 |
| $335^{\circ}$ | 1200.23 | 2625.91 | 3826.14 |
| $340^{\circ}$ | 786.09 | 2203.40 | 2989.49 |
| $345^{\circ}$ | 450.15 | 1713.94 | 2164.09 |
| $350^{\circ}$ | 202.63 | 1172.40 | 1375.03 |
| $355^{\circ}$ | 51.04 | 595.24 | 646.29 |
| $360^{\circ}$ | 0 | 0 | 0 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Appendix C - Demonstration that Total Power of an Unbalanced Delta Circuit is Equivalent to that of an Assumed Wye Circuit

It is demonstrated in this appendix that the power of a delta circuit can be determined from line parameters by assuming that the user is a wye circuit instead of a delta circuit. The first step of the demonstration is to calculate the power of a delta circuit when the phase values of the delta circuit are known. Next, the line parameters of the delta circuit are calculated. It is then shown that if only the line currents are known, and not the phase currents, the total power consumption of the circuit may be determined. This end is accomplished by assuming the user to be a delta circuit. Two demonstrations are performed in the form of a confirming exercise and these are shown below, first, in Part 1, and secondly, in Part 2.

## Part 1 - First Demonstration that Power of an Unbalanced Delta Circuit is Equivalent to that of an Assumed Wye Circuit

The following conditions are assumed for an unbalanced delta circuit:
Line potential: 480-3-60 volts
$\mathrm{I}_{\mathrm{ab}}=5 \mathrm{amps} @ \mathrm{PF}=1.0$
$\mathrm{I}_{\mathrm{bc}}=10 \mathrm{amps} @ \mathrm{PF}=0.9$ lagging
$\mathrm{I}_{\mathrm{ca}}=15 \mathrm{amps} @ \mathrm{PF}=0.8$ leading
Find line currents in conductors A, B and C.
Solution:
$\theta_{\mathrm{P}-\mathrm{AB}}=\cos ^{-1} 1.0=0$
$\theta_{\text {P-BC }}=\cos ^{-1} 0.9=-25.84193^{\circ}$
$\theta_{\text {P-CA }}=-\cos ^{-1} 0.8=+36.86989^{\circ}$

These are the same line parameters that were assumed in Example 8 in Section 4C3b. Calculations repeated in that section determined that the line
parameters corresponding to the assumed phase parameters shown in the following summary.

Summary of calculated values for the delta circuit:

| Line Current | Lead/lag |
| :--- | :--- |
| $\mathrm{I}_{\mathrm{A}}=19.6962506 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}=+42.5929^{\circ}$ |
| $\mathrm{I}_{\mathrm{B}}=14.413495 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}=+22.9263^{\circ}$ |
| $\mathrm{I}_{\mathrm{C}}=12.762278 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}=+55.6244^{\circ}$ |

The power of the delta circuit is:
$\mathrm{P}_{\mathrm{C}-\mathrm{A}}=\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}-\mathrm{A} / \mathrm{CA}}=(480)(15)(.8)=5760$
$\mathrm{P}_{\mathrm{A}-\mathrm{B}}=\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}-\mathrm{B} / \mathrm{AB}}=(480)(5)(1)=2400$
$\mathrm{P}_{\mathrm{B}-\mathrm{C}}=\mathrm{V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}-\mathrm{C} / \mathrm{BC}}=(480)(10)(.9)=4320$
$\mathrm{P}_{\mathrm{T}}=5760+2400+4320=12480$ watts
As postulated, assume that the user is a wye circuit instead of a delta circuit:

Equation 4C3b is applicable.

$$
\begin{array}{r}
\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)\right. \\
\\
\left.+\mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \quad \ldots \quad \text { Eq. 4C3b }
\end{array}
$$

$$
\begin{aligned}
\text { Assume } \mathrm{V}_{\mathrm{L}}= & 480 \mathrm{VAC} \\
\mathrm{P}_{\mathrm{T}}= & (480)(1 / \sqrt{3}) \\
& {\left[(19.6962506) \cos \left(42.5929^{\circ}-30^{\circ}\right)\right.} \\
& (14.41349) \cos \left(22.9263^{\circ}-30^{\circ}\right) \\
& \left.+(12.76227) \cos \left(55.6244^{\circ}-30^{\circ}\right)\right] \\
\mathrm{P}_{\mathrm{T}}=(480)(1 / \sqrt{3}) & {\left[(19.696250) \cos 12.5929^{\circ}\right.} \\
& +(14.41349) \cos -7.0737^{\circ} \\
& \left.+(12.76227) \cos 25.6244^{\circ}\right] \\
\mathrm{P}_{\mathrm{T}}= & (277.1281)[(19.696250)(.97594) \\
& +(14.41349)(.992388) \\
& +(12.76227)(.90165)] \\
\mathrm{P}_{\mathrm{T}}= & (277.1281)[(19.22235)+14.30377+11.50710] \\
\mathrm{P}_{\mathrm{T}}= & (277.1281)[45.03322]=12479.97 \approx 12480 \text { watts }
\end{aligned}
$$

Thus, it is seen that the power calculation yields the same value as that which was determined by adding the wattages of the three parts of a delta circuit. This calculation tends to confirm that the power of a delta circuit can be determined by assuming the user to be a wye circuit. A second calculation is performed below as a second confirmation.

## Part 2 - Second Demonstration that Power of an Unbalanced Delta Circuit is Equivalent that of an Assumed Wye Circuit

Assume the following conditions for an unbalanced delta circuit:
Potential: 480-3-60 volts
$\mathrm{I}_{\mathrm{ab}}=10 \mathrm{amps}, \mathrm{PF}=.5$, lagging, $\theta_{\mathrm{P}-\mathrm{AB}}=-60.0000^{\circ}$
$\mathrm{I}_{\mathrm{bc}}=20 \mathrm{amps}, \mathrm{PF}=.6$, lagging, $\theta_{\mathrm{P}-\mathrm{BC}}=-53.1301^{\circ}$
$\mathrm{I}_{\mathrm{ca}}=30 \mathrm{amps}, \mathrm{PF}=.7$, lagging, $\theta_{\mathrm{P}-\mathrm{CA}}=-45.5729^{\circ}$
$\mathrm{P}_{\mathrm{ab}}=\mathrm{VI}(\mathrm{PF})=(480)(10)(.5)=2,400$ watts
$\mathrm{P}_{\mathrm{bc}}=\mathrm{VI}(\mathrm{PF})=(480)(20)(.6)=5,760$ watts
$\mathrm{P}_{\mathrm{ca}}=\mathrm{VI}(\mathrm{PF})=(480)(30)(.7)=10,080$ watts
Total delta power: 18,240 watts

Use the stated currents for the delta circuit and calculate the power if the currents and voltages were descriptive of a wye circuit. The first step is to calculate line currents for the delta circuit.
Reference: Equations 9-1 to 9-6 of Appendix F.
Determine $\mathrm{I}_{\mathrm{A}}$ :
$\mathrm{X}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}=-(10)(.5)=-5$
$\mathrm{X}_{\mathrm{ca}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$=-(30)(1 / 2)\left[(\sqrt{3}) \sin -45.5729^{\circ}+\cos -45.5729^{\circ}\right]$
$=-(30)(1 / 2)[(\sqrt{3})(-.714142843)+.700000]$
$=-(15)[-1.2369316+.70000]=-(15)[-.5369316]=8.05397531$
$\mathrm{X}_{\mathrm{A}}=\mathrm{X}_{\mathrm{ba}}+\mathrm{X}_{\mathrm{ca}}=-5+8.05397531=3.05397531$
$\mathrm{Y}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}=-(10) \sin -60.00^{\circ}=8.66025404$
$\mathrm{Y}_{\mathrm{ca}}=\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right]$

$$
\begin{aligned}
&=(30)(1 / 2)\left[(\sqrt{3}) \cos -45.5729^{\circ}-\sin -45.5729^{\circ}\right] \\
&=(15)[(\sqrt{3})(.7)-(-.714142843)]=(15)[1.21243+(.71414)] \\
&=(15)[1.9265784]=28.89867612 \\
& \mathrm{Y}_{\mathrm{A}}=\mathrm{Y}_{\mathrm{ba}}+\mathrm{Y}_{\mathrm{ca}}=8.66025404+28.89867612=37.55893016 \\
& \mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}^{1 / 2}=\left\{(3.05397531)^{2}+(37.55893016)^{2}\right\}^{1 / 2} \\
&=37.68288736 \mathrm{amps} \\
& \lambda= \sin ^{-1}\left(\mathrm{Y}_{\mathrm{A}} \div \mathrm{I}_{\mathrm{A}}\right)=\sin ^{-1}(37.55893016 \div 37.68288736)=\sin ^{-1}(.9967105) \\
& \mathrm{I}_{\mathrm{A}} \text { is in } \mathrm{Quadrant} \mathrm{I} \\
& \lambda= 85.3514173^{\circ} \\
& \theta_{\mathrm{L}-\mathrm{A}}=\left(\lambda-120^{\circ}\right)=\left(85.3514173^{\circ}-120^{\circ}\right)=-34.6485827^{\circ}
\end{aligned}
$$

Determine $\mathrm{I}_{\mathrm{B}}$ :
$\mathrm{X}_{\mathrm{ab}}=\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}=(10) \cos -60^{\circ}=(10)(.5)=5$
$\mathrm{X}_{\mathrm{cb}}=-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right]$

$$
=-(20)(1 / 2)\left[(\sqrt{3}) \sin -53.1301^{\circ}-\cos -53.1301^{\circ}\right]
$$

$$
=-(10)[(\sqrt{3})(-.8000)-(.600)]
$$

$$
=-(10)[-1.38564-(.60000)]=-(10)[-1.985640]=19.856406
$$

$\mathrm{X}_{\mathrm{B}}=\mathrm{X}_{\mathrm{ab}}+\mathrm{X}_{\mathrm{cb}}=5+19.85640=24.8564064$
$\mathrm{Y}_{\mathrm{ab}}=\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}=(10) \sin -60.000^{\circ}=(10)(-.86602)=-8.6602540$
$\mathrm{Y}_{\mathrm{cb}}=\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CB}}+\sin \theta_{\mathrm{P}-\mathrm{CB}}\right]$

$$
=(20)(1 / 2)\left[(\sqrt{3}) \cos -53.1301^{\circ}+\sin -53.1301^{\circ}\right]
$$

$$
=(10)[(\sqrt{3})(.6)+(-.8)]=(10)[1.0392304-.80000]=2.392304
$$

$\mathrm{Y}_{\mathrm{B}}=\mathrm{Y}_{\mathrm{ab}}+\mathrm{Y}_{\mathrm{cb}}=-8.660254+2.39230=-6.267954$
$\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2}=\left\{(24.856406)^{2}+(-6.267954)^{2}\right\}^{1 / 2}=25.634511 \mathrm{amps}$
$\theta_{\mathrm{L}-\mathrm{B}}=-\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right)=-\sin ^{-1}(-6.267954 \div 25.634511)$
$=-\sin ^{-1}(-.2445123)$
$\mathrm{I}_{\mathrm{B}}$ is in Quadrant IV
$\sin ^{-1}(-.2445)=-14.15301^{\circ}$
$\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$

Determine $\mathrm{I}_{\mathrm{C}}$ :
$\mathrm{X}_{\mathrm{bc}}=\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right]$

$$
=(20)(1 / 2)\left[(\sqrt{3}) \sin -53.1301^{\circ}-\cos -53.1301^{\circ}\right]
$$

$$
\begin{aligned}
&=(10)[(\sqrt{3})(-.80000)-(.60000)] \\
&=(10)[-1.385640-(-.60000)]=(10)[-1.98564]=-19.856406 \\
& \mathrm{X}_{\mathrm{ac}}=\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right] \\
&=(30)(1 / 2)\left[(\sqrt{3}) \sin -45.5729^{\circ}+\cos -45.5729^{\circ}\right] \\
&=(15)[(\sqrt{3})(-.7141428)+.70000]=(15)[-1.23693+.70000] \\
&=(15)[-.53693]=-8.053975 \\
& \mathrm{X}_{\mathrm{C}}= \mathrm{X}_{\mathrm{bc}}+\mathrm{X}_{\mathrm{ac}}=-19.856406-8.053975=-27.91038 \\
& \mathrm{Y}_{\mathrm{ac}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right] \\
&=-(30)(1 / 2)\left[(\sqrt{3}) \cos -45.5729^{\circ}-\sin -45.5729^{\circ}\right] \\
&=-(30)(1 / 2)[(\sqrt{3})(.70000)-(-.7141428)] \\
&=-(15)[1.2124355+.7141428]=-(15)[1.926578]=-28.898676 \\
& \mathrm{Y}_{\mathrm{bc}}=-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{BC}}+\sin \theta_{\mathrm{P}-\mathrm{BC}}\right] \\
&=-(20)(1 / 2)\left[(\sqrt{3}) \cos -53.130^{\circ}+\sin -53.1301^{\circ}\right] \\
&=-(10)[(\sqrt{3})(.60000)+(-.80000)]= \\
&=-(10)[1.039230-(.80000)]=-(10)[.2392304]=-2.392304 \\
& \mathrm{Y}_{\mathrm{C}}= \mathrm{Y}_{\mathrm{bc}}+\mathrm{Y}_{\mathrm{ac}}=-28.898676-2.392304=-31.29098 \\
& \mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2}=\left\{(-27.91038)^{2}+(-31.29098)^{2}\right\}^{1 / 2}=41.929878 \mathrm{amps} \\
& \varphi= \sin ^{-1}\left(\mathrm{Y}_{\mathrm{C}} \div \mathrm{I}_{\mathrm{C}}\right)=\sin ^{-1}(-31.29098 \div 41.929878)=\sin ^{-1}(-.746269) \\
& \mathrm{I}_{\mathrm{C}} \text { is } \text { in }_{\mathrm{Quadrant}} \mathrm{III} \\
& \varphi= \sin ^{-1}(-.746269)=228.2682^{\circ} \\
& \theta_{\mathrm{L}-\mathrm{C}}=\left(\varphi-240^{\circ}\right)=\left(228.2682^{\circ}-240^{\circ}\right)=-11.73176^{\circ}
\end{aligned}
$$

Summary of calculated values for the assumed delta circuit:

| Line Current | Lead/lag |
| :--- | :--- |
| $\mathrm{I}_{\mathrm{A}}=37.682887 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{A}}=-34.64858^{\circ}$ |
| $\mathrm{I}_{\mathrm{B}}=25.634511 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$ |
| $\mathrm{I}_{\mathrm{C}}=41.929878 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{C}}=-11.73176^{\circ}$ |

Assume that the calculated line currents and the respective lead/lag angles are for a wye circuit, and calculate the power.

Equation 4C3b is applicable.
$\mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{L}}(1 / \sqrt{3})\left[\mathrm{I}_{\mathrm{L}-\mathrm{A}} \cos \left(\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ}\right)+\mathrm{I}_{\mathrm{L}-\mathrm{B}} \cos \left(\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ}\right)\right.$

$$
\left.+\mathrm{I}_{\mathrm{L}-\mathrm{C}} \cos \left(\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}\right)\right] \quad \text {... } \quad \text { Eq. 4C3b }
$$

$\mathrm{P}_{\mathrm{T}}=(480)(1 / \sqrt{3})\left[(37.682887) \cos \left(-34.64858^{\circ}-30^{\circ}\right)\right.$
$\left.+(25.634511) \cos \left(-14.15301^{\circ}-30^{\circ}\right)+(41.929878) \cos \left(-11.73176^{\circ}-30^{\circ}\right)\right]$
$\mathrm{P}_{\mathrm{T}}=(277.1281)\left[(37.682887) \cos -64.64858^{\circ}\right)$
$\left.\left.+(25.634511) \cos \left(-44.15301^{\circ}\right)+(41.929878) \cos -41.73176^{\circ}\right)\right]$
$\mathrm{P}_{\mathrm{T}}=(277.1281)[(37.682887)(.42817)+(25.634511)(.71748)+$ (41.929878) (.746269]
$\mathrm{P}_{\mathrm{T}}=(277.1281)[(16.13468)+(18.392248)+(31.290968]$
$\mathrm{P}_{\mathrm{T}}=(277.1281)[65.81789]=18239.98$ watts $\approx 18240$ watts

The power calculation yields the same value as that which was calculated by adding the wattages of the three parts of a delta circuit. As was the case with the first calculation in Part 1 of this Appendix, this second calculation likewise tends to confirm that the power of a delta circuit can be determined by assuming the user to be a wye circuit.

The phasors representative of the voltages and currents in the Part 2
calculation are shown drawn to scale in Fig. 21.


## Appendix D - Proof of Two Wattmeter and Three Wattmeter Methods.

In Section 6C it was stated that the two wattmeter method is a valid method for measuring the total power of a three phase three wire circuit. And, in Section 6D it was stated that the three wattmeter method is also a valid method of measuring the total power of a three phase three wire circuit. It is demonstrated here that both are valid methods. In fact, proof of the two wattmeter method follows from the proof of the three wattmeter method. For this reason, the three wattmeter method is treated first.

Reference is made to Fig. 17 where the negative leads of the three wattmeters are shown joined together. The positive leads of the respective meters connect to the phase in which the current coils are located. It is demonstrated that the voltage of Point "x" of Fig. 17 is immaterial and can be at any level.

As mentioned in Section 2B the expression for instantaneous power in a single phase circuit is,
$\mathrm{P}_{i}^{i}=v^{i} \cdot i^{i}$, where
$v_{i}^{i}=$ instantaneous value of voltage, and
$i=$ instantaneous value of current
The average power throughout a specific period of time for each phase is, $\mathrm{P}=(1 / \mathrm{T}) \int \boldsymbol{v}^{i} \cdot i^{i} d t$ evaluated throughout the period from $\mathrm{t}_{1}$ to $\mathrm{t}_{2}$.

In Fig. 17 the three single phase wattmeters are configured to measure the currents of all three lines, i.e. Phase A, Phase B and Phase C. The total of the readings of the three meters is,
$\mathrm{P}=(1 / \mathrm{T}) \int v_{\mathrm{A}-\mathrm{X}}^{i} \cdot i_{\mathrm{A}}^{i} d t+(1 / \mathrm{T}) \int v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{i}}^{i} d t+(1 / \mathrm{T}) \int v_{\mathrm{C}-\mathrm{X}}^{i} \cdot i_{\mathrm{C}}^{i} d t$
$\mathbf{P}=(1 / \mathrm{T}) \int\left[v_{\mathrm{A}-\mathrm{X}}^{i} \cdot i_{\mathrm{A}}^{i}+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{B}}^{i}+v_{\mathrm{C}-\mathrm{X}}^{i} \bullet i_{\mathrm{C}}^{i}\right] d t$
Consider a delta circuit of the configuration represented in Fig. 7:
$i_{\mathrm{A}}^{i}=i_{\mathrm{C}-\mathrm{A}}^{i}-i_{\mathrm{A}-\mathrm{B}}^{i}$
$i_{\mathrm{B}}^{i}=i_{\mathrm{A}-\mathrm{B}}^{i}-i_{\mathrm{B}-\mathrm{C}}^{i}$
$i_{\mathrm{C}}=i_{\mathrm{B}-\mathrm{C}}-i_{\mathrm{C}-\mathrm{A}}$
Then,
$\left[v_{{ }_{\mathrm{A}-\mathrm{X}}}^{i} \cdot i_{\mathrm{A}}^{i}+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{B}}^{i}+v_{\mathrm{C}-\mathrm{X}}^{i} \cdot i_{\mathrm{C}}^{i}\right]=$
$\left[v_{\mathrm{A}-\mathrm{X}}^{i} \cdot\left(i_{\mathrm{CA}}^{i}-i_{\mathrm{A}-\mathrm{B}}^{i}\right)+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot\left(i_{\mathrm{A}-\mathrm{B}}^{i}-i_{\mathrm{B}-\mathrm{C}}^{i}\right)+v_{\mathrm{C}-\mathrm{X}}^{i} \cdot\left(i_{\mathrm{BBC}}^{i}-i_{\mathrm{CA}}^{i}\right)\right]=$
$\left[i_{\mathrm{AB}}^{i}\left(v_{\mathrm{B}-\mathrm{X}}^{i}-v_{\mathrm{A}-\mathrm{X}}^{i}\right)+i_{\mathrm{B}-\mathrm{C}}\left(v_{\mathrm{C}-\mathrm{X}}^{i}-v_{\mathrm{B}-\mathrm{X}}^{i}\right)+i_{\mathrm{CA}}^{i}\left(v_{\mathrm{A}-\mathrm{X}}^{i}-v_{\mathrm{C}-\mathrm{X}}^{i}\right)\right]$
According to Kirchoff's Voltage Law, the sum of the voltages around a circuit must equal zero. Accordingly, it is recognized that,
$v_{i \mathrm{~A}-\mathrm{B}}^{i}=v_{\mathrm{B}-\mathrm{X}}^{i}-v_{\mathrm{A}-\mathrm{X}}^{i}$
$v_{\mathrm{B}-\mathrm{C}}^{i}=v_{\mathrm{C}-\mathrm{X}-}^{i}-v_{\mathrm{B}-\mathrm{X}}^{i}$, and
$v_{\mathrm{C}-\mathrm{A}}^{i}=v_{\mathrm{A}-\mathrm{X}}^{i}-v_{\mathrm{C}-\mathrm{X}}^{i}$
It follows that,
$\mathbf{P}=(1 / \mathrm{T}) \int\left[v_{\mathrm{A}-\mathrm{B}}^{i} \cdot i_{\mathrm{A}-\mathrm{B}}+v_{\mathrm{B}-\mathrm{C}}^{i} \cdot i_{\mathrm{B}-\mathrm{C}}^{i}+v_{\mathrm{C}-\mathrm{A}}^{i} \cdot i_{\mathrm{C}-\mathrm{A}}\right] d t$, evaluated from $\mathrm{t}_{1}$ to $\mathrm{t}_{2}$.

This expression is the equivalent to the power that would be determined by measuring the currents and voltages of each of the three phases, i.e. Phase A$\mathrm{B}, \mathrm{B}-\mathrm{C}$ and $\mathrm{C}-\mathrm{A}$, and adding the three readings to obtain a total. Thus the three wattmeter method is proven valid for a delta circuit. A similar proof can be established for a wye connected circuit.

In a wye connected circuit, as with the delta circuit, the expression for instantaneous power is,
$\mathrm{P}^{i}=v^{i} \cdot i^{i}$
Let $v^{i}{ }_{\text {X }}$ be the elevation of the neutral point voltage, which is the voltage of the common negative leads of the three wattmeters.
$\boldsymbol{v}_{\mathrm{A}-\mathrm{X}}^{i}=\boldsymbol{v}_{\mathrm{A}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}$
$\mathrm{P}=(1 / \mathrm{T}) \int v_{\mathrm{A}-\mathrm{N}}^{i} \cdot i_{\mathrm{A}}^{i} d t+(1 / \mathrm{T}) \int \boldsymbol{v}_{\mathrm{B}-\mathrm{N}}^{i} \cdot i_{\mathrm{B}}^{i} d t+(1 / \mathrm{T}) \int v_{\mathrm{C}-\mathrm{N}}^{i} \cdot i_{\mathrm{C}}^{i} d t$
$\left.\mathrm{P}=(1 / \mathrm{T}) \int_{\left[v^{i}\right.}{ }_{\mathrm{A}-\mathrm{N}} \bullet i_{\mathrm{A}}^{i}+v^{i}{ }_{\mathrm{B}-\mathrm{N}} \bullet i_{\mathrm{B}}^{i}+v_{\mathrm{C}-\mathrm{N}}^{i} \cdot i_{\mathrm{C}}^{i}\right] d t$

With the three wattmeters connected so that the negative leads are common and at voltage " $x$ " above the neutral voltage.
$\left.\mathrm{P}=(1 / \mathrm{T}) \int_{[ } v_{\mathrm{A}-\mathrm{X}}^{i} \cdot i_{\mathrm{A}}^{i}+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{B}}^{i}+v_{\mathrm{C}-\mathrm{X}}^{i} \bullet i_{\mathrm{C}}^{i}\right] d t$
$v_{\mathrm{A}-\mathrm{X}}^{i}=\boldsymbol{v}_{\mathrm{A}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}$
$v_{\mathrm{A}-\mathrm{X}}^{i} \cdot i_{\mathrm{A}}^{i}=i_{\mathrm{A}}^{i}\left(v_{\mathrm{A}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)$
$v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{B}}^{i}=i_{\mathrm{B}}^{i}\left(v_{\mathrm{B}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)$
$v^{i}{ }_{\mathrm{C}-\mathrm{X}} \cdot i_{\mathrm{C}}^{i}=i_{\mathrm{C}}^{i}\left(v_{\mathrm{C}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)$
$\left[v^{i}{ }_{\mathrm{A}-\mathrm{X}} \cdot i_{\mathrm{A}}^{i}+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{B}}^{i}+v_{\mathrm{C}-\mathrm{X}}^{i} \cdot i_{\mathrm{C}}^{i}\right]=$
$\left[i_{\mathrm{A}}^{i}\left(v_{\mathrm{A}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)+i_{\mathrm{B}}^{i}\left(v_{\mathrm{B}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)+i_{\mathrm{C}}^{i}\left(v_{\mathrm{C}-\mathrm{N}}^{i}+v_{\mathrm{X}}^{i}\right)\right]=$
$\left[i_{\mathrm{A}}^{i} v_{\mathrm{A}-\mathrm{N}}^{i}+i_{\mathrm{B}-\mathrm{N}}^{i} v_{\mathrm{B}}^{i}+i_{\mathrm{C}-\mathrm{N}}^{i} v_{\mathrm{C}}^{i}\right]+v_{\mathrm{X}}^{i}\left[i_{\mathrm{A}}^{i}+i_{\mathrm{B}}^{i}+i_{\mathrm{C}}^{i}\right]$
According to Kirchoff's Current Law,
$i_{\mathrm{A}}^{i}+i_{\mathrm{B}}^{i}+i_{\mathrm{C}}^{i}=0$
Then,
$\mathrm{P}=(1 / \mathrm{T}) \int\left[\nu_{\mathrm{A}-\mathrm{X}}^{i} \cdot i_{\mathrm{A}}^{i}+v_{\mathrm{B}-\mathrm{X}}^{i} \cdot i_{\mathrm{i}}^{i}+v_{\mathrm{C}-\mathrm{X}}^{i} \cdot i_{\mathrm{C}}^{i}\right] d t=$
$\mathrm{P}=(1 / \mathrm{T}) \int v_{\mathrm{A}-\mathrm{N}}^{i} \cdot i_{\mathrm{A}}^{i} d t+(1 / \mathrm{T}) \int v_{\mathrm{B}-\mathrm{N}}^{i} \cdot i_{\mathrm{B}}^{i} d t+(1 / \mathrm{T}) \int v_{\mathrm{C}-\mathrm{N}}^{i} \bullet i_{\mathrm{C}}^{i} d t$
Thus, it is apparent that the three wattmeter method as applied to a wye circuit provides the same results that would be obtained by measuring the power of each phase and then adding the three measurements.
Here it was demonstrated that the three wattmeter method is applicable to either a delta or a wye circuit. Therefore it may be concluded that the three wattmeter method as depicted in Fig. 17 is applicable to any form of a three phase circuit, delta or wye.
Given that the three wattmeter method is valid for a three phase three wire circuit, it will become apparent that the two wattmeter method is likewise a valid method of measuring power in a three phase three wire circuit. As mentioned above with respect to the three wattmeter method, the three
negative leads may be placed at any voltage. Obviously, if the negative leads were placed common to one of the phase lines, i.e. Phase A, Phase B or Phase $C$, the wattmeter which measures the current in that phase would read " 0 ." In other words, the meter with the " 0 " readings can be removed and the total power of the circuit would be the total of the readings on the remaining two wattmeters. Thus the two wattmeter method is also a valid means to measure the total power of a three phase three wire circuit.
(Note: Some authors on the subject of three phase power state that the two wattmeter wattmeter method is valid only for balanced circuits. In fact, the method is applicable to both balanced and unbalanced circuits. The below calculations corroborate this position.)

## Computations to Corroborate the Two Wattmeter Method for Three Phase Three Wire Circuits

The computations of this Appendix corroborate that the two wattmeter method is applicable to a three wire three phase circuit. According to the method only two currents of a three wire three phase circuit need to be measured in order to determine the power consumption of the circuit. The currents to be measured may be currents $\mathrm{A} \& \mathrm{~B}, \mathrm{~B} \& \mathrm{C}$ or $\mathrm{C} \& A$. Potentials are to be measured according to a specific order that is determined by the currents that selected for the measurement.

## Part 1 of Appendix D

This part of Appendix D considers the scenario in which two wattmeters are configured as depicted in Fig. 13.
The configuration of Fig. 13 requires measurement of currents A and B as well as voltages C-A and C-B. The currents and voltages of Part 2 of Appendix C are assumed. The phasors for the voltages and currents are represented in Fig. 21.
The voltage leads of the two meters must be arranged as described in Fig. 13 with care taken to position the positive and negative leads as shown.

For this computation, assume that Meter \#1 has its voltage connections with the negative lead on Phase C and the positive lead on Phase A. Meter \#2 has the negative voltage lead on Phase C and the positive lead on Phase B.

As demonstrated in the example of part 2 of Appendix C, the following line values were determined:
Computed values of line currents:

| Line Current | Lead $/$ lag |
| :--- | :--- |
| $\mathrm{I}_{\mathrm{A}}=37.682887 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{A}}=-34.64858^{\circ}$ |
| $\mathrm{I}_{\mathrm{B}}=25.634511 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$ |

Since only two meters are used and only the currents in Conductors A and B are measured, Current $\mathrm{I}_{\mathrm{C}}$ and its lead/lag angle $\theta_{\mathrm{L}-\mathrm{C}}$ are not required and are not repeated here.

Reference is made to Fig. 22 which shows the respective currents and the pertinent leads/lags of currents $\mathrm{I}_{\mathrm{A}}$ and $\mathrm{I}_{\mathrm{B}}$. Meter $\# 1$ measures $\mathrm{I}_{\mathrm{A}}, \mathrm{V}_{\mathrm{CA}}$ and $\theta_{\mathrm{L}-\mathrm{A}}$. Meter $\# 2$ measures $\mathrm{I}_{\mathrm{B}}, \mathrm{V}_{\mathrm{CB}}$ and $\theta_{\mathrm{L}-\mathrm{B}}$. As determined in Appendix C, $\mathrm{I}_{\mathrm{A}}=37.682887 \mathrm{amps}$
$\theta_{\mathrm{L}-\mathrm{A}}=-34.64858^{\circ}$, and
$\mathrm{V}_{\mathrm{CA}}=480 \mathrm{VAC}$
Thus the reading of Meter \#1 would be:
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{CA}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{L}-\mathrm{A}}$
$\mathrm{W}_{1}=(480)(37.682887) \cos -34.64858^{\circ}$
$\mathrm{W}_{1}=14879.9936$ watts $\approx 14,880$ watts
Meter $\# 2$ measures $\mathrm{I}_{\mathrm{B}}, \mathrm{V}_{\mathrm{cb}}$ and the angle between these two parameters. As illustrated in Fig. 18 the angle of lag between $I_{B}$ and $V_{c b}$ is $\theta_{L-B}+60^{\circ}$. Since $\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$, the angle of lag between $\mathrm{I}_{\mathrm{B}}$ and $\mathrm{V}_{\mathrm{cb}}$ is $\left(60^{\circ}+14.15301^{\circ}\right)$, or $+74.15302^{\circ}$.
Therefore, the reading of Meter \#2 would be:
$\mathrm{W}_{2}=(480)(25.634511) \cos -74.15302^{\circ}$
$\mathrm{W}_{2}=3359.999$ watts $\approx 3360$ watts
The net measured wattage then is:
$\mathrm{W}_{\mathrm{T}}=\mathrm{W}_{1}+\mathrm{W}_{2}$
$\mathrm{W}_{\mathrm{T}}=14880$ watts + 3360 watts $=18240$ watts
Thus, the sum of the measured wattage is in agreement with the calculated net wattage as determined in Part 2 of Appendix C. This computation corroborates the two wattmeter theory as applicable to a three
 wire three phase circuit.

## Part 2 of Appendix D

This part of Appendix D considers the scenario in which the wattmeters are arranged as depicted in Fig. 14.

The configuration of Fig. 14 requires measurement of currents A and C as well as voltages B-C and B-A. The currents and voltages of Part 2 of Appendix C are assumed. The phasors for the voltages and currents are represented in Fig. 21.

The voltage leads of the two meters would be arranged as described in Fig. 14 with care taken to position the positive and negative leads as shown.

For this computation, assume that Meter \#1 has its voltage connections with the negative lead on Phase B and the positive lead on Phase C. Meter \#2 has the negative voltage lead on Phase B and the positive lead on Phase A.

As demonstrated in the example of Part 2 of Appendix C, the following line values were determined:

Computed values of line currents:

| Line Current | Lead/lag |
| :--- | :--- |
| $\mathrm{I}_{\mathrm{A}}=37.682887 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{A}}=-34.64858^{\circ}$ |
| $\mathrm{I}_{\mathrm{C}}=41.929878 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{C}}=-11.73176^{\circ}$ |

Since only two meters are used and only the currents in Conductors A and C are measured, Current $\mathrm{I}_{\mathrm{B}}$ and its lead/lag angle $\theta_{\mathrm{L}-\mathrm{B}}$ are not required and are not repeated here.

Reference is made to Fig. 23 which shows the currents and the pertinent leads/lags of currents $\mathrm{I}_{\mathrm{A}}$ and $\mathrm{I}_{\mathrm{C}}$. Meter $\# 1$ measures $\mathrm{I}_{\mathrm{C}}, \mathrm{V}_{\mathrm{BC}}$ and $\theta_{\mathrm{L}-\mathrm{C}}$, Meter $\# 2$ measures $\mathrm{I}_{\mathrm{A}}, \mathrm{V}_{\mathrm{BA}}$ and $\theta_{\mathrm{L}-\mathrm{A}}$. As determined in
Appendix C,
$\mathrm{I}_{\mathrm{C}}=41.929878 \mathrm{amps}$
$\theta_{\mathrm{L}-\mathrm{C}}=-11.73176^{\circ}$, and
$\mathrm{V}_{\mathrm{BC}}=480 \mathrm{VAC}$
Thus the reading of Meter \#1 would be:
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{CA}} \mathrm{I}_{\mathrm{A}} \cos \theta_{\mathrm{L}-\mathrm{C}}$
$\mathrm{W}_{1}=(480)(41.929878) \cos -11.73176^{\circ}$
$\mathrm{W}_{1}=19705.907$ Watts $\approx 19706$ watts
Meter $\# 2$ measures $\mathrm{I}_{\mathrm{A}}, \mathrm{V}_{\mathrm{ba}}$ and the angle between these two parameters. As
illustrated in Fig. 19
the angle of lag between $\mathrm{I}_{\mathrm{A}}$ and $\mathrm{V}_{\mathrm{ca}}$ is $\left(\theta_{\mathrm{L}-\mathrm{A}}+60^{\circ}\right)$. Since $\theta_{\mathrm{L}-\mathrm{A}}=-34.64^{\circ}$, the angle of lag between $I_{B}$ and $V_{b a}$ is $-94.64^{\circ}$.
Therefore, the reading of Meter \#2 would be:
$\mathrm{W}_{2}=(480)$

(37.682887) $\cos -94.64858^{\circ}$
$\mathrm{W}_{2}=-1465.907$ watts $\approx-1466$ watts
The net measured wattage then is:
$\mathrm{W}_{\mathrm{T}}=\mathrm{W}_{1}+\mathrm{W}_{2}$
$\mathrm{W}_{\mathrm{T}}=19706$ watts -1466 watts $=18240$ watts
Thus, the sum of the measured wattage is in exact agreement with the calculated net wattage as determined in Appendix C. This computation corroborates the two wattmeter theory as applicable to a three wire three phase circuit.

## Part 3 of Appendix D

This part of Appendix D considers the scenario in which two wattmeters are arranged in a configuration similar to that of Fig. 13 except that the current measurements are in Phase B and Phase C and the potential measurements are of A-B and A-C. The phasors for the voltages and currents are represented in Fig. 21. (There is no figure to represent the arrangement.)
For this computation, assume that Meter \#1 measures the current in Phase B and has its voltage connections with the negative lead on Phase A and the positive lead on Phase B. Meter \#2 measures the current in Phase C and has the negative voltage lead on Phase A and the positive lead on Phase C.
As demonstrated in the example of Part 2 of Appendix C, the following line values were determined:

Computed values of line currents:

| Line Current | Lead/lag |
| :--- | :--- |
| $\mathrm{I}_{\mathrm{B}}=25.634511 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$ |
| $\mathrm{I}_{\mathrm{C}}=41.929878 \mathrm{amps}$ | $\theta_{\mathrm{L}-\mathrm{C}}=-11.73176^{\circ}$ |

Since only two meters are used and only the currents in Conductors B and C are measured, Current $\mathrm{I}_{\mathrm{B}}$ and its lead/lag angle $\theta_{\mathrm{L}-\mathrm{B}}$ are not required and are not repeated here.
Reference is made to Fig. 24 which shows the currents and the pertinent leads/lags of currents $\mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$. Meter $\# 1$ measures $\mathrm{I}_{\mathrm{B}}, \mathrm{V}_{\mathrm{AB}}$ and $\theta_{\mathrm{L}-\mathrm{B}}$. Meter $\# 2$ measures $\mathrm{I}_{\mathrm{C}}, \mathrm{V}_{\mathrm{AC}}$ and $\theta_{\mathrm{L}-\mathrm{C}}$. As determined in Appendix C,
$\mathrm{I}_{\mathrm{B}}=25.634511 \mathrm{amps}$
$\theta_{\mathrm{L}-\mathrm{B}}=-14.15301^{\circ}$
$\mathrm{V}_{\mathrm{AB}}=480 \mathrm{VAC}$
Thus the reading of Meter \#1 would be:
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{ab}} \mathrm{I}_{\mathrm{B}} \cos \theta_{\mathrm{L}-\mathrm{B}}$
$\mathrm{W}_{1}=(480)(25.634511) \cos -14.15301^{\circ}$
$\mathrm{W}_{1}=11931.07$ Watts $\approx 11931$ watts
Meter $\# 2$ measures $\mathrm{I}_{\mathrm{C}}, \mathrm{V}_{\mathrm{ac}}$ and the angle between these two parameters. As illustrated in Fig. 20 the angle of lag between $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\mathrm{ac}}$ is $\theta_{\mathrm{L}-\mathrm{C}}+60^{\circ}$. Since $\theta_{\mathrm{L}-\mathrm{AC}}=-11.73176^{\circ}$, the angle of lag between $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\mathrm{ac}}$ is $\theta_{\mathrm{L}-\mathrm{C}}=-71.73176^{\circ}$.
Therefore, the reading of Meter \#2 would be:
$\mathrm{W}_{2}=(480)(41.929878) \cos \left(-71.73176^{\circ}\right)$
$\mathrm{W}_{2}=6308.92$ watts $\approx 6309$ watts
The net measured wattage then is:
$\mathrm{W}_{\mathrm{T}}=\mathrm{W}_{1}+\mathrm{W}_{2}$
$\mathrm{W}_{\mathrm{T}}=11931$ watts $+6309=18240$ watts
Thus, the sum of the measured wattage is in exact agreement with the calculated net wattage as determined in Appendix C. This computation corroborates the two wattmeter theory as applicable to a three wire three phase circuit.


## Appendix E - Table of Typical Three Phase Power Values vs. $\omega t$ for One Cycle

The values tabulated below are for the assumed conditions of Example 2. Column I is Phase AB power. Column II is Phase BC power. Column III is Phase CA power. Column IV is the total three phase power, i.e. Column I plus Column II plus Column III.

| $\omega t$ | I <br> Phase AB <br> Power | II <br> Phase BC <br> Power | III <br> Phase CA <br> Power | IV <br> Total <br> Power |
| :--- | :--- | :--- | :--- | :--- |
| $0^{\circ}$ | 0 | 2071.36 | 8008.63 | 10080.00 |
| $5^{\circ}$ | -544.20 | 2893.85 | 7730.34 | 10080.00 |
| $10^{\circ}$ | -969.77 | 3730.50 | 7319.26 | 10080.00 |
| $15^{\circ}$ | -1263.78 | 4555.90 | 6787.88 | 10080.00 |
| $20^{\circ}$ | -1417.31 | 5344.96 | 6152.35 | 10080.00 |
| $25^{\circ}$ | -1425.67 | 6073.70 | 5431.97 | 10080.00 |
| $30^{\circ}$ | -1288.63 | 6720.00 | 4648.63 | 10080.00 |
| $35^{\circ}$ | -1010.34 | 7264.20 | 3826.14 | 10080.00 |
| $40^{\circ}$ | -599.26 | 7689.77 | 2989.49 | 10080.00 |
| $45^{\circ}$ | -67.88 | 7983.78 | 2164.09 | 10080.00 |
| $50^{\circ}$ | 567.64 | 8137.31 | 1375.03 | 10080.00 |
| $55^{\circ}$ | 1288.02 | 8145.67 | 646.29 | 10080.00 |
| $60^{\circ}$ | 2071.36 | 8008.63 | 0 | 10080.00 |
| $65^{\circ}$ | 2893.85 | 7730.34 | -544.20 | 10080.00 |
| $70^{\circ}$ | 3730.50 | 7319.26 | -969.77 | 10080.00 |
| $75^{\circ}$ | 4555.90 | 6787.88 | -1263.78 | 10080.00 |
| $80^{\circ}$ | 5344.96 | 6152.35 | -1417.31 | 10080.00 |
| $85^{\circ}$ | 6073.70 | 5431.97 | -1425.67 | 10080.00 |
| $90^{\circ}$ | 6720.00 | 4648.63 | -1288.63 | 10080.00 |
| $95^{\circ}$ | 7264.20 | 3826.14 | -1010.34 | 10080.00 |
| $100^{\circ}$ | 7689.77 | 2989.49 | -599.26 | 10080.00 |
| $105^{\circ}$ | 7983.78 | 2164.09 | -67.88 | 10080.00 |
| $110^{\circ}$ | 8137.31 | 1375.03 | 567.64 | 10080.00 |
| $115^{\circ}$ | 8145.67 | 646.29 | 1288.02 | 10080.00 |
| $120^{\circ}$ | 8008.63 | 0 | 2071.36 | 10080.00 |
| $125^{\circ}$ | 7730.34 | -544.20 | 2893.85 | 10080.00 |
| $130^{\circ}$ | 7319.26 | -969.77 | 3730.50 | 10080.00 |
| $135^{\circ}$ | 6787.88 | -1263.78 | 4555.90 | 10080.00 |
| $140^{\circ}$ | 6152.35 | -1417.31 | 5344.96 | 10080.00 |
| $145^{\circ}$ | 5431.97 | -1425.67 | 6073.70 | 10080.00 |
| $150^{\circ}$ | 4648.63 | -1288.63 | 6720.00 | 10080.00 |
| $155^{\circ}$ | 3826.14 | -1010.34 | 7264.20 | 10080.00 |
|  |  |  |  |  |


| $160^{\circ}$ | 2989.49 | -599.26 | 7689.77 | 10080.00 |
| :--- | :--- | :--- | :--- | :--- |
| $165^{\circ}$ | 2164.09 | -67.88 | 7983.78 | 10080.00 |
| $170^{\circ}$ | 1375.03 | 567.64 | 8137.31 | 10080.00 |
| $175^{\circ}$ | 646.29 | 1288.02 | 8145.67 | 10080.00 |
| $180^{\circ}$ | 0 | 2071.36 | 8008.63 | 10080.00 |
| $185^{\circ}$ | -544.20 | 2893.85 | 7730.34 | 10080.00 |
| $190^{\circ}$ | -969.77 | 3730.50 | 7319.26 | 10080.00 |
| $195^{\circ}$ | -1263.78 | 4555.90 | 6787.88 | 10080.00 |
| $200^{\circ}$ | -1417.31 | 5344.96 | 6152.35 | 10080.00 |
| $205^{\circ}$ | -1425.67 | 6073.70 | 5431.97 | 10080.00 |
| $210^{\circ}$ | -1288.63 | 6720.00 | 4648.63 | 10080.00 |
| $215^{\circ}$ | -1010.34 | 7264.20 | 3826.14 | 10080.00 |
| $220^{\circ}$ | -599.26 | 7689.77 | 2989.49 | 10080.00 |
| $225^{\circ}$ | -67.88 | 7983.78 | 2164.09 | 10080.00 |
| $230^{\circ}$ | 567.64 | 8137.31 | 1375.03 | 10080.00 |
| $235^{\circ}$ | 1288.02 | 8145.67 | 646.29 | 10080.00 |
| $240^{\circ}$ | 2071.36 | 8008.63 | 0 | 10080.00 |
| $245^{\circ}$ | 2893.85 | 7730.34 | -544.20 | 10080.00 |
| $250^{\circ}$ | 3730.50 | 7319.26 | -969.77 | 10080.00 |
| $255^{\circ}$ | 4555.90 | 6787.88 | -1263.78 | 10080.00 |
| $260^{\circ}$ | 5344.96 | 6152.35 | -1417.31 | 10080.00 |
| $265^{\circ}$ | 6073.70 | 5431.97 | -1425.67 | 10080.00 |
| $270^{\circ}$ | 6720.00 | 4648.63 | -1288.63 | 10080.00 |
| $275^{\circ}$ | 7264.20 | 3826.14 | -1010.34 | 10080.00 |
| $280^{\circ}$ | 7689.77 | 2989.49 | -599.26 | 10080.00 |
| $285^{\circ}$ | 7983.78 | 2164.09 | -67.88 | 10080.00 |
| $290^{\circ}$ | 8137.31 | 1375.03 | 567.64 | 10080.00 |
| $295^{\circ}$ | 8145.67 | 646.29 | 1288.02 | 10080.00 |
| $300^{\circ}$ | 8008.63 | 0 | 2071.36 | 10080.00 |
| $305^{\circ}$ | 7730.34 | -544.20 | 2893.85 | 10080.00 |
| $310^{\circ}$ | 7319.26 | -969.77 | 3730.50 | 10080.00 |
| $315^{\circ}$ | 6787.88 | -1263.78 | 4555.90 | 10080.00 |
| $320^{\circ}$ | 6152.35 | -1417.31 | 5344.96 | 10080.00 |
| $325^{\circ}$ | 5431.97 | -1425.67 | 6073.70 | 10080.00 |
| $330^{\circ}$ | 4648.63 | -1288.63 | 6720.00 | 10080.00 |
| $335^{\circ}$ | 3826.14 | -1010.34 | 7264.20 | 10080.00 |
| $340^{\circ}$ | 2989.49 | -599.26 | 7689.77 | 10080.00 |
| $345^{\circ}$ | 2164.09 | -67.88 | 7983.78 | 10080.00 |
| $350^{\circ}$ | 1375.03 | 567.64 | 8137.31 | 10080.00 |
| $355^{\circ}$ | 646.29 | 1288.02 | 8145.67 | 10080.00 |
| $360^{\circ}$ | 0 | 2071.36 | 8008.63 | 10080.00 |
|  |  |  |  |  |
|  |  |  |  |  |

Note: the values in Columns I, II and III are shown to two significant places but were actually calculated to four significant places.

The above computations of instantaneous power (which determined a power level of $10,080.00$ watts) can be verified with Equation 3C3b, which is applicable to three phase delta circuits using phase parameters. The equation states:
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
Using the given parameters,
$\mathrm{V}_{\mathrm{P}}=480 \mathrm{VAC}$
$\mathrm{I}_{\mathrm{P}}=10 \mathrm{amps}$
$\mathrm{PF}=\cos \theta_{\mathrm{P}}=0.70$
$\mathrm{P}=3 \mathrm{~V}_{\mathrm{P}} \mathrm{I}_{\mathrm{P}} \cos \theta_{\mathrm{P}}$
$\mathrm{P}=3(480)(10)(.7)=10,080.00$ watts
Thus, the computation of power using Equation 3C3b confirms the above computations of instantaneous power.

## Appendix F - Equations for Calculating Currents

The below listed equations will be found convenient in calculating currents. These equations are repeated from PDH Online Course E336, Calculating Currents in Balanced and Unbalanced Three Phase Circuits. The symbols used in these equations are consistent with the use in this course (Course E344).

These equations used in this course follow the more easily understood method of "vector algebra" to determine the values of currents. As such, the equations avoid the use of complex variables that, although sometimes more convenient, can be baffling to persons not comfortable with the use of complex variables. The equation identification numbers used below and throughout this course (Course E344) are the same as those used in Course E336.

## Equation 5A

For single phase circuits (Reference Fig. 11) : The magnitude and lead/lag of line current $\mathrm{I}_{\mathrm{a}}$ resulting from the addition of line current $\mathrm{I}_{1}$ at lag/lead angle $\theta_{l}$ (to line voltage) and line current $\mathrm{I}_{2}$ at lead/lag angle $\theta_{2}$ (to line voltage) and ...line current $\mathrm{I}_{\mathrm{n}}$ at lead/lag angle $\theta_{\mathrm{n}}$ (to line voltage):
$\mathrm{I}_{\mathrm{a}}=\left\{\left(\mathrm{X}_{\mathrm{a}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{a}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 5A
where
$\mathrm{X}_{\mathrm{a}}=\mathrm{I}_{1} \cos \theta_{1}+\mathrm{I}_{2} \cos \theta_{2}+\ldots \mathrm{I}_{\mathrm{n}} \cos \theta_{\mathrm{n}}$, and
$\mathrm{Y}_{\mathrm{a}}=\left[\mathrm{I}_{1} \sin \theta_{1}+\mathrm{I}_{2} \sin \theta_{2}+\ldots \mathrm{I}_{\mathrm{n}} \sin \theta_{\mathrm{n}}\right]$
$\theta_{\mathrm{a}}=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{a}} \div \mathrm{I}_{\mathrm{a}}\right)$

## Equation 5B

For balanced three phase circuits: The magnitude and lead/lag of line current $\mathrm{I}_{\mathrm{B}}$ resulting from the addition of line current $\mathrm{I}_{1}$ at lag/lead angle $\theta_{l}$ (to line voltage) and line current $\mathrm{I}_{2}$ at lead/lag angle $\theta_{2}$ (to line voltage) and up to $\ldots$...ine current $\mathrm{I}_{\mathrm{n}}$ at lead/lag angle $\theta_{\mathrm{n}}$ (to line voltage):
$\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 5B
where,
$\mathrm{X}_{\mathrm{B}}=\mathrm{I}_{1} \cos \theta_{1}+\mathrm{I}_{2} \cos \theta_{2}+\ldots \mathrm{I}_{\mathrm{n}} \cos \theta_{\mathrm{n}}$, and
$\mathrm{Y}_{\mathrm{B}}=\left[\mathrm{I}_{1} \sin \theta_{1}+\mathrm{I}_{2} \sin \theta_{2}+\ldots \mathrm{I}_{\mathrm{n}} \sin \theta_{\mathrm{n}}\right]$
$\theta_{\mathrm{B}}=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right)$
(Note: For a balanced three phase circuit, the three line currents are equal. $\mathrm{I}_{\mathrm{B}}=\mathrm{I}_{\mathrm{A}}=\mathrm{I}_{\mathrm{C}}$ )

## Equation 6

Power consumption of a balanced three phase wye or balanced three phase delta load:

$$
\mathrm{P}=(\sqrt{3}) \mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \theta_{\mathrm{P}} \quad \ldots \quad \text { Equation } 6
$$

## Equation 7

Line current of a balanced three phase delta load:

$$
\mathrm{I}_{\mathrm{L}}=(\sqrt{3}) \mathrm{I}_{\mathrm{P}} \quad \ldots \text { Equation } 7
$$

## Equation 8

Phase voltage in a balanced three phase wye load or an unbalanced three phase load with a grounded neutral:

$$
V_{L}=(\sqrt{3}) V_{P} \quad \ldots \text { Equation } 8
$$

Equations 9-1 to 9-6: Equations for calculating line currents when the phase currents in an unbalanced delta circuit are known.

With reference to Fig. 13 - where $\mathrm{I}_{\mathrm{ab}}$ is the current in phase $\mathrm{a}-\mathrm{b}, \mathrm{I}_{\mathrm{bc}}$ is the current in phase b-c, $\mathrm{I}_{\mathrm{ac}}$ is the current in phase a-c, $\theta_{\mathrm{P}-\mathrm{AB}}$ is the lead/lag of current in phase A-B, $\theta_{\mathrm{P}-\mathrm{BC}}$ is the lead/lag of current in phase B-C, $\theta_{\mathrm{P}-\mathrm{CA}}$ is the lead/lag of current in phase C-A, $\mathrm{I}_{\mathrm{A}}, \mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ are the line currents in, respectively, conductors $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}, \theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}$, is the lead/lag of the line current in phase A with respect to line voltage $\mathrm{C}-\mathrm{A}, \theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}$ is the lead/lag of the line current in phase B with respect to line voltage A-B,
and $\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}$ is the lead/lag of the line current in phase C with respect to line voltage B-C.
$\mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 9-1
$\theta_{\text {L-A }}=\left(\lambda-120^{\circ}\right) \quad \ldots$ Equation 9-2
where
$\mathrm{X}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}$
$\mathrm{X}_{\mathrm{ca}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{X}_{\mathrm{A}}=\mathrm{X}_{\mathrm{ba}}+\mathrm{X}_{\mathrm{ca}}$
$\mathrm{Y}_{\mathrm{ba}}=-\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}$
$\mathrm{Y}_{\mathrm{ca}}=\mathrm{I}_{\mathrm{Ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{Y}_{\mathrm{A}}=\mathrm{Y}_{\mathrm{ba}}+\mathrm{Y}_{\mathrm{ca}}$
$\lambda=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{A}} \div \mathrm{I}_{\mathrm{A}}\right)$
Valid range of $\theta_{\mathrm{P}-\mathrm{AB}} \& \theta_{\mathrm{P}-\mathrm{CA}}: \pm 90^{\circ}$; valid range of $\theta_{\mathrm{L}-\mathrm{A}}:+120^{\circ}$ to $-60^{\circ}$
$\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 9-3
$\theta_{\mathrm{L}-\mathrm{B}}=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right) \quad \ldots$ Equation 9-4
where
$\mathrm{X}_{\mathrm{ab}}=\mathrm{I}_{\mathrm{ab}} \cos \theta_{\mathrm{P}-\mathrm{AB}}$
$\mathrm{X}_{\mathrm{cb}}=-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$X_{B}=X_{a b}+X_{c b}$
$\mathrm{Y}_{\mathrm{ab}}=\mathrm{I}_{\mathrm{ab}} \sin \theta_{\mathrm{P}-\mathrm{AB}}$
$\mathrm{Y}_{\mathrm{cb}}=\mathrm{I}_{\mathrm{cb}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{BC}}+\sin \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$Y_{B}=Y_{a b}+Y_{c b}$
Valid range of $\theta_{\mathrm{P}-\mathrm{AB}} \& \theta_{\mathrm{P}-\mathrm{CB}}: \pm 90^{\circ}$; valid range of $\theta_{\mathrm{L}-\mathrm{B}}:+120^{\circ}$ to $-60^{\circ}$
$\mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 9-5
$\theta_{\text {L-C }}=\left(\varphi-240^{\circ}\right) \quad \ldots$ Equation 9-6
where
$\mathrm{X}_{\mathrm{bc}}=\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{BC}}-\cos \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$\mathrm{X}_{\mathrm{ac}}=\mathrm{I}_{\mathrm{Ca}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{P}-\mathrm{CA}}+\cos \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{X}_{\mathrm{C}}=\mathrm{X}_{\mathrm{bc}}+\mathrm{X}_{\mathrm{ac}}$
$\mathrm{Y}_{\mathrm{bc}}=-\mathrm{I}_{\mathrm{bc}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{BC}}+\sin \theta_{\mathrm{P}-\mathrm{BC}}\right]$
$\mathrm{Y}_{\mathrm{ac}}=-\mathrm{I}_{\mathrm{ca}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{P}-\mathrm{CA}}-\sin \theta_{\mathrm{P}-\mathrm{CA}}\right]$
$\mathrm{Y}_{\mathrm{C}}=\mathrm{Y}_{\mathrm{bc}}+\mathrm{Y}_{\mathrm{ac}}$
$\varphi=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{C}} \div \mathrm{I}_{\mathrm{C}}\right)$
Valid range of $\theta_{\mathrm{P}-\mathrm{BC}} \& \theta_{\mathrm{P}-\mathrm{AC}}: \pm 90^{\circ}$; valid range of $\theta_{\mathrm{L}-\mathrm{C}}:+120^{\circ}$ to $-60^{\circ}$

Equations 10-1 to 10-6: Equations for determining line currents in an unbalanced delta circuit when the loads on all three phases are resistive. With reference to Fig. 13 - where $\mathrm{I}_{\mathrm{ab}}$ is the current in phase $\mathrm{a}-\mathrm{b}, \mathrm{I}_{\mathrm{bc}}$ is the current in phase b-c, $\mathrm{I}_{\mathrm{ac}}$ is the current in phase a-c, $\theta_{\mathrm{P}-\mathrm{AB}}$ is the lead/lag of current in phase A-B with respect to line voltage A-B, $\theta_{\mathrm{P}-\mathrm{BC}}$ is the lead/lag of current in phase $\mathrm{B}-\mathrm{C}$ with respect to line voltage $\mathrm{BC}, \theta_{\mathrm{P}-\mathrm{CA}}$ is the lead/lag of current in phase C -A with respect to line voltage $\mathrm{C}-\mathrm{A}, \mathrm{I}_{\mathrm{A}}$, $\mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ are the line currents in, respectively, conductors $\mathrm{A}, \mathrm{B}$ and C , $\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}$ is the lead/lag of the line current in phase A with respect to line voltage $\mathrm{C}-\mathrm{A}, \theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}$ is the lead/lag of the line current in phase B with respect to line voltage AB , and $\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}$ is the lead/lag of the line current in phase C with respect to line voltage $\mathrm{B}-\mathrm{C}$.
$\mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}^{1 / 2} \ldots \quad$ Equation 10-1
$\theta_{\mathrm{L}-\mathrm{A}}=\left(\lambda-120^{\circ}\right) \quad \ldots$ Equation 10-2
where,
$\mathrm{X}_{\mathrm{A}}=-\mathrm{I}_{\mathrm{ab}}-(1 / 2) \mathrm{I}_{\mathrm{ca}}$
$\mathrm{Y}_{\mathrm{A}}=\mathrm{I}_{\mathrm{ca}}(\sqrt{3} / 2)$
$\lambda=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{A}} \div \mathrm{I}_{\mathrm{A}}\right)$
$\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2} \ldots \quad$ Equation 10-3
$\theta_{\mathrm{L}-\mathrm{B}}=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{B}} \div \mathrm{I}_{\mathrm{B}}\right) \quad \ldots \quad$ Equation 10-4
where,
$\mathrm{X}_{\mathrm{B}}=\mathrm{I}_{\mathrm{ab}}+(1 / 2) \mathrm{I}_{\mathrm{bc}}$
$\mathrm{Y}_{\mathrm{B}}=\mathrm{I}_{\mathrm{bc}}(\sqrt{3} / 2)$
$\mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2} \ldots \quad$ Equation 10-5
$\theta_{\mathrm{L}-\mathrm{C}}=\left(\varphi-240^{\circ}\right) \quad \ldots$ Equation 10-6
where,
$\mathrm{X}_{\mathrm{C}}=(1 / 2) \mathrm{I}_{\mathrm{ca}}-(1 / 2) \mathrm{I}_{\mathrm{bc}}$
$\mathrm{Y}_{\mathrm{C}}=-\mathrm{I}_{\mathrm{bc}}(\sqrt{3} / 2)-\mathrm{I}_{\mathrm{ca}}(\sqrt{3} / 2)$
$\varphi=\sin ^{-1}\left(Y_{C} \div I_{C}\right)$

Equations 11-1 to 11-6: Equations for adding the vectors (phasors) of balanced or unbalanced line currents
Equations for determining the currents in a feeder that delivers power to two or more three phase loads. Conductor A-1 is the conductor connecting common conductor A to phase A of load \#1, A-2 is the conductor connecting conductor A to phase A of load $\# 2$, etc. $\mathrm{I}_{\mathrm{A}}, \mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ are the line currents in, respectively, feeder conductors $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}, \theta_{\mathrm{L}-\mathrm{A}}$, is the lead/lag of the line current in phase $\mathrm{A}, \theta_{\mathrm{L}-\mathrm{B}}$ is the lead/lag of the line current in phase B , and $\theta_{\mathrm{L}-\mathrm{C}}$ is the lead/lag of the line current in phase C.
$\mathrm{I}_{\mathrm{A}}=\left\{\left(\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{A}}\right)^{2}\right\}^{1 / 2} \ldots$ Equation 11-1, and
$\theta_{\mathrm{L}-\mathrm{A}}=\left(\kappa-120^{\circ}\right) \ldots$ Equation 11-2, where
$\kappa=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{A}} \div \mathrm{I}_{\mathrm{A}}\right)$, and
$\mathrm{X}_{\mathrm{A}}=\Sigma\left(\mathrm{X}_{\mathrm{A}-1}+\mathrm{X}_{\mathrm{A}-2}+\mathrm{X}_{\mathrm{A}-3} \ldots+\mathrm{X}_{\mathrm{A}-\mathrm{N}}\right)$
$\mathrm{Y}_{\mathrm{A}}=\Sigma\left(\mathrm{Y}_{\mathrm{A}-1}+\mathrm{Y}_{\mathrm{A}-2}+\mathrm{Y}_{\mathrm{A}-3} \ldots+\mathrm{Y}_{\mathrm{A}-\mathrm{N}}\right)$
$\mathrm{X}_{\mathrm{A}-1}=-\mathrm{I}_{\mathrm{A}-1}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{A}-1}+\cos \theta_{\mathrm{A}-1}\right]$
$\mathrm{X}_{\mathrm{A}-2}=-\mathrm{I}_{\mathrm{A}-2}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{A}-2}+\cos \theta_{\mathrm{A}-2}\right] \ldots$ to
$\mathrm{X}_{\mathrm{A}-\mathrm{N}}=-\mathrm{I}_{\mathrm{A}-\mathrm{N}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{A}-\mathrm{N}}+\cos \theta_{\mathrm{A}-\mathrm{N}}\right]$
$\mathrm{Y}_{\mathrm{A}-1}=\mathrm{I}_{\mathrm{A}-1}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{A}-1}-\sin \theta_{\mathrm{A}-1}\right] \ldots$ to
$\mathrm{Y}_{\mathrm{A}-\mathrm{N}}=\mathrm{I}_{\mathrm{A}-\mathrm{N}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{A}-\mathrm{N}}-\sin \theta_{\mathrm{A}-\mathrm{N}}\right]$
$\mathrm{I}_{\mathrm{A}-1}=$ line current in branch of conductor A to load \#1
$\mathrm{I}_{\mathrm{A}-2}=$ line current in branch of conductor A to load \#2 $\ldots$ to
$\mathrm{I}_{\mathrm{A}-\mathrm{N}}=$ line current in branch of conductor A to load \#N
$\theta_{\mathrm{A}-1}=$ lead/lag of line current $\mathrm{I}_{\mathrm{A}-1}$ with respect to line voltage $\mathrm{V}_{\mathrm{ca}}$
$\theta_{\mathrm{A}-2}=$ lead/lag of line current $\mathrm{I}_{\mathrm{A}-2}$ with respect to line voltage $\mathrm{V}_{\mathrm{ca}} \quad .$. to
$\theta_{\text {A-N }}=$ lead/lag of line current $\mathrm{I}_{\mathrm{A}-\mathrm{N}}$ with respect to line voltage $\mathrm{V}_{\mathrm{ca}}$
$\mathrm{I}_{\mathrm{B}}=\left\{\left(\mathrm{X}_{\mathrm{B}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}\right)^{2}\right\}^{1 / 2}$, and $\ldots$ Equation 11-3
$\theta_{L-B}=\sin ^{-1}\left(Y_{B} \div I_{B}\right)$, where $\ldots$ Equation 11-4
$\mathrm{X}_{\mathrm{B}}=\Sigma\left(\mathrm{X}_{\mathrm{B}-1}+\mathrm{X}_{\mathrm{B}-2}+\mathrm{X}_{\mathrm{B}-3} \ldots+\mathrm{X}_{\mathrm{B}-\mathrm{N}}\right)$
$\mathrm{Y}_{\mathrm{B}}=\Sigma\left(\mathrm{Y}_{\mathrm{B}-1}+\mathrm{Y}_{\mathrm{B}-2}+\mathrm{Y}_{\mathrm{B}-3} \ldots+\mathrm{Y}_{\mathrm{B}-\mathrm{N}}\right)$
$\mathrm{X}_{\mathrm{B}-1}=\mathrm{I}_{\mathrm{B}-1} \cos \theta_{\mathrm{B}-1} \quad \ldots$ to
$\mathrm{X}_{\mathrm{B}-\mathrm{N}}=\mathrm{I}_{\mathrm{B}-\mathrm{N}} \cos \theta_{\mathrm{B}-\mathrm{N}}$
$\mathrm{Y}_{\mathrm{B}-1}=\mathrm{I}_{\mathrm{B}-1} \sin \theta_{\mathrm{B}-1} \quad \ldots$ to
$\mathrm{Y}_{\mathrm{B}-\mathrm{N}}=\mathrm{I}_{\mathrm{B}-\mathrm{N}} \sin \theta_{\mathrm{L}-\mathrm{N}}$
$\mathrm{I}_{\mathrm{B}-1}=$ line current in branch of conductor B to load \#1
$\mathrm{I}_{\mathrm{B}-2}=$ line current in branch of conductor B to load \#2 ... to
$\mathrm{I}_{\mathrm{B}-\mathrm{N}}=$ line current in branch of conductor B to load $\# \mathrm{~N}$
$\theta_{\mathrm{B}-1}=$ lead/lag of line current $\mathrm{I}_{\mathrm{B}-1}$ with respect to line voltage $\mathrm{V}_{\mathrm{ca}}$
$\theta_{\mathrm{B}-2}=$ lead/lag of line current $\mathrm{I}_{\mathrm{A}-2}$ with respect to line voltage $\mathrm{V}_{\mathrm{ca}} \ldots$ to
$\theta_{\mathrm{B}-\mathrm{N}}=$ lead/lag of line current $\mathrm{I}_{\mathrm{A}-\mathrm{N}}$ with respect to line voltage $\mathrm{V}_{\mathrm{Ca}}$
$\mathrm{I}_{\mathrm{B}-1}=$ line current in branch of conductor B to load \#1
$\mathrm{I}_{\mathrm{B}-2}=$ line current in branch of conductor B to load \#2 $\ldots$ to
$\mathrm{I}_{\mathrm{B}-\mathrm{N}}=$ line current in branch of conductor B to load \#N
$\theta_{\mathrm{B}-1}=$ lead/lag of line current $\mathrm{I}_{\mathrm{B}-1}$ with respect to line voltage $\mathrm{V}_{\mathrm{ab}}$
$\theta_{\mathrm{B}-2}=$ lead/lag of line current $\mathrm{I}_{\mathrm{B}-2}$ with respect to line voltage $\mathrm{V}_{\mathrm{ab}} \ldots$ to
$\theta_{\mathrm{B}-\mathrm{N}}=$ lead/lag of line current $\mathrm{I}_{\mathrm{B}-\mathrm{N}}$ with respect to line voltage $\mathrm{V}_{\mathrm{ab}}$
$\mathrm{I}_{\mathrm{C}}=\left\{\left(\mathrm{X}_{\mathrm{C}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{C}}\right)^{2}\right\}^{1 / 2}$, and $\ldots$ Equation 11-5
$\theta_{\mathrm{L}-\mathrm{C}}=\zeta-240^{\circ} \ldots$ Equation 11-6
$\zeta=\sin ^{-1}\left(\mathrm{Y}_{\mathrm{C}} \div \mathrm{I}_{\mathrm{C}}\right)$
$\mathrm{X}_{\mathrm{C}}=\Sigma\left(\mathrm{X}_{\mathrm{C}-1}+\mathrm{X}_{\mathrm{C}-2}+\mathrm{X}_{\mathrm{C}-3} \ldots+\mathrm{X}_{\mathrm{C}-\mathrm{N}}\right)$
$\mathrm{Y}_{\mathrm{C}}=\Sigma\left(\mathrm{Y}_{\mathrm{C}-1}+\mathrm{Y}_{\mathrm{C}-2}+\mathrm{Y}_{\mathrm{C}-3} \ldots+\mathrm{Y}_{\mathrm{C}-\mathrm{N}}\right)$
$\mathrm{I}_{\mathrm{C}-1}=$ current in branch of conductor C to load \#1
$\mathrm{I}_{\mathrm{C}-2}=$ current in branch of conductor C to load \#2 $\ldots$ to
$\mathrm{I}_{\mathrm{C}-\mathrm{N}}=$ current in branch of conductor C to load \#N
$\mathrm{X}_{\mathrm{C}-1}=\mathrm{I}_{\mathrm{C}-1}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{C}-1}-\cos \theta_{\mathrm{C}-1}\right]$
$\mathrm{X}_{\mathrm{C}-2}=\mathrm{I}_{\mathrm{C}-2}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{C}-2}-\cos \theta_{\mathrm{C}-2}\right] \ldots$ to
$\mathrm{X}_{\mathrm{C}-\mathrm{N}}=\mathrm{I}_{\mathrm{C}-\mathrm{N}}(1 / 2)\left[(\sqrt{3}) \sin \theta_{\mathrm{C}-\mathrm{N}}-\cos \theta_{\mathrm{C}-\mathrm{N}}\right]$
$\mathrm{Y}_{\mathrm{C}-1}=-\mathrm{I}_{\mathrm{C}-1}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{C}-1}+\sin \theta_{\mathrm{C}-1}\right]$
$\mathrm{Y}_{\mathrm{C}-2}=-\mathrm{I}_{\mathrm{C}-2}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{C}-2}+\sin \theta_{\mathrm{C}-2}\right] \ldots$ to
$\mathrm{Y}_{\mathrm{C}-\mathrm{N}}=-\mathrm{I}_{\mathrm{C}-\mathrm{N}}(1 / 2)\left[(\sqrt{3}) \cos \theta_{\mathrm{C}-\mathrm{N}}+\sin \theta_{\mathrm{C}-\mathrm{N}}\right]$
$\theta_{\mathrm{C}-1}=$ lead/lag of line current $\mathrm{I}_{\mathrm{C}-1}$ with respect to line voltage $\mathrm{V}_{\mathrm{bc}}$
$\theta_{\mathrm{C}-2}=$ lead/lag of line current $\mathrm{I}_{\mathrm{C}-2}$ with respect to line voltage $\mathrm{V}_{\mathrm{bc}} \ldots$ to
$\theta_{\mathrm{C}-\mathrm{N}}=$ lead/lag of line current $\mathrm{I}_{\mathrm{C}-\mathrm{N}}$ with respect to line voltage $\mathrm{V}_{\mathrm{bc}}$

Equations 12-1: Equation for the relationship of phase current and line current lead/lag angle in a balanced delta and a balanced or unbalanced wye circuit
$\theta_{\mathrm{P}}=\theta_{\mathrm{L}}-30^{\circ} \quad \ldots \quad$ Equation 12-1
where
$\theta_{\mathrm{L}}=$ line lead/lag angle between line current and line voltage (degrees or radians)(for lagging current, $\theta_{\mathrm{L}}<0$; for leading current, $\theta_{\mathrm{L}}>0$ )
$\theta_{\mathrm{P}}=$ phase lead/lag angle between phase current and phase voltage (degrees or radians) (for lagging current, $\theta_{\mathrm{P}}<0$; for leading current, $\theta_{\mathrm{P}}>0$ )
Specifically,
For balanced delta circuits:

$$
\begin{aligned}
& \theta_{\mathrm{P}-\mathrm{CA}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{AB}}=\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{BC}}=\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}
\end{aligned}
$$

For balanced or unbalanced wye circuits:

$$
\begin{aligned}
& \theta_{\mathrm{P}-\mathrm{A} / \mathrm{AD}}=\theta_{\mathrm{L}-\mathrm{A} / \mathrm{CA}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{B} / \mathrm{BD}}=\theta_{\mathrm{L}-\mathrm{B} / \mathrm{AB}}-30^{\circ} \\
& \theta_{\mathrm{P}-\mathrm{C} / \mathrm{CD}}=\theta_{\mathrm{L}-\mathrm{C} / \mathrm{BC}}-30^{\circ}
\end{aligned}
$$

