



PDHonline Course E483 (4 PDH)

Calculating Nonlinear Power

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Calculating Nonlinear Power

1.0 Background

The terms “linear” and “nonlinear” are often used in reference to the potentials and currents of both electronic circuits and power circuits. In this course, only alternating current (AC) power circuits are considered.

The term “linear” can have various definitions. As used with regard to AC power circuits the term is generally accepted to designate a function that may be described as a sinusoidal function. Most AC power circuits are of this description. However, in recent years more and more circuits can be characterized as nonlinear. A nonlinear circuit is one that is other than linear. In other words, all of the properties of a nonlinear circuit cannot be described by a sine function. Often the effects of nonlinearity in a circuit are relatively innocuous and of no consequence. Other times corrective measures may be warranted.

Sometimes it becomes desirable to quantify the power of a nonlinear circuit. The reasons for a determining the actual power in a nonlinear circuit might vary. One cause might be to rectify actual power consumption with meter readings.

Calculating power in nonlinear circuits can be a challenge but there are methods. Where to start? A good place to begin is with an understanding of root mean square values and average power.

2.00 Power of a Linear Single Phase Circuit

When the values of a linear AC circuit are cited it is automatically assumed that, in the absence of any mentioned exceptions, stated potentials and currents may be assumed to be root mean square (RMS) values. For example, a circuit could be described as one that has a potential of 120 VAC and current of 10 amps. The terms “potential” and “current” are the variables. The “VAC” is short for “volts-alternating current” and the “amps” is short for “amperage.” The “volts” and “amps” are the units and the “120” and “10” are the values, respectively, of the potential and the current. The acronym “AC” distinguishes the circuit from a DC (direct current) circuit. It may safely be assumed that the voltage of “120” and the

current of “10” are RMS values. But, why RMS values?

It would appear that the short explanation of the use of RMS values in AC circuits is that AC power computations using RMS values simulate those of DC computations. For example, the power of an AC circuit with a voltage of 120 VAC, a current of 10 amps and current in-phase with voltage is: $P = (120)(10) = 1200$ watts. If a DC circuit has 120 VDC and 10 amps, the power is likewise: $P = (120)(10) = 1200$ watts. Thus, the similarity of the two computations. The similarity is due solely because RMS values were used in the calculation of AC power rather than some other property as, for example, peak voltage.

Instantaneous power in any circuit, DC or AC, is defined by the expression,

$$p^i = v^i \cdot i^i, \text{ where}$$

$$p^i = \text{instantaneous power (watts)}$$

$$v^i = \text{instantaneous potential (volts)}$$

$$i^i = \text{instantaneous current (amps)}$$

If the voltage of a circuit can be described by a sinusoidal function the applicable expression for voltage is,

$$v^i = (v_{PK}) \sin \omega t, \text{ where}$$

$$v_{PK} = \text{peak potential (volts)}$$

$$\omega = 2\pi f \text{ (rads/sec)}$$

$$f = \text{frequency (hz)}$$

$$t = \text{time (sec)}$$

Likewise, if the current can be described by a sinusoidal function, the common expression for current is:

$$i^i = i_{PK} \sin (\omega t + \theta), \text{ where}$$

$$i_{PK} = \text{peak current (amps)}$$

$$\theta = \text{lead/lag of current with respect to applied voltage (radians or degrees)}$$

$$\text{for a lagging power factor, } \theta < 0$$

$$\text{for a leading power factor, } \theta > 0$$

The instantaneous power in an AC circuit with both a linear potential and a linear current can be described by the expression,

$$p^i = [(v_{PK}) \sin \omega t] [i_{PK} \sin (\omega t + \theta)], \text{ or}$$

$$p^i = (v_{PK}) (i_{PK}) (\sin \omega t) [\sin (\omega t + \theta)]$$

Since, $\sin (\omega t + \theta) = \sin \omega t \cos \theta + \cos \omega t \sin \theta$

$$p^i = (v_{PK}) (i_{PK}) (\sin \omega t) [\sin \omega t \cos \theta + \cos \omega t \sin \theta], \text{ or}$$

$$p^i = (v_{PK}) (i_{PK}) (\cos \theta) (\sin \omega t)^2 + (v_{PK}) (i_{PK}) (\sin \theta) (\sin \omega t) (\cos \omega t) \dots$$

(Equation 1)

Equation 1 is often presented in different forms. One common version is sometimes more convenient to use in computations is:

$$p^i = [(v_{PK}) (i_{PK})/2] \cos \theta - [(v_{PK}) (i_{PK})/2] \cos (2\omega t + \theta) \dots \quad \text{Equation 2}$$

Example 1

It will be informative to plot the computed values of power according to Equation 1 for one complete cycle ($\omega t = 0^\circ$ to $\omega t = 360^\circ$) of the variable ωt for a typical single phase application. The plot provides a pictorial view of the instantaneous value of power as a function of time. In the way of illustration, consider a specific case having the following parameters:

Power Factor = 0.70, lagging

Thus,

$$v_{PK} = 480 \sqrt{2} = 678.82 \text{ VAC, and}$$

$$i_{PK} = 10 \sqrt{2} = 14.14 \text{ amps}$$

where

v_{PK} = peak voltage

i_{PK} = peak current

Power Factor = 0.70 = $\cos \theta = 0.70$, and $\theta = -45.57^\circ$

Equation 1 states,

$$p^i = (v_{PK}) (i_{PK}) (\cos \theta) (\sin^2 \omega t) + (v_{PK}) (i_{PK}) (\sin \theta) (\sin \omega t) (\cos \omega t)$$

Let,

$$A = (v_{PK}) (i_{PK}) \cos \theta \sin^2 \omega t, \text{ and } B = (v_{PK}) (i_{PK}) \sin \theta \sin \omega t \cos \omega t$$

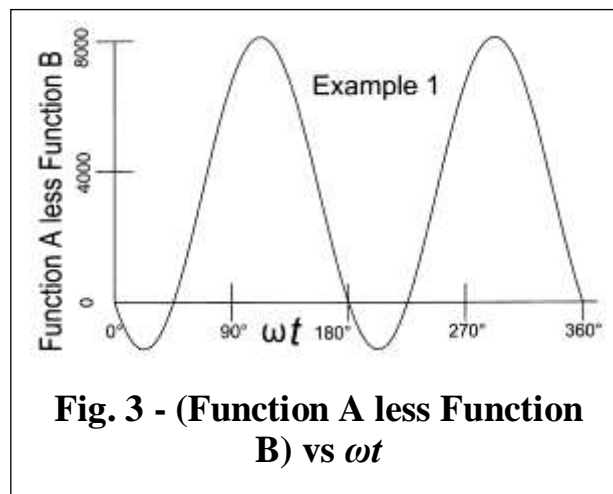
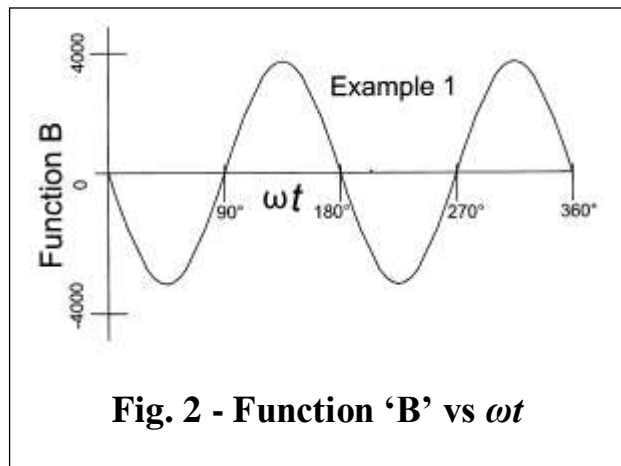
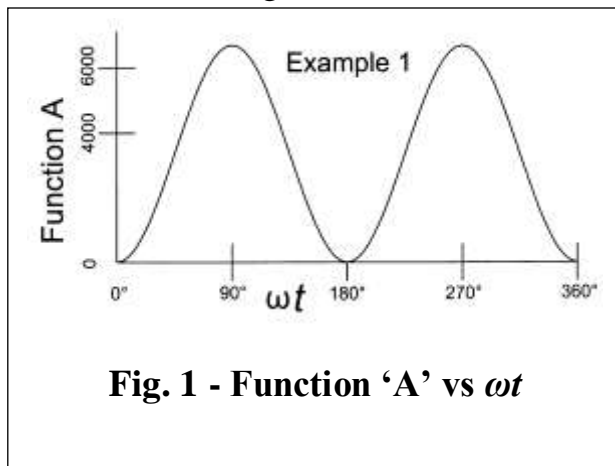
$$p^i = A + B$$

$$A = (480 \sqrt{2}) (10 \sqrt{2}) (0.7) \sin^2 \omega t = (6720.00) \sin^2 \omega t, \text{ and}$$

$$B = (480 \sqrt{2}) (10 \sqrt{2}) (\sin -45.57^\circ) \sin \omega t \cos \omega t$$

$$B = (9600) (-.7141) \sin \omega t \cos \omega t = (-6855.36) \sin \omega t \cos \omega t$$

The values of Function A in this example were computed and are plotted to-scale in Fig. 1. Likewise the values of Function B were computed and are shown in Fig. 2, also drawn to-scale. A plot of Function A plus Function B is shown in Fig. 3.



According to Equation 1, the instantaneous power in a single phase circuit in which both the potential and the current is described by a sinusoidal equation is:

$$p^i = (v_{PK}) (i_{PK}) (\cos \theta) (\sin^2 \omega t) + (v_{PK}) (i_{PK}) (\sin \theta) (\sin \omega t) (\cos \omega t)$$

Generally the average power throughout a specific period of time is of the most interest. To determine the average power a single cycle of ωt is considered (the value of $\omega t = 0$ to $\omega t = 360^\circ$). To determine average power, the net power over a single cycle is determined and then divided by the time of one period.

Let T = the time for a single cycle (from $\omega t = 0^\circ$ to $\omega t = 360^\circ$). The net power in the sample period is determined from integration of the equation for instantaneous power (Equation 1) with respect to t .

Let,

$$p^i = A(t) + B(t), \text{ whereby}$$

$$A(t) = (v_{PK}) (i_{PK}) \cos \theta \sin^2 \omega t, \text{ and}$$

$$B(t) = (v_{PK}) (i_{PK}) \sin \theta \sin \omega t \cos \omega t$$

$$\int A(t) = \int (v_{PK}) (i_{PK}) \cos \theta \sin^2 \omega t dt = (v_{PK}) (i_{PK}) \cos \theta \int \sin^2 \omega t dt$$

$$\int A(t) = (v_{PK}) (i_{PK}) \cos \theta [t/2 - (\sin 2 \omega t)/4\omega]$$

The integral of the function A(t) is evaluated from $t = 0$ to $t = T$, (with value "T" occurring at $\omega t = 360^\circ$)

$$\int A(t) = [(v_{PK}) (i_{PK}) \cos \theta] \{ [T/2 - (\sin 2 \omega t)/4\omega] - [0/2 - (\sin 0)/4\omega] \}$$

$$\int A(t) = [(v_{PK}) (i_{PK}) \cos \theta] (T/2)$$

It will be apparent that the function B(t) throughout the period $t = 0$ to $t = T$ provides no net addition to the value of function P. This is the case since half of the function is positive during the selected period of time and half is negative, with the result that there is no net contribution of the function to the value of power throughout the period. Consequently, the function A(t) solely determines the value of power throughout one cycle.

Therefore, the average value of $\int A(t)$, and power, throughout the period is:

$$P = [(v_{PK}) (i_{PK}) \cos \theta] [T/2] / T = [(v_{PK}) (i_{PK}) \cos \theta] / 2$$

This equation may be written as,

$$P = [v_{PK}/\sqrt{2}] [i_{PK}/\sqrt{2}] \cos \theta$$

The term $[v_{PK}/\sqrt{2}]$ is recognized as the root mean square of the sine function of AC voltage. More specifically,

$$V_{RMS} = v_{PK}/\sqrt{2}$$

Also, the term $[i_{PK}/\sqrt{2}]$ is recognized as the root mean square of the sine function of AC current, or

$$I_{RMS} = i_{PK}/\sqrt{2}$$

Voltages and currents are commonly used in RMS (root mean squared) values. Accordingly the “RMS” may be dropped with the understanding that the term “V” and “I” designates respectively RMS values of potential and current.

Thus,

$$P = VI \cos \theta \dots \text{Equation 3}$$

Where,

V = root mean square of AC voltage

$$= v_{PK}/\sqrt{2}$$

I =root mean square of AC current

$$= i_{PK}/\sqrt{2}$$

In Example 1 it is demonstrated that the values of root mean square of potential and current may be used to calculate the power throughout a given period of time in a linear AC circuit, the relationship being described by Equation 3.

The root mean values of sinusoidal AC potentials and currents are useful in the computations of AC power. It is pertinent to review the importance of the root mean square value of a function.

3.0 Root Mean Square

The root mean square of a function is also known as the “quadratic mean.” A value of root means square is defined as the mean of the squares of a function. Root mean square values are commonly used in electrical calculations but root mean square values are also computed for a number of other purposes many of which have nothing to do with electrical circuits. A value of root means square can be

determined by one of two possible methods. If values of a function are defined by an equation, the root means square can be calculated with precision. Alternatively, the root means square can be determined from discrete values distributed throughout the region of consideration. Generally the accuracy obtained by this later mentioned method is a function of the number of samples taken throughout the region of consideration. The larger the number of samples the better the accuracy. Of course, the greater the number of samples the greater the time and effort to complete the calculations. For most engineering purposes, a moderate number of samples will usually provide adequate accuracy. For a function that is given by an equation from time T_1 to time T_2 , the root mean square is defined as:

$$f(t)_{\text{RMS}} = \left\{ 1/(T_2 - T_1) \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2} \dots \text{Equation 4}$$

An alternate method of determining root mean square used discrete values. If discrete values are known throughout a region, the root mean square is defined as:

$$f(x)_{\text{RMS}} = \left\{ [1/n][x_1^2 + x_2^2 + \dots x_n^2] \right\}^{1/2} \dots \text{Equation 5}$$

The values of x_1, x_2, x_n used in Equation 5 must be evenly spaced values within the region of the variable x under consideration.

3.1 Root Mean Square and Power

If both the voltage and the current are linear functions the power throughout a period of time can be computed using the root mean square values of the voltage and the current. However, if either the voltage or the current are nonlinear, the root mean square of either function cannot be used to compute power. The product of the RMS values of a nonlinear circuit would yield incorrect values. To compute power when either the voltage or the current is nonlinear a method independent of root mean square values is needed.

4.0 Root Mean Square vs Average

When computing power, it is important to understand the difference between root mean square and average. The root mean square of a function is not the average value of a function. The average value of a function throughout a region is defined as the area under the function's curve divided by the base value. A few examples using various shapes will illustrate the principles involved.

4.1 Rectangles

4.1.1 Root Mean Square of Rectangle

A function of a rectangular shape is shown in Fig. 4. The representation could be, for example, a DC potential or a DC current from time T_1 to time T_2 . The value of the function throughout the period of consideration is: $f(t) = a$

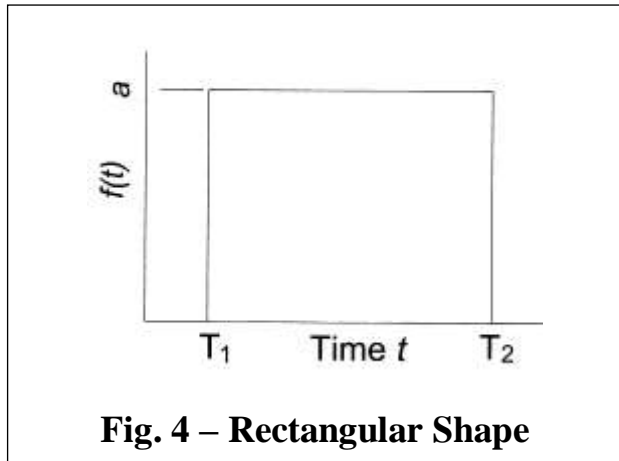


Fig. 4 – Rectangular Shape

From Equation 4 the root mean square of the function is determined by the expression,

$$f(t)_{\text{RMS}} = \left\{ 1/(T_2 - T_1) \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2}$$

Let,

$$f(t) = a; T_1 = 0 \text{ \& } T_2 = T$$

$$\begin{aligned} f(t)_{\text{RMS}} &= \left\{ 1/(T) \int_0^T [f(a)]^2 dt \right\}^{1/2} \\ &= \{ Ta^2/T \}^{1/2} = a \end{aligned}$$

So, the root mean square of a rectangle of height “a” is a.

If a DC voltage is defined by: $f(v) = A$, and the DC current as $f(i) = B$

Using the RMS values, power may be computed as: $P = AB$

Example 2

A plot of a DC potential is essentially in the shape of a rectangle as represented in Fig. 5. The top of the rectangle is the value of the potential. The left side is the time when the plot begins and the right side is the time when the plot ends. The bottom is the “0” value of the voltage. The same may be said

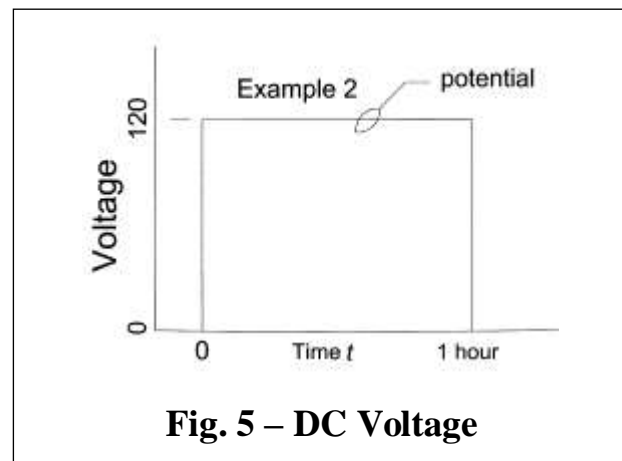


Fig. 5 – DC Voltage

of a DC current. The DC potential is assumed to be 120 VDC. A typical DC current plot is represented in Fig. 6 and is assumed to be 10 amps. The plots assume a beginning time and an end time. The plot is assumed to start at time $t = 0$ and to end at time $t = 1$ hour.

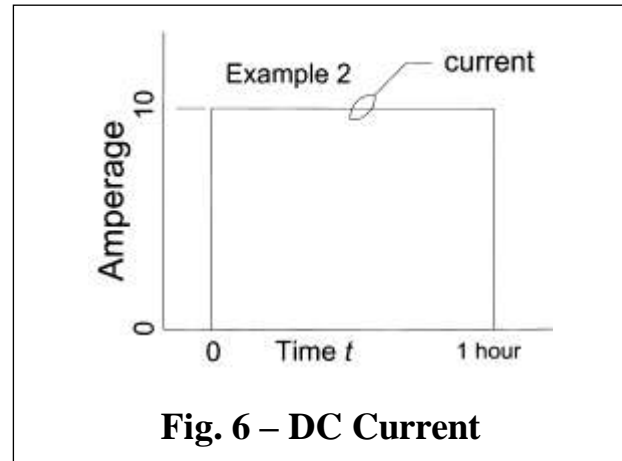


Fig. 6 – DC Current

It is apparent that the root mean square of voltage is 120 VDC, the root mean square current is 10 amps. The instantaneous power at any time between time $t = 0$ and time $t = 1$ hour is $p = (120)(10)$, or 1200 watts, and the average power throughout the period is also $P = 1200$ watts.

4.1.2 Average of a Rectangle

With the application of Equation 4, it is apparent that the root mean square of a rectangle shape of height A is: A . Likewise if a rectangle is of height A and base T , the average is $AT/T = A$. In this case the average is the same as the root mean square. The RMS and average values of a typical rectangle shape are shown in Fig. 7

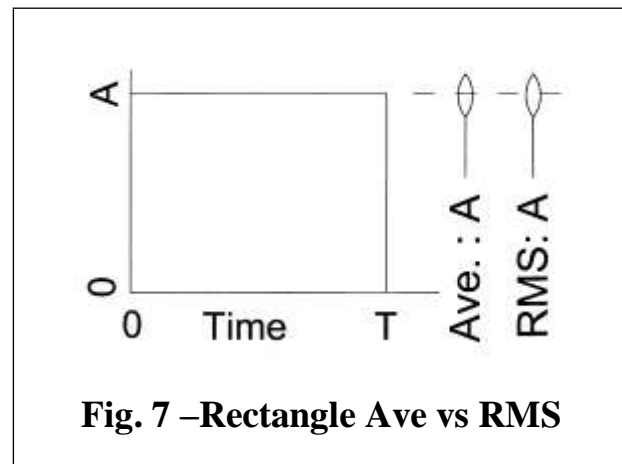


Fig. 7 –Rectangle Ave vs RMS

Example 3

Consider an application in which DC power at 2 kW is constant throughout a period of 1 hour. The average is:

$$(2 \text{ kW})(1 \text{ hour}) / (1 \text{ hour}) = 2 \text{ kW}.$$

4.2 Triangles

4.2.1 Root Mean Square of a Triangle

If $f(t) = at$, the configuration with time takes the shape of a triangle as represented

in Fig. 8. This function is commonly described as a ramp and could be representative of voltage or current. The RMS of the triangular function becomes:

$$f(t)_{\text{RMS}} = \left\{ \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2}$$

$$f(t)_{\text{RMS}} = \left\{ \frac{1}{(T)} \int_0^T [f(at)]^2 dt \right\}^{1/2}$$

$$= \left\{ \left[\frac{1}{(T)} \right] [a^2 T^3] \left[\frac{1}{3} \right] \right\}^{1/2}$$

$$= (aT/\sqrt{3})$$

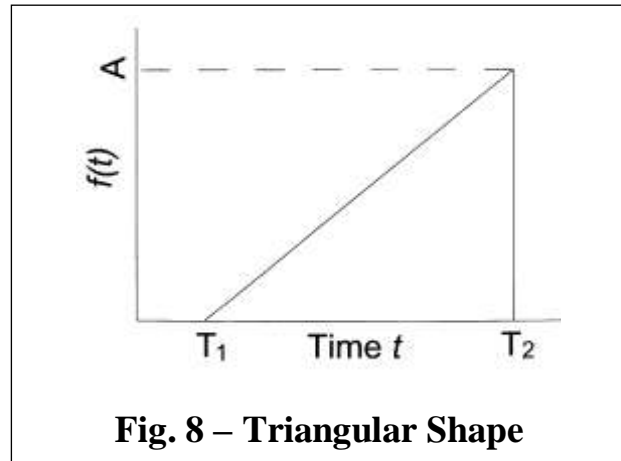


Fig. 8 – Triangular Shape

It is apparent that the value (aT) is the peak value at time T.

Let,

$$(aT) = A$$

Then,

$$f(t)_{\text{RMS}} = A/\sqrt{3} = .5773 A$$

Example 4

Consider a ramp function example is which

$$f(v) = at \text{ (volts)}$$

$$f(i) = bt \text{ (amps)}$$

$$T = 4 \text{ hours}$$

Let $a = 1$ and $b = 3$. Then $v = (1) t$ and $i = 3t$.

The RMS values are:

$$V_{\text{RMS}} = (aT) (1/\sqrt{3}) = (1 \cdot 4) (1/\sqrt{3}) = 4/\sqrt{3} = 2.309$$

$$I_{\text{RMS}} = (bT) (1/\sqrt{3}) = (3 \cdot 4) (1/\sqrt{3}) = 12/\sqrt{3}$$

Since both $f(v)$ and $f(i)$ are of the same shape, namely triangular, power can be calculated by use of the respective RMS values.

$$P = (V_{\text{RMS}}) (I_{\text{RMS}})$$

$$P = [(4) (1/\sqrt{3})] [(12) (1/\sqrt{3})] = 48/3 = 16 \text{ watts}$$

4.2.2 Average of a Triangle

If a triangle is of height A at the end of a period of time T , the average is: $AT/2T = A/2$. Typical RMS and average values of a triangle are shown in Fig. 9, drawn to-scale.

Example 5

Consider a period of time of 1 hour during which voltage increases at the rate of $v = 2$ (volts per hour). At the end of $t = 1$ hour, $v = (2)(1) = 2$ volts.

Average: $(1/2)(2 \cdot 1) = 1$ volt

4.3 Sine Function

4.3.1 Sine Function Root Mean Square

If $f(t) = a \sin \omega t$, the configuration is that of a sine function. Since the function in the period $\omega t = \pi/2$ to $\omega t = \pi$ is a mirror image of the function from $\omega t = 0$ to $\omega t = \pi/2$ it is necessary to consider only the region from $\omega t = 0$ to $\omega t = \pi/2$. The function takes the shape of a sin wave as represented in Fig. 10. The RMS of the function becomes:

$$f(t)_{\text{RMS}} = \left\{ \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2}$$

For the period $\omega t = 0$ to $\omega t = \pi/2$

$$f(t)_{\text{RMS}} = \left\{ \frac{1}{(T)} \int_0^{\pi/2} [a \sin \omega t]^2 dt \right\}^{1/2}$$

$$= a \left\{ \frac{1}{(\pi/2)} [\pi/4 - \sin \pi/4] \right\}^{1/2} = a \left\{ \frac{1}{(\pi/2)} [\pi/4] \right\}^{1/2} = a/\sqrt{2}$$

Example 6

Consider an example is which,

$$f(v) = a \sin \omega t \text{ (volts)}$$

$$f(i) = b \sin (\omega t + \theta) \text{ (amps)}$$

$$\theta = 10^\circ$$

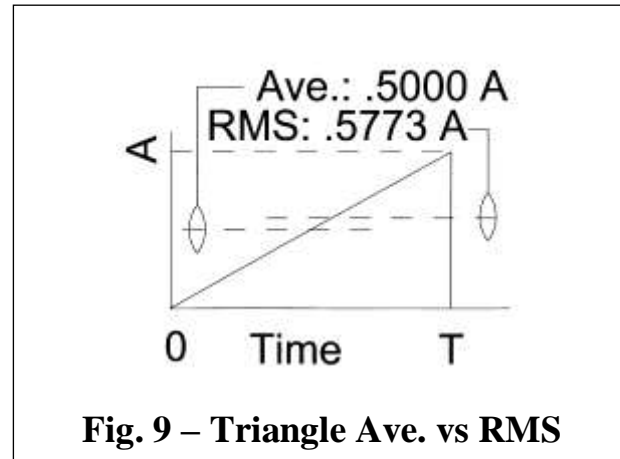


Fig. 9 – Triangle Ave. vs RMS

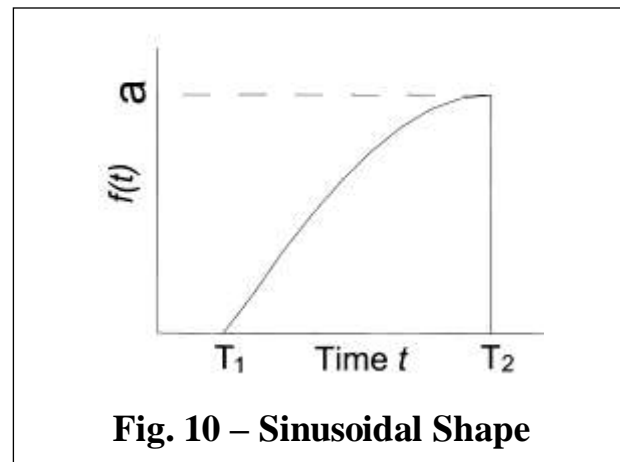


Fig. 10 – Sinusoidal Shape

Determine the root means square of the potential, the root mean square of the current and the circuit power.

Let $a = 2$ and $b = 3$. Then $v = a \sin \omega t$ and $i = b \sin (\omega t + \theta)$.

Consider first the voltage function. As demonstrated, the root mean square of a sine function originating at $t = 0^\circ$ is:

$$V_{\text{RMS}} = a/\sqrt{2}, \text{ where } a = \text{peak value}$$

The root mean square of the current function can be derived from the general expression for root mean square:

$$I_{\text{RMS}} = \left\{ 1/(T_2 - T_1) \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2}$$

Here, $f(t) = b \sin (\omega t + \theta)$

$$\int_{T_1}^{T_2} [f(t)]^2 dt = b^2 \int_{T_1}^{T_2} [\sin (\omega t + \theta)]^2 dt$$

$$= b^2 \int_{T_1}^{T_2} (1) [\sin \omega t \cos \theta + \cos \omega t \sin \theta]^2 dt$$

$$= b^2 \int_{T_1}^{T_2} (1) [(\sin \omega t \cos \theta)^2 + (2) (\sin \omega t \cos \theta) (\cos \omega t \sin \theta) + (\cos \omega t \sin \theta)^2] dt$$

Integrating in parts from $t = 0$ to $t = \pi/\omega$ yields

$$\int_{T_1}^{T_2} [f(t)]^2 dt = (\pi/2\omega) [(\cos \theta)^2 + (\sin \theta)^2] = (\pi/2\omega)$$

$$I_{\text{RMS}} = \left\{ 1/(T_2 - T_1) \int_{T_1}^{T_2} [f(t)]^2 dt \right\}^{1/2} = \left\{ [1/(\pi/\omega)] (b^2) (\pi/2\omega) \right\}^{1/2} = b/\sqrt{2}$$

From Equation 3,

$$P = VI \cos \theta = (a/\sqrt{2}) (b/\sqrt{2}) \cos 10^\circ = [(2 \cdot 3)/2] (.9848) = 2.95 \text{ watts}$$

4.3.2 Sine Function Average

Consider a sine function that is described by the expression, $y = a \sin \omega t$.

The area under the function from $t = 0$ to $t = \pi/2$ is the integral:

$$\begin{aligned} f(t) &= \int_0^{\pi/2} [A \sin \omega t] dt \\ &= A(-1) \{ [\cos \pi/2] - [\cos 0] \} \\ &= -A \{ 0 - (1) \} = A \end{aligned}$$

The base was taken as $\pi/2$ (radians). If the peak value of the function is A , the average is:

$$\text{Ave: } A / \pi/2 = 2A / \pi = A(.6366).$$

Typical RMS and average values are shown in Fig. 11, drawn to-scale.

Example 7

Consider the function $f(t) = 4 \sin \omega t$. Determine the average throughout the period $t = 0$ to $t = \pi$.

Since the time from $t = \pi/2$ to $t = \pi$ is the mirror image of the time from $t = 0$ to $t = \pi/2$, the average from $t = 0$ to $t = \pi$ is the same as that from $t = 0$ to $t = \pi/2$ the average is: $2a / \pi = 2 \cdot 4 / \pi = 2.546$

5.0 Non-RMS Power Computations

In a conventional, linear AC circuit, both the voltage and the current can be described with a sine function. In addition, the power consumption throughout a period of time is the RMS voltage times the RMS current times the displacement of current with respect to the applied potential. This, of course, is the accepted and standard method of calculating the power of a linear AC circuit. Nevertheless, there are other methods of determining power.

If instantaneous power vs time relationship is known throughout a period of time, the average power for that period of time may be computed. The average power is the area under the curve of the function divided by the time base. Below, power is determined for several known functions without use of the respective root mean square values. The method used here first computes the area under the curve of the expression of instantaneous power. The area so determined is divided by the time base to calculate the average power throughout the selected time period.

5.1 Rectangle

If throughout a period of time,

$$v = a, i = b \text{ \& } p = v \cdot i = ab$$

These expressions essentially describe a DC potential, a DC current and DC

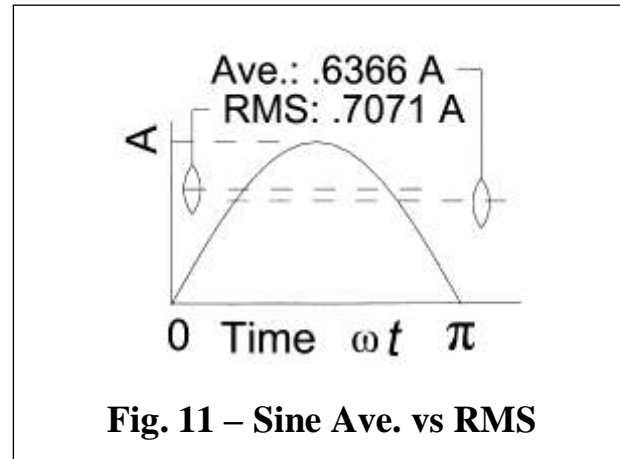


Fig. 11 – Sine Ave. vs RMS

power. From time $t = T_1$ to $t = T_2$ the area under the instant power p vs time t plot is (abT) where $T = (T_2 - T_1)$. The area divided by the base is $(abT)/T = ab$. So the average power throughout the period is: $P = ab$. The area under the p vs t plot is represented by the crosshatched area represented in Fig. 12.

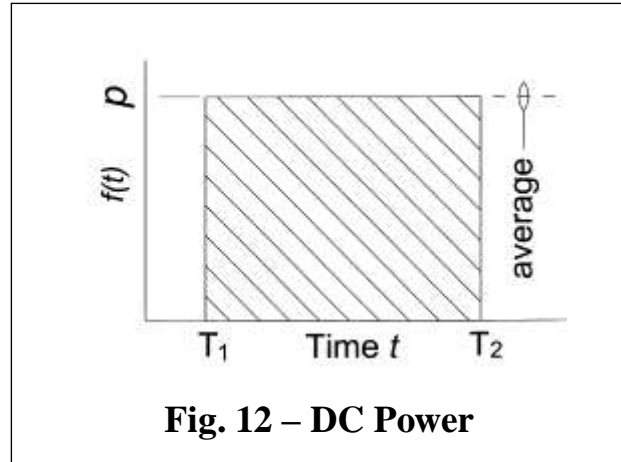


Fig. 12 – DC Power

5.2 Triangle

If throughout a period of time,

$$v = at, \quad i = bt \quad \& \quad p = v \cdot i = abt^2$$

If $v = at$ it could be said that the voltage is ramped up with time. This is not a common scenario but it is one that is possible. If the voltage were applied across a resistive load the current would likewise ramp up with time.

For the period from $t = 0$ to $t = T$, the area under the instantaneous power p vs time plot is determined by integration of the function “ $v \cdot i$ ” with time.

$$f(t) = \int_{T_1}^{T_2} f(t) dt = \int_0^T abt^2 dt = ab[T^3] / 3$$

The area under the curve $p = abt^2$ is $ab[T^3] / 3$

The base was given as “ T .” Therefore the average power is:

$$P = ab[T^3] / 3T = abT^2 / 3$$

Example 8

Consider an example in which the assumed values are:

$$p = abt^2$$

$$a = 2, \quad b = 3 \quad \text{from } t = 0 \text{ to } t = 2$$

The area under the curve $p = abt^2$ is

$$\text{Area} = ab[T^3] / 3 = 2 \cdot 3(2)^3 / 3$$

$$= 16 \text{ watt} \cdot \text{hours}$$

Peak value of power:

$$p = abt^2 = 2 \cdot 3(2)^2 = 24 \text{ watts}$$

Average power: Area under curve/base = 16 watt•hours/ 2 hours = 8 watts.

A plot of the instantaneous power is shown in Fig. 13. The area under the curve is represented by the crosshatched area.

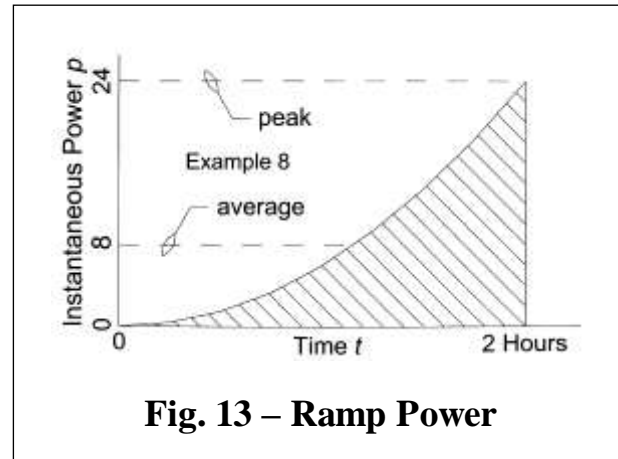


Fig. 13 – Ramp Power

5.3 Sine Function

If, throughout a period of time,

$$v = a \sin \omega t, \text{ and}$$

$$i = b \sin (\omega t + \theta)$$

$$p = v \cdot i = (a \sin \omega t) [b \sin (\omega t + \theta)]$$

This expression is the common equation of the instantaneous power of an AC circuit. As explained in above Section 1.0, the instantaneous power can be represented by the expression:

$$p = (v_{PK}) (i_{PK}) \cos \theta \sin^2 \omega t, \text{ or}$$

$$p = ab \cos \theta \sin^2 \omega t$$

The area under the power P vs time t plot is determined by integration of the function with time.

$$\int A(t) = \int ab \cos \theta \sin^2 \omega t dt = ab \cos \theta \int \sin^2 \omega t dt$$

The function is evaluated from $\omega t = 0$ to $\omega t = \pi$ for the reasons explained in above Section 1.0.

$$\int A(t) = ab (\cos \theta) [\omega t/2 - (1/2) (\sin \omega t) (\cos \omega t)]$$

Evaluated from $\omega t = 0$ to $\omega t = \pi$,

$$\int A(t) = ab (\cos \theta) \{ [\pi/2 - (1/2) (0) (-1)] - [0 - (0) (-1)] \}$$

$$\int A(t) = ab (\cos \theta) (\pi/2) \text{ (The area under the A versus } \omega t \text{ curve.)}$$

The average power from

$\omega t = 0$ to $\omega t = \pi$ is:

$$\begin{aligned} P &= [ab (\cos \theta) (\pi/2)] / \pi \\ &= [ab (\cos \theta)] / 2 \end{aligned}$$

Example 9

Determine average power using the parameters of above Example 1.

$$V_{\text{RMS}} = 480 \text{ VAC, single phase}$$

$$v_{\text{PK}} = a = 480 \sqrt{2} = 678.82 \text{ VAC}$$

$$I_{\text{RMS}} = 10 \text{ amps}$$

$$i_{\text{PK}} = b = 10 \sqrt{2} = 14.14 \text{ amps}$$

A plot of instantaneous power with time is shown in Fig. 3. The power for one cycle or one half cycle can be represented by the expression:

$$\begin{aligned} p &= ab \cos \theta \sin^2 \omega t \\ &= (678.82) (14.14) (.70) \sin^2 \omega t \\ &= 6720 \sin^2 \omega t \end{aligned}$$

For the period from $\omega t = 0^\circ$ to $\omega t = \pi$ the area, A , under the plot of instantaneous power from is:

$$\begin{aligned} A &= ab \cos \theta (T/2), \text{ or} \\ &= [ab \cos \theta] [\pi/2\omega] \end{aligned}$$

The average is the area under the curve divided by the time ($t = \pi/\omega$) is:

$$\begin{aligned} P &= \{[ab \cos \theta] [\pi/2\omega]\} / (\pi/\omega) \\ &= (ab/2) \cos \theta \end{aligned}$$

Power Factor = 0.70, lagging & $\cos \theta = 0.70$

$$\begin{aligned} P &= [ab (\cos \theta)] / 2 \\ &= (678.82) (14.14) (0.7) / 2 \\ &= 3360 \text{ watts} \end{aligned}$$

The plot representative of instantaneous power from $\omega t = 0$ to $\omega t = \pi$ is shown in Fig. 14. The plot shows a peak power of 6720 watts and the average power of 3,360 watts.

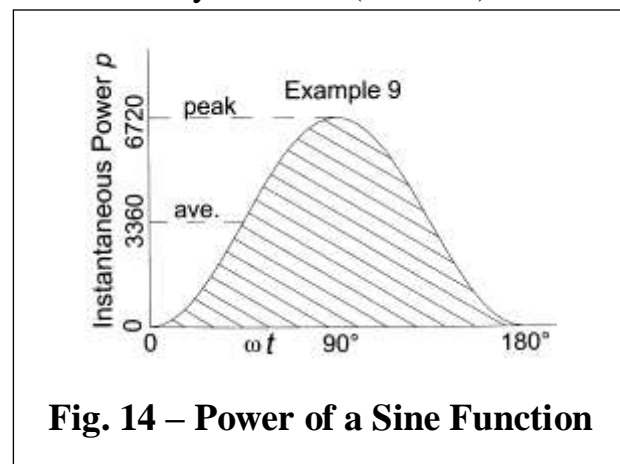


Fig. 14 – Power of a Sine Function

6.0 Graphical Plots

Nonlinear currents and potentials come in a variety of shapes but most of these shapes cannot readily be described by a mathematical expression. Nevertheless, there is a viable means to determine power when the potential or current plot is of

an irregular shape. One method involves the determination of power using a graphical plot of power with time. By this technique, power of a circuit can be determined with relatively good accuracy. A digital, recording oscilloscope is particularly useful for obtaining values for a plot of this type. After the plot of instantaneous power is established the area under the instantaneous power plot is then determined. Average power is the division of the area so determined by the respective time base.

In general there are several graphical methods that may be used to calculate the area under a curve. With most of these graphical techniques it is assumed that the area under consideration can be approximated by using either a number of narrow rectangles or trapezoids positioned side-by-side to simulate the area under consideration. Four of the more common methods are: (1) right endpoint approximation, (2) left endpoint approximation, (3) midpoint approximation, and (4) Riemann sums.

The technique of the midpoint approximation is relatively intuitive and is used here. With the midpoint approximation method the area under consideration is assumed to be made up of numerous rectangles. The rectangles are to have equal widths throughout the region of consideration and the midpoint of the top of each rectangle is bisected by the curve of the function under consideration. Typical positioning of the rectangles is shown in Fig. 15. Generally, the larger the number of rectangles the better the accuracy.

The base divided by the number of rectangles should be a whole number. For example, if the base is 90° , then rectangle widths of 2° , 2.5° or 5° would all result in the number of rectangles being a whole number. The number of rectangles corresponding to these mentioned widths would be $90/2 = 45$, $90/2.5 = 36$, and $90/5 = 18$. For a base that extends from $\omega t = 0$ to $\omega t = \pi/2$ the number of rectangles could typically be between 15 and 30. If the curve does not change direction quickly and moderate accuracy is acceptable then approximately 15 rectangles will suffice. If a curve has short radius bends of if

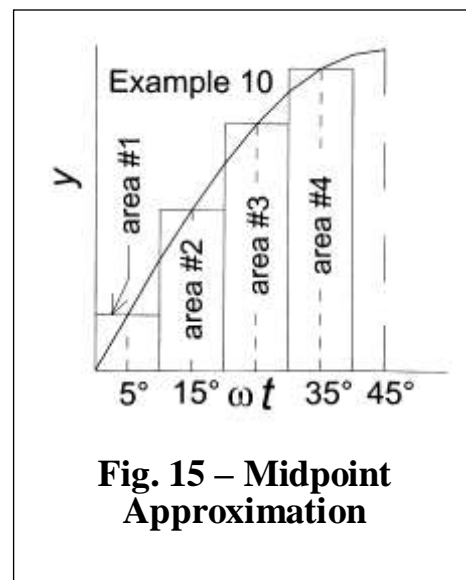


Fig. 15 – Midpoint Approximation

better accuracy is needed then a number of rectangles greater than 15 would be more appropriate. The sum of the areas of the rectangles becomes the area under the curve.

The selection of rectangles is governed by the following rules. If M = the base width and N = the number of rectangles, then the width of each rectangle is $a = M/N$. The center of the first rectangle is positioned at $a/2$. The center of the second rectangle is positioned at $(a/2 + a)$. The center of the third rectangle is positioned at $(a/2 + 2a)$, etc. The center of the n th rectangle will automatically be positioned at $(M - a/2)$.

Example 10

In this example the midpoint approximation method is used to estimate the area under the curve for a known function. The results obtained by the graphical method are compared to the true, calculated value. A typical midpoint approximation is used as shown in Fig. 15. The function under consideration in Fig. 15 is: $y = 50 \sin 2 \omega t$. For the purposes of illustration, the area under the curve from $\omega t = 0^\circ$ to $\omega t = 40^\circ$ is estimated using the method of midpoint approximation. As represented in Fig. 15, only four rectangles are used for the purposes of illustration. For better accuracy far more than four rectangles would normally be used. The four rectangular areas are identified in Fig. 15 as #1, #2, #3 and #4. Each of the rectangles has a base width of $\omega t = 10^\circ$. The midpoints for the four rectangles are:

#1 @ 5° ; #2 @ 15° ; #3 @ 25° ; #4 @ 35°

The heights of the four rectangles at midpoint are calculated as:

$$\#1 @ 5^\circ: y = 50 \sin 2 (5) = 8.68240$$

$$\#2 @ 15^\circ: y = 50 \sin 2 (15) = 25.0000$$

$$\#3 @ 25^\circ: y = 50 \sin 2 (25) = 38.3022$$

$$\#4 @ 35^\circ: y = 50 \sin 2 (35) = 46.9846$$

The base width of each of the four rectangles is: 10° (0.17453) rad and the respective areas are:

$$\#1: 8.68240 \cdot (0.17453) \text{ rad} = 1.51536$$

$$\#2: 25.0000 \cdot (0.17453) \text{ rad} = 4.36332$$

$$\#3: 38.3022 \cdot (0.17453) \text{ rad} = 6.68499$$

$$\#4: 46.9846 \cdot (0.17453) \text{ rad} = 8.20036$$

Sum of the areas: 20.76403

Since the function is defined by an equation, the area may be calculated precisely by integration of the function with time. The function is:

$$y = 50 \sin 2 \omega t$$

Area under the curve is:

$$A = \int_{T_1}^{T_2} f(t) dt, \text{ evaluated from } \omega t = 0 \text{ to } \omega t = 40^\circ$$

$$A = (50) \{ [-(1/2) (\cos 80^\circ) - [-(1/2) (\cos 0^\circ)]] \}$$

$$= (50) \{ [-(1/2) (.173648) + (1/2) (1)] \}$$

$$A = 50 [-.086824 + .5000] = (.41317) (50) = 20.6587$$

Using four rectangles and the midpoint approximation method resulted in an error of only 0.51%

(Recalculating with eight rectangles instead of four rectangles yields an area of: 20.6850 which is an error of only 0.127%.)

The computations of this example demonstrate that the midpoint approximation method is a satisfactory means of calculating the area under a curve.

From the above it may be concluded that, given the values of instantaneous power with time, the average power may be determined for a given period of time from a graphical plot of the function.

Example 11

In this example circuit power, P , is determined for known values of a linear potential, linear current and power factor. Power is computed using the above-mentioned midpoint graphical approximation method.

Assume a circuit for which:

$$V = 240 \text{ VAC (rms)}$$

$$I = 10 \text{ amp (rms)}$$

PF = 0.70, leading

Throughout one cycle the power computes to exactly:

$$P = (240) (10) (0.70) = 1680 \text{ watts}$$

Determine the value of power from a plot of instantaneous power with time.

$$v^i = (V_{PK}) \sin \omega t = (240) (\sqrt{2}) \sin \omega t = 339.4112 \sin \omega t$$

$$i^i = I_{PK} \sin (\omega t + \theta) = (10) (\sqrt{2}) \sin (\omega t + \theta)$$

$$\theta = \cos^{-1}(.70) = +45.57^\circ$$

$$i^i = I_{PK} \sin (\omega t + \theta) = (10) (\sqrt{2}) \sin (\omega t + 45.57)$$

$$= 14.12 \sin (\omega t + 45.57)$$

$$P^i = v^i \cdot i^i$$

A plot of instantaneous power, P^i , for these assumed values is shown in Fig. 16. The computed values are also used to estimate the area under the curve of P^i . The area under the plot of P^i is determined in the following manner. Only the

first half (0° to 180°) of a full cycle is used since the second half of the cycle would be a repeat of the first half and the average power of the second half of the cycle would be the same as that of the first half of the cycle. (A full cycle is considered as extending from $\omega t = 0^\circ$ to $\omega t = 360^\circ$.)

Separate sections of a width equal to $\omega t = 5^\circ$ are considered throughout the period from $\omega t = 0^\circ$ to $\omega t = 180^\circ$ and values of P^i are computed at every 5° starting at the value of ωt centered at $\omega t = 2.5^\circ$. Consequently there is a total of 36 points. It is assumed that at each point there is an area that is of a width of 5° and of a height equal to the computed value of P^i . The sum of the areas was determined to be equal to $(60443.20) (5^\circ)$. The average value of P^i throughout the period from $\omega t = 0^\circ$ to $\omega t = 180^\circ$ is:

$$P = (60443.20) (5^\circ) \div (36) (5^\circ) = 1678.97 \text{ watts}$$

In this exercise the value of power computed by estimating the area under the instantaneous power curve was within 0.061% of the value computed using the mathematically precise values of RMS voltage, RMS current and power factor.

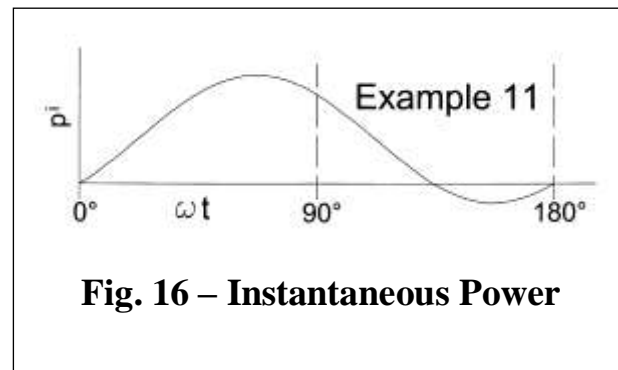


Fig. 16 – Instantaneous Power

The degree of accuracy obtained in this example would generally be considered acceptable for most applications. Nevertheless, better accuracy could generally be obtained by assuming a larger number of rectangles, each of a smaller width. It may be concluded that the described midpoint approximation method described here offers a viable method of determining the average power of functions for which instantaneous power with time is known.

With the above described method, the value of power may be determined for not only linear systems but nonlinear systems as well. Following are examples of the method used with nonlinear currents.

7.0 Nonlinear Electric Circuits

If a system has a “stiff” voltage source the voltage would be less likely to be effected by a nonlinear current. A stiff voltage source, for example, would be a situation in which a transformer has a relatively high VA rating and the nonlinear currents are relatively small. If the voltage source has a relatively low VA rating and the current is relatively large it very well could be that a nonlinear current could cause the voltage trace to deviate from the shape of a sine wave. In short, there can be both nonlinear voltages and nonlinear currents.

The increasingly greater use of thyristors has been the primary cause of the appearance of nonlinear circuits although a smaller percentage of nonlinear circuits were present before the prevalence of thyristors. Some of the more common, contemporary causes of nonlinear circuits are:

1. Electric arc furnaces
2. Fluorescent lighting
3. Switch mode power supplies for personal computers
4. Battery chargers
5. Variable frequency drives
6. Solid state inverters
7. Electric welders
8. Uninterruptible power supplies
9. Untuned capacitor banks

Although there are a variety of causes of nonlinearity in circuits, in most instances there are no resulting problems. Generally, nonlinear currents represent a small percentage of the net current consumption at a facility and most often corrective measures are unwarranted. On the other hand, some circumstances might merit ameliorating measures. A nonlinear voltage can cause overheating of transformers and motors. Protective devices are mostly designed to guard against overheating resulting from linear voltages and may not respond adequately to a nonlinear voltage.

In some circumstances a wattmeter may not indicate the true power of a nonlinear circuit. Most wattmeters are designed to measure power that is in the form of a sine wave. In consequence, some forms of nonlinear currents will cause high readings whereas others will cause low readings. If the utility's meter is found to be reading high then savings can be realized if the customer takes corrective action.

Nonlinear circuits appear in a variety of forms. The number of different shapes of possible nonlinear circuits is practically unlimited as there are a great number of possible causes of nonlinearity. Two possible forms of nonlinear currents are represented in Fig. 17 and Fig. 18. The nonlinear current of Fig. 17 is called a "spiked" current and is typical of a current trace that might result when a circuit contains capacitive elements. The representation of Fig. 18 shows a "flattened" current and is typical of a 50 hz or 60 hz circuit that is distorted by the presence of 3rd and 5th harmonic currents.

7.1 Nonlinear Power Computations

The best method to calculate the power of a nonlinear circuit is by use of a graphical method. Values of circuit

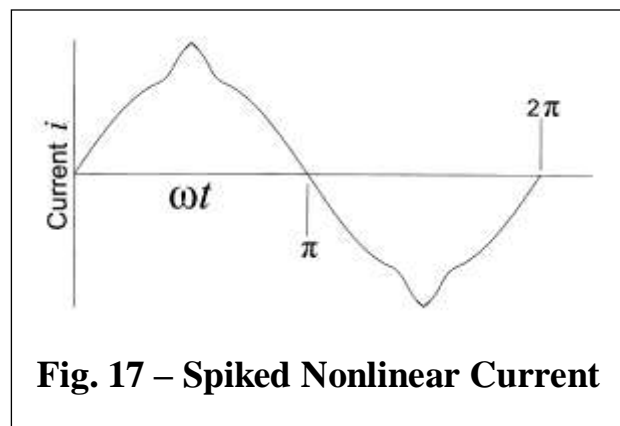


Fig. 17 – Spiked Nonlinear Current

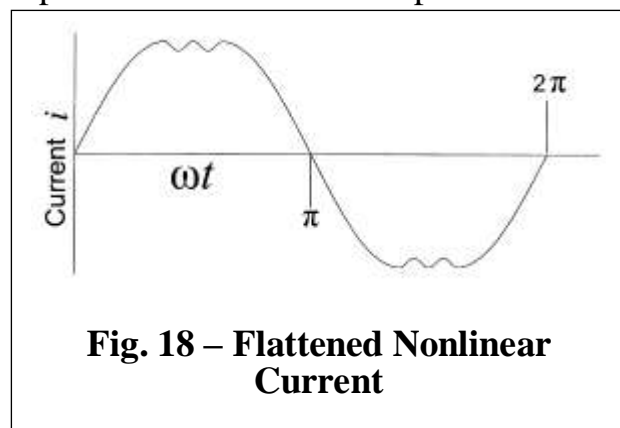


Fig. 18 – Flattened Nonlinear Current

current and potential are established as a function of time and the product of the two variables, which is instantaneous power, is plotted. The area under the instantaneous power curve divided by the time interval becomes the average power for the period under consideration. The graphical method of midpoint approximation described in above Section 6 is used.

Example 12

In this example a “spiked” nonlinear current of the general configuration of Fig. 17 is considered and the associated power is calculated. The voltage is assumed to be in-phase with the current and in the configuration of a sine wave. The voltage is assumed to be a “stiff” source and to have the profile of a sine function. The following parameters are assumed:

Current peak of $i^i = (10) (\sqrt{2})$. (This is the same peak current that would be found if the current were a sine wave and 10 amps rms.) The circuit voltage is assumed to be a linear voltage of $V = 480$ (RMS) with a peak of $v^i = (480) (\sqrt{2})$, or 678.82. The method of midpoint approximation discussed in above Section 6.0 is used to calculate the area under the instantaneous power curve. To calculate the area under the instantaneous power curve, two sets of calculations are performed: (1) one set with rectangle widths of 5° , and (2) one set with rectangle widths of 2.5° . It is assumed that the current can be described by the following equations:

$$\text{From } 0^\circ \text{ to } 65^\circ: (i) = (10.2608053) \sin [(1.15) x]$$

$$\text{From } 65^\circ \text{ to } 75^\circ: (i) = 12.77222 + 2.917055 \{ \sin [(280) + 5(x - 65)] \}$$

$$\text{From } 75^\circ \text{ to } 90^\circ: (i) = -8.21643 + 22.55161 \{ \sin [(420) + 1.5(x - 75)] \}$$

It is assumed that the potential can be described by the following equation:

$$\text{From } 0^\circ \text{ to } 90^\circ:$$

$$(v) = 678.822 \sin x$$

A plot of a nonlinear current described by these equations is depicted in Fig. 19 and drawn to scale. Particulars of the computations are tabulated in

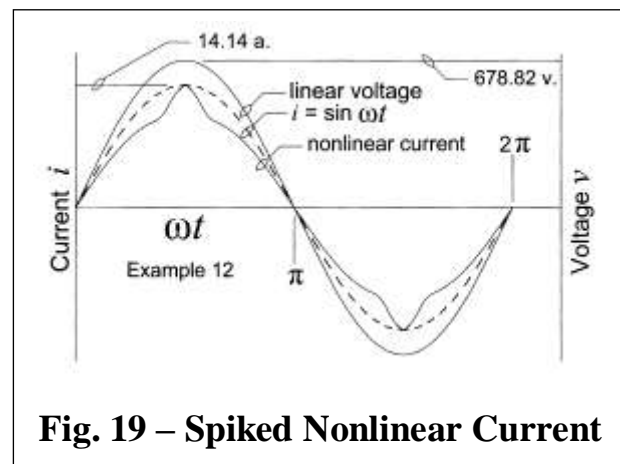


Fig. 19 – Spiked Nonlinear Current

Appendix 1 and Appendix 2. A plot of the instantaneous power is shown in Fig. 20.

Computations using rectangle widths of 5° (0.08726 rads) are shown in Appendix 1. The area under the instantaneous power curve is assumed to be comprised of total of 18 rectangles. The total area under the curve becomes

(72587.63) (0.08726 rads). The time period is (18) (0.08726 rads), and the average power is $(72587.63) / 18 = 4032.64$. A scaled drawing of the area under the instantaneous power curve is shown in Fig. 21.

Computations were also performed using a rectangle width of 2.5° .

The results of the computations are shown in Appendix 2. In this set of calculations the area under the instantaneous power curve is assumed to be comprised of total of 36 rectangles. The total area under the curve becomes:

(145,178.50) (0.04363 rads). The period of time is: (36) (0.04363 rads).

Therefore the average power in the period is: $(145,178.50) / 36 = 4032.73$ watts. Obviously, in this instance the smaller rectangles made little difference in the computation of average power. A value of 4032.7 watts is assumed to be more accurate.

If the current were a linear current described by a sine function of magnitude 10 amps. RMS (or 14.142 amps peak), then the RMS values could be used to calculate power for the period under consideration. That value would be: $P = 480 \cdot 10 = 4800$ watts. A plot of a 10 amp RMS current is shown in Fig. 19.

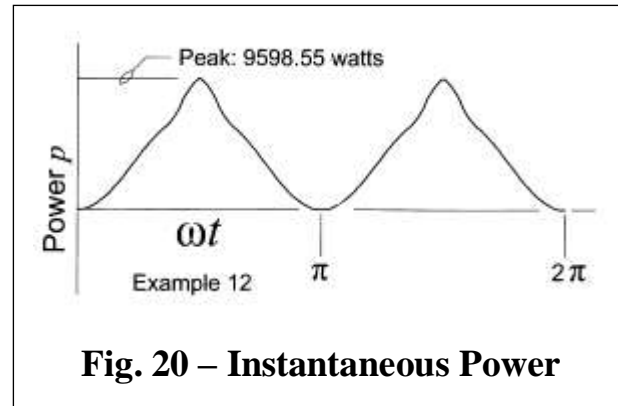


Fig. 20 – Instantaneous Power

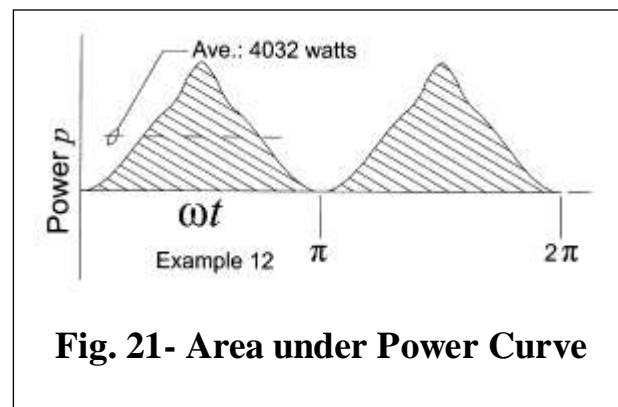


Fig. 21- Area under Power Curve

It is noted that the power of the circuit with the nonlinear current of Fig. 19 is 16% less than what it would be if the current were described by a perfect sine wave of 10 amps, RMS. If the power of the circuit were measured by a wattmeter that based readings on peak current the meter would read high by 16%.

Example 13

In this example consideration is given to a “flattened” nonlinear current of a configuration similar to that of Fig. 18. As in above Example 12, the voltage is assumed to be in-phase with the current and in the shape of a sine wave. The voltage is assumed to be a “stiff” source. The following parameters are assumed:

The circuit voltage is assumed to be a linear voltage of $V = 480$ (RMS) with a peak of $v^i = (480)(\sqrt{2})$. The specific nonlinear current selected for this example is shown in Fig. 22. The plot of nonlinear current is assumed to be of the following description:

For the period from 0° to 67.5° :

$$(i) = (12.00925) \sin [(1.3) x]$$

For the period from 67.5° to 90° :

$$(i) = 11.5 + (.5) \{ \cos [16(x - 67.5)] \}$$

A plot of instantaneous power is shown in Fig. 23.

To calculate the area under the instantaneous power curve, two sets of calculations are performed:

(1) one set with rectangle widths of 5° , and (2) one set with rectangle widths of 2.5° . Computations using rectangle widths of 5° (0.08726

rads) are shown in Appendix 3. The area under the instantaneous power curve

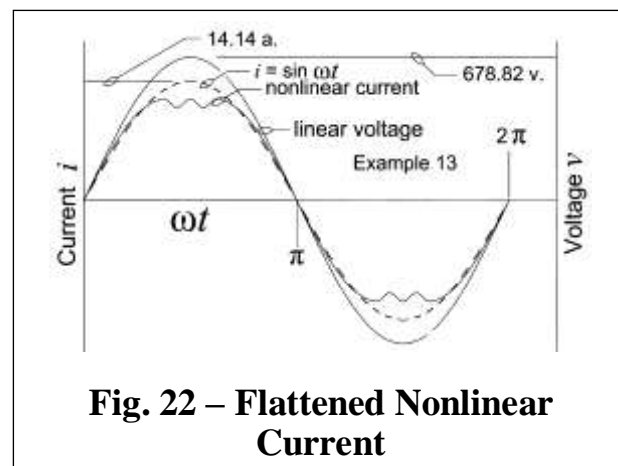


Fig. 22 – Flattened Nonlinear Current

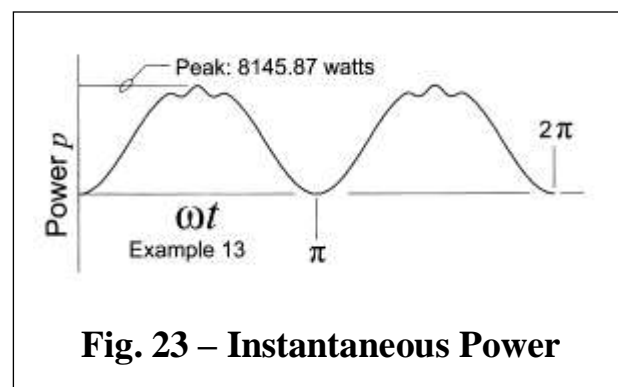


Fig. 23 – Instantaneous Power

is assumed to be comprised of total of 18 rectangles. The sum of the products of current and potential throughout the period from 0° to 90° is (79305.001) (0.08726 rads). The time period is (18) (0.08726 rads). The average power is $(79305.001) / 18 = 4405.83$. A scaled drawing of the area under the instantaneous power curve is shown in Fig. 24.

Computations were also performed using a rectangle width of 2.5° . The results of the computations are shown in Appendix 4. The area under the instantaneous power curve is assumed to be comprised of total of 36 rectangles. The total area under the curve becomes:

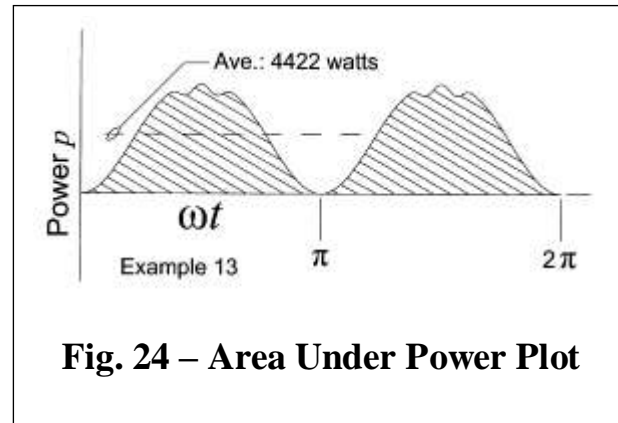


Fig. 24 – Area Under Power Plot

(159,227.13) (0.04363 rads). The period of time is: (36) (0.04363 rads).

Therefore the average power in the period is: $(159,227.13) / 36 = 4422.97$

watts. In this instance the smaller rectangles made a slight difference from the results using a rectangle width of 2.5° . If the computations using the smaller width rectangles is assumed to be correct then the rectangles of the wider width resulted in an error of $(4422.97 - 4405.83) / 4422.97 \cdot 100 = 0.38\%$.

The nonlinear, flattened current of this example is assumed to have a peak of 12.00 amps. As indicated in Fig. 22, if the current were not flattened it would probably have a peak of approximately 14.14 amps (corresponding to a current of 10 amps RMS). In other words, the power of the circuit less the flattening effect would be: $P = 480 \cdot 10 = 4800$ watts.

It is noted that the power of the circuit with the nonlinear current of Fig. 22 is 7.8% less than what it would be if the current were described by a perfect sine wave of 10 amps, RMS. With the flattened curve of this example a wattmeter that determines power from the peak reading would indicate a power of only 4073 watts whereas the actual power was determined be 4422 watts. The meter would be reading 7.9% low.

7.2 Linear Power Factor and Displacement Power Factor

A linear, instantaneous AC voltage can very nearly be described by the expression,

$$v^i = (V_{PK}) \sin \omega t$$

A linear instantaneous AC current can be described by the expression,

$$i^i = (I_{PK}) \sin (\omega t + \theta)$$

Instantaneous AC power, linear or otherwise, is defined as,

$$p^i = v^i \cdot i^i$$

The more common expressions of AC voltage and AC current use RMS voltage and RMS current. According to the accepted and proven relationship,

$$V_{RMS} = (V_{PK}) \div \sqrt{2}, \text{ and}$$

$$I_{RMS} = (I_{PK}) \div \sqrt{2}$$

If the current leads or lags voltage in an AC circuit then a power factor is needed in the computation of power namely as,

$$P = VI \cos \theta, \text{ where}$$

$$\theta = \text{lead/lag of current with respect to voltage}$$

The power factor of a linear AC circuit is mostly known simply as the “power factor” but it is also known as a “displacement power factor.” In a linear AC circuit that has a displacement power factor,

the peak of the current trace is displaced to some extent from the peak of the voltage trace. The peak of the current trace represented in Fig. 25 can be described as being displaced from the peak of the voltage trace. Actually, a displacement power factor is a special case of the more generic definition of power factor. A more

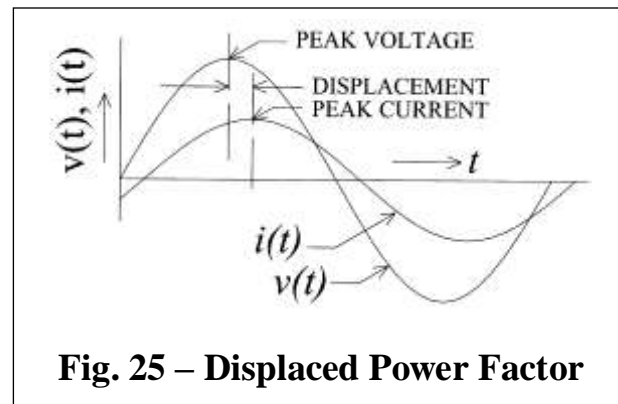


Fig. 25 – Displaced Power Factor

encompassing definition of power factor defines power factor as the ratio of real power to total power. This later mention definition of power factor is applicable to both linear and nonlinear circuits. The terms applicable to the generic definition of power factor are:

Real power, also called active power, P

Reactive power, volt-ampere, Q (VAR)

Total power, or complex power, volt-amperes, S

Apparent power, $|S|$ volt-amperes

In a linear, single phase AC circuit the angle between real power and total power is equivalent to θ and the power factor is $PF = \cos \theta$ which is also equal to the ratio of real power to total power, or

$PF = |P| \div |S|$. The values of P, Q and S are typically represented as shown in Fig. 26.

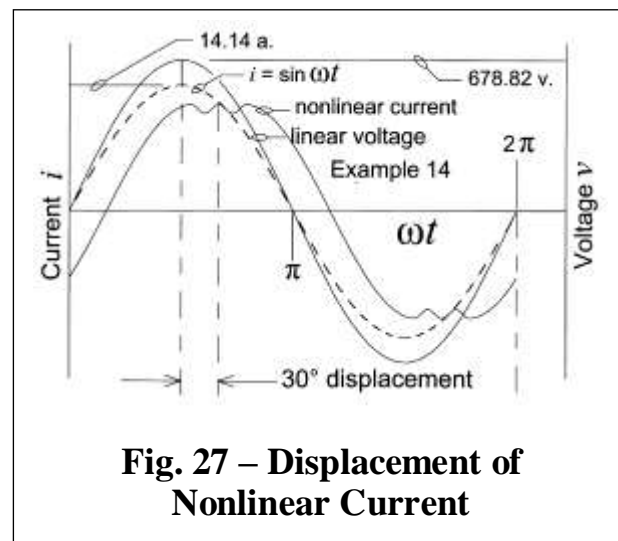
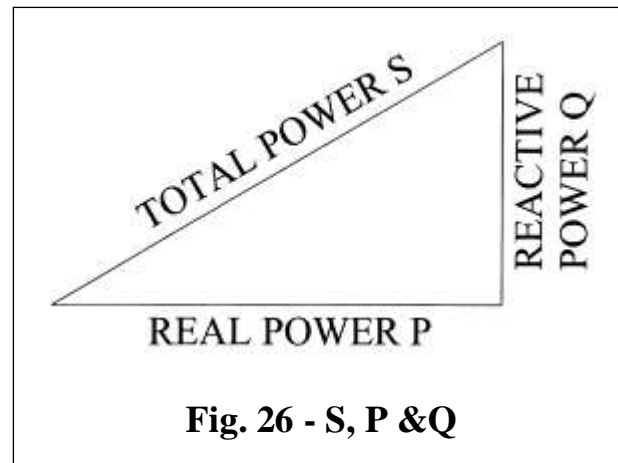
7.3 Lagging or Leading Nonlinear Current

Consideration is given to nonlinear currents in above Section 7.1 which are in-phase with the applied voltage. Much as linear currents, nonlinear currents can also lag or lead the applied voltage. However, some of the expressions applicable to linear circuits are not applicable to the nonlinear circuits and special rules apply. In the way of illustration consider a nonlinear current that lags the voltage.

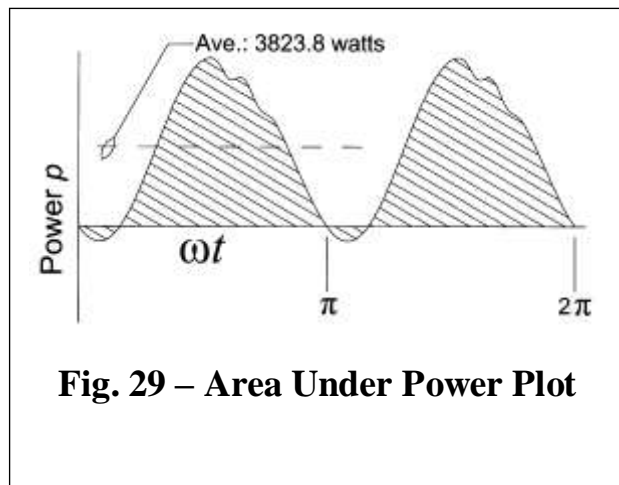
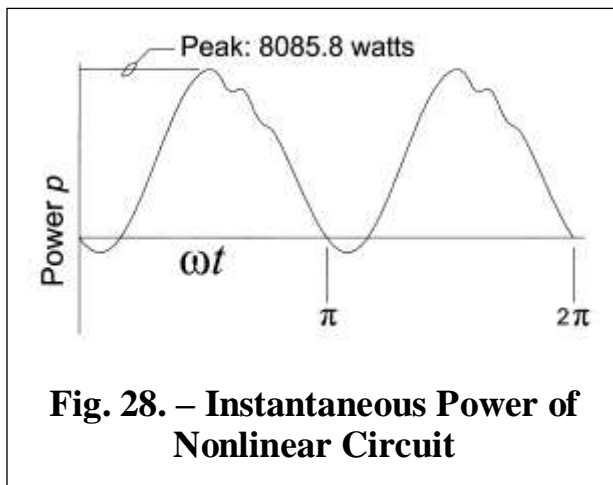
Example 14

In above Example 12 and Example 13 consideration is given to nonlinear currents that are in-phase with the voltage. On this example, consideration is given to the current of Example 13 with the exception that it is assumed the current lags voltage by 30° . Particulars applicable to the associated calculations are presented in Appendix 5. A plot of current with time is shown in Fig. 27.

A plot of instantaneous power is shown in Fig. 28. To calculate the area under the instantaneous power curve rectangle widths of 2.5° are used for the region from $\omega t = 0^\circ$ to $\omega t = 180^\circ$. Computations are shown in Appendix 5. The area under the instantaneous power curve is comprised of a



total of 72 rectangles. The total area under the curve throughout the period from 0° to 180° becomes: (275,312.83) (0.08726 rads). The period of time is: (72) (0.04363 rads). The average power is (275,312.83) / 72 = 3,823.8 watts. A scaled drawing of the area under the instantaneous power curve is shown in Fig. 29. (The power from 180° to 360° is a duplicate of the region from 0° to 180° .)



7.4 Nonlinear Power Factor

Above, consideration is given to the common definition of power factor as applicable to linear circuits. When nonlinear currents are involved, determination of the power factor is a more complicated and involved procedure. For this reason nonlinear power factors are merely mentioned here but not treated or explained in detail.

When nonlinear currents are present there are also currents resulting from harmonics. These harmonic currents are essentially reactive and, except for the I^2R losses, in general are not associated with any power consumption. In this regard the harmonic currents are similar to the VAR's of linear circuits. In linear circuits a power factor defines that part of the circuit's current that does no work but which current must be accommodated in the design of equipment. Accordingly a power factor of a nonlinear circuit should also define those reactive currents that do no work. The usual method of achieving this end for nonlinear currents is by use of a "total power factor." According to accepted practice, a total power factor consists of two parts: (1) a "displacement power factor" and, (2) a "distortion power

factor.”

According to common practice, if

Total power factor = PF_{TOT}

Displacement power factor = PF_{DISP} , and,

Distortion power factor = PF_{DIST} , then,

$PF_{TOT} = PF_{DISP} \cdot PF_{DIST}$

The displacement power factor of a nonlinear current is easily determined, as explained above. The calculation of the distortion power factor requires a more extensive effort and is considered beyond the scope of this course.

END

Appendix 1

The nonlinear current (i) of Fig. 19 is generated from the following expressions:

From 0° to 65° : (i) = $(10.2608053) \sin [(1.15) x]$

From 65° to 75° : (i) = $12.77222 + 2.917055 \{ \sin [(280) + 5(x - 65)] \}$

From 75° to 90° : (i) = $-8.21643 + 22.55161 \{ \sin [(420) + 1.5(x - 75)] \}$

The linear voltage (v) of Fig. 19 is generated from the following expression:

From 0° to 90° : (v) = $678.822 \sin x$

The instantaneous power is shown in Fig. 20 and the area under the instantaneous power curve is represented in Fig. 21. The area under the instantaneous power curve is determined using midpoint approximation and a rectangle width of 5° .

For the period from 0° to 65° :

N	(x)	(i)	(v)	(p)
1	2.5°	.514	29.609	15.238
2	7.5	1.538	88.604	136.342
3	12.5	2.547	146.924	374.277
4	17.5	3.530	204.125	720.651
5	22.5	4.477	259.774	1163.245
6	27.5	5.380	313.445	1686.439
7	32.5	6.228	364.731	2271.766
8	37.5	7.014	413.240	2898.557
9	42.5	7.729	458.605	3544.666
10	47.5	8.366	500.480	4187.249
11	52.5	8.919	538.546	4803.565
12	57.5	9.382	572.513	5371.772
13	62.5	9.751	602.122	5871.701
14	67.5°	10.077	627.150	6319.926
15	72.5	10.801	647.404	6992.931
16	77.5°	12.009	662.731	7959.072
17	82.5	13.138	673.015	8842.320
18	87.5	13.901	678.176	9427.911

Total area under the instantaneous power curve: $(72587.63) (.08726 \text{ rad})$

Average power: $(72587.63) (.08726 \text{ rad}) / (18) (.08726 \text{ rad}) = 4032.64 \text{ watts}$

Appendix 2

The nonlinear current (i) of Fig. 19 is generated from the following expressions:

From 0° to 65° : (i) = $(10.2608053) \sin [(1.15) x]$

From 65° to 75° : (i) = $12.77222 + 2.917055 \{ \sin [(280) + 5(x - 65)] \}$

From 75° to 90° : (i) = $-8.21643 + 22.55161 \{ \sin [(420) + 1.5(x - 75)] \}$

The linear voltage (v) of Fig. 19 is generated from the following expression:

From 0° to 90° : (v) = $678.822 \sin x$

The instantaneous power is shown in Fig. 20 and the area under the [power curve is represented in Fig. 21. The area under the instantaneous power curve is determined using midpoint approximation and a rectangle width of 2.5° .

For the period from 0° to 65° :

N	(x)	(i)	(v)	(p)
1	1.25°	.2574	14.808	3.811
2	3.75	.77157	44.397	34.255
3	6.25	1.2837	73.901	94.874
4	8.75	1.7927	103.264	185.132
5	11.25	2.29727	132.431	304.231
6	13.75	2.7959	161.346	451.119
7	16.25	3.2876	189.954	624.498
8	18.75	3.7710	218.200	822.835
9	21.25	4.2448	246.031	1044.377
10	23.75	4.7080	273.393	1287.164
11	26.25	5.15945	300.235	1549.051
12	28.75	5.5978	326.506	1827.721
13	31.25	6.0220	352.154	2120.709
14	33.75	6.4312	377.133	2425.423
15	36.25	6.8241	401.394	2739.165
16	38.75	7.1998	424.890	3059.160
17	41.25	7.5574	447.578	3382.573
18	43.75	7.8960	469.414	3706.539
19	46.25	8.2148	490.356	4028.187
20	48.75	8.5128	510.365	4344.665
21	51.25	8.7894	529.403	4653.166
22	53.75	9.0439	547.432	4950.949

23	56.25	9.2756	564.420	5235.369
24	58.75	9.4840	580.333	5503.897
25	61.25	9.6685	595.141	5754.140
26	63.75	9.8286	608.817	5983.867
27	66.25°	9.9717	621.334	6195.757
28	68.75	10.2147	632.667	6462.550
29	71.25	10.5790	642.797	6800.194
30	73.75	11.0473	651.703	7199.587
31	76.25°	11.67231	659.368	7696.358
32	78.75	12.32501	665.779	8205.738
33	81.25	12.88975	670.922	8648.021
34	83.75	13.36411	674.787	9017.942
35	86.25	13.74606	677.369	9311.158
36	88.75	14.03396	678.660	9524.304

Total area under the instantaneous power curve: (145,178.50) (.04363 rad)

Average power: (145,178.50) (.04363 rad) / (36) (.04363 rad) = 4,032.73 watts

Appendix 3

The nonlinear current (i) of Fig. 22 is generated from the following expressions:

For the period from 0° to 67.5° : (i) = (12.00925) $\sin [(1.3) x]$

For the period from 67.5° to 90° : (i) = 11.5 + (.5) { $\cos [16(x - 67.5)]$ }

The linear voltage (v) of Fig. 19 is generated from the following expression:

From 0° to 90° : (v) = 678.822 $\sin x$

The instantaneous power is shown in Fig. 23 and the area under the [power curve is represented in Fig. 24. The area under the instantaneous power curve is determined using midpoint approximation and a rectangle width of 5° .

For the period from 0° to 90° :

N	(x)	(i)	(v)	(p)
1	2.5°	.680	29.609	20.159
2	7.5	2.033	88.604	180.199
3	12.5	3.360	146.924	493.743
4	17.5	4.644	204.125	947.982
5	22.5	5.867	259.774	1524.347
6	27.5	7.016	313.445	2199.255
7	32.5	8.074	364.731	2945.064
8	37.5	9.029	413.240	3731.165
9	42.5	9.867	458.605	4525.230
10	47.5	10.578	500.480	5294.500
11	52.5	11.154	538.546	6007.108
12	57.5	11.586	572.513	6633.348
13	62.5	11.869	602.122	7146.886
14	67.5	12.000	627.150	7525.802
15	72.5	11.586	647.404	7501.359
16	77.5	10.560	662.731	6998.645
17	82.5	11.250	673.015	7571.418
18	87.5	11.883	678.176	8058.783
19	90 (n/a)	12.000	678.822	

Total area under the instantaneous power curve: (79305.001) (.08726 rads)

Average power: (79305.001) (.08726 rads) / (18) (.08726 rads) = 4405.833 watts

Appendix 4

The nonlinear current (i) of Fig. 22 is generated from the following expressions:

From 0° to 67.5° : (i) = (12.00925) $\sin [(1.3) x]$

For the period from 67.5° to 90° : (i) = 11.5 + (.5) { $\cos [16(x - 67.5)]$ }

The linear voltage (v) of Fig. 22 is generated from the following expression:

From 0° to 90° : (v) = 678.822 $\sin x$

The area under the instantaneous power curve of Fig. 23 is determined using midpoint approximation and a rectangle width of 2.5° .

For the period from 0° to 67.5° : (i) = (12.00925) $\sin [(1.3) x]$

N	(x)	(i)	(v)	(p)
1	1.25°	.340	14.808	5.043
2	3.75	1.020	44.397	45.310
3	6.25	1.697	73.901	125.432
4	8.75	2.368	103.264	244.589
5	11.25	3.032	132.431	401.562
6	13.75	3.686	161.346	594.744
7	16.25	4.328	189.954	822.154
8	18.75	4.956	218.200	1081.466
9	21.25	5.568	246.031	1370.019
10	23.75	6.162	273.393	1684.853
11	26.25	6.737	300.235	2022.274
12	28.75	7.289	326.506	2380.215
13	31.25	7.819	352.154	2753.598
14	33.75	8.323	377.133	3139.052
15	36.25	8.800	401.394	3532.610
16	38.75	9.249	424.890	3930.215
17	41.25	9.669	447.578	4327.760
18	43.75	10.057	469.414	4721.142
19	46.25	10.413	490.356	5106.277
20	48.75	10.735	510.365	5479.169
21	51.25	11.023	529.403	5835.939
22	53.75	11.276	547.432	6172.857
23	56.25	11.492	564.420	6486.391
24	58.75	11.671	580.333	6773.233
25	61.25	11.812	595.141	7030.351
26	63.75	11.916	608.817	7254.978

27	66.25	11.981	621.334	7444.693
28	68.75	11.969	632.667	7572.933
29	71.25	11.750	642.797	7552.870
30	73.75	11.413	651.703	7438.002
31	76.25	11.116	659.368	7330.183
32	78.75	11.000	665.779	7323.570
33	81.25	11.116	670.922	7458.620
34	83.75	11.413	674.787	7701.467
35	86.25	11.750	677.369	7959.086
36	88.75	11.969	678.660	8123.463

Total area under the instantaneous power curve: (159,227.134) (.04363)

Average power: (159,227.134) (.04363) / (36) (.04363) = 4422.97 watts

Appendix 5

The nonlinear current (i) of Fig. 27 is generated from the following expressions:

For the period from 0° to 97.5° : (i) = (12.00925) sin [(1.3) (x - 30)]

For the period from 97.5° to 142.5° : (i) = 11.50000 + (.5) {cos [16(x - 97.5)]}

For the period from 142.5° to 180° : (i) = (12.00925) sin [(1.3) (x - 71.5384)]

The linear voltage (v) of Fig. 27 is generated from the following expression:

From 0° to 90° : (v) = 678.822 sin x

The instantaneous power is shown in Fig. 28 and the area under the [power curve is represented in Fig. 29. The area under the instantaneous power curve is determined using midpoint approximation and a rectangle width of 2.5° .

For the period from 0° to 97.5° :

N	(x)	(i)	(v)	(p)
	0	-7.557	0	0
1	1.25°	-7.289	14.808	-107.95
2	3.75	-6.737	44.397	-299.11
3	6.25	-6.162	73.901	-455.43
4	8.75	-5.568	103.264	-575.02
5	11.25	-4.956	132.431	-656.37
6	13.75	-4.328	161.346	-698.33
7	16.25	-3.686	189.954	-700.19
8	18.75	-3.032	218.200	-661.63
9	21.25	-2.368	246.031	-582.74
10	23.75	-1.697	273.393	-464.03
11	26.25	-1.050	300.235	-306.41
12	28.75	-.340	326.506	-111.19
13	31.25	.340	352.154	119.92
14	33.75	1.020	377.133	384.89
15	36.25	1.697	401.394	681.28
16	38.75	2.368	424.890	1006.38
17	41.25	3.032	447.578	1357.16
18	43.75	3.686	469.414	1730.32
19	46.25	4.328	490.356	2122.35
20	48.75	4.956	510.365	2529.52
21	51.25	5.568	529.403	2947.97
22	53.75	6.162	547.432	3373.69
23	56.25	6.737	564.420	3802.60
24	58.75	7.289	580.333	4230.61

25	61.25	7.819	595.141	4653.58
26	63.75	8.323	608.817	5067.46
27	66.25	8.800	621.334	5468.27
28	68.75	9.249	632.667	5852.14
29	71.25	9.669	642.797	6215.38
30	73.75	10.057	651.703	6554.51
31	76.25	10.413	659.368	6866.26
32	78.75	10.735	665.779	6657.79
33	81.25	11.023	670.922	7395.99
34	83.75	11.276	674.787	7608.91
35	86.25	11.492	677.369	7784.41
36	88.75	11.671	678.660	7920.84
37	91.25	11.812	678.660	8016.95
38	93.75	11.916	677.369	8071.87
39	96.25	11.981	674.787	8085.16
40	98.75	11.969	670.922	8030.83
41	101.25	11.750	665.779	7822.90
42	103.75	11.413	659.368	7525.49
43	106.25	11.116	651.703	7244.97
44	108.75	11.000	642.797	7070.77
45	111.25	11.116	632.667	7033.35
46	113.75	11.413	621.334	7091.39
47	116.25	11.750	608.817	7153.60
48	118.75	11.969	595.141	7123.75
49	121.25	11.969	580.333	6946.50
50	123.75	11.750	564.420	6631.93
51	126.25	11.413	547.432	6247.94
52	128.75	11.116	529.403	5885.36
53	131.25	11.019	510.365	5623.90
54	133.75	11.116	490.356	5451.12
55	136.25	11.413	469.414	5357.51
56	138.75	11.750	447.578	5259.05
57	141.25	11.969	424.890	5085.87
58	143.75	11.981	401.394	4809.42
59	146.25	11.916	377.133	4494.11
60	148.75	11.812	352.154	4164.75
61	151.25	11.671	326.506	3810.74

62	153.75	11.492	300.235	3450.34
63	156.25	11.276	273.393	3082.27
64	158.75	11.023	246.031	2712.15
65	161.25	10.735	218.200	2342.54
66	163.75	10.413	189.954	1978.06
67	166.25	10.057	161.346	1622.74
68	168.75	9.669	132.431	1280.51
69	171.25	9.249	103.264	955.19
70	173.75	8.800	73.901	650.39
71	176.25	8.323	44.397	369.53
72	178.75	7.819	14.808	115.79

Total area under the instantaneous power curve: (275,312.83) (.04363 rads)

Average power: (275,312.83) (.04363 rads) / (72) (.04363 rads) = 3823.78 watts