



PDHonline Course G294 (4 PDH)

Explosion Standoff Distances

Instructor: Marvin Liebler, P.E.

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PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone & Fax: 703-988-0088
www.PDHonline.org
www.PDHcenter.com

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1. INTRODUCTION

1.1 EXPLOSIVES -- DEFINITION

The dictionary defines an explosion as a rapid chemical or nuclear reaction with production of heat and violent expansion of gas. For blast loads under consideration here, the explosive must be detonated to produce an explosion, rather than a fire. Detonation is a rapid and stable reaction which proceeds through the explosive, at speeds of approximately 25000 ft/sec (hypersonic). This detonation rapidly converts the solid or liquid explosive into hot, dense, high pressure gas and this gas volume is the source of strong blast waves in air.

1.2 EXPLOSIVE MATERIALS

Explosive materials are characterized, for study purposes, as TNT equivalents, as shown in the following charts in Reference 5.1, showing single and multi-component explosives.

1.3 BLAST LOADING CATEGORIES

The three unconfined (as contrasted with confined explosions which are not covered here) loading categories are free air burst, air burst, and surface burst. A free air burst is one in which the shock wave proceeds outward from its origin with no intermediate amplification. Air bursts occur such that the ground reflections impact the structure prior to the arrival of the blast wave. When the explosion is located close to the ground surface such that the initial shock is amplified at the point of detonation, this is called a surface burst. The surface burst will be the type considered here, as it is the most common.

1.4 SURFACE BLAST SCALING LAWS

The pertinent properties of surface blast explosions are given for a range of standoff distances and explosive weight by the dimensionless scaled distance parameter $Z=R/W^{(1/3)}$. Pertinent parameters are defined on the page 90 of Reference 5.2

TERM	DEFINITION	UNITS
-----	-----	-----
Pr	peak positive reflected normal pressure	psi
Pso	peak positive incident pressure. This is the basic incident loading characteristic. It is not uniform over the surface, but varies with vertical and horizontal distance.	psi
Pt	Pr+Pso	psi
R	standoff distance from explosion	ft

to element
 to duration of positive phase of ms
 blast wave
 W explosive weight, converted to TNT lbf
 Z scaled ground distance= $R/W^{(1/3)}$ ft/lbf^(1/3)

It is very convenient to automate the selection of $P_t=Pr+P_{so}$ and t_o , using a polynomial approximation.

This is done, in general, by:

- (1) Carefully select points of (x,y) where x is the independent variable and y the dependent variable.
- (2) Express each dependent variable in the form:
 $y = c_n*x^n + c_{(n-1)}*x^{(n-1)} + \dots + c_1*x + c_0$
 There will be n+1 equations in n+1 unknowns.
 Eight points at $Z=.5,.9,2,5,10,20,50,$ and 100 were chosen. So that the equations are:
 $y = c_7*x^7 + c_6*x^6 + \dots + c_1*x + c_0.$
 Note that for each dependent variable there are eight equations in 8 unknowns.
 In this case, $x = \log_{10}(Z)$ and $y = P_t=Pr+P_{so}$, or $\log_{10}(t_o/W^{(1/3)})$.
- (3) Solve for the 'c' coefficients, enabling solution of The dependent variable at any point. See the heading of source code "interpolation.c" for details of solution.

PROCEDURE FOR FINDING BLAST WAVE PARAMETERS

1. Find point of interest on ground → R.
2. Assume charge weight W (TNT or TNT equivalent)
3. Apply safety factor to W, if required.
4. Find scaled standoff distance Z.
5. Find desired scaled parameter values from chart.
6. Multiply scaled values by $W^{(1/3)}$ to obtain absolute values for t_o , but not P_t , which is Given directly as $Pr+P_{so}$

Using these methods, construct a table for a weight of 10 000 lbf of TNT for standoff distances of 50,100, and 200 feet. Find P_t , t_o , and $\frac{1}{2} P_t*t_o$ for each case Here $W^{(1/3)} = 21.544 \text{ lbf}^{(1/3)}$

R	Z	$P_t(\text{psi})$	$t_o(\text{sec})$	$\frac{1}{2} P_t*t_o$ (psi-sec)
---	-----	-----	-----	-----
50	2.321	1744.439	.015164	26.453
100	4.642	249.526	.028212	7.040
200	9.283	38.611	.051168	1.976

Notice how P_t falls off with increasing standoff distance. This is somewhat balanced by the increase in t_o . $\frac{1}{2} P_t*t_o$ is the area of the triangle formed by coordinates (0,0),($t_o,0$), and (0, P_t).

2. MATHEMATICAL MODELS OF EXPLOSION

- 2.1 IMPULSE INPUT -- This model uses an input of finite area and zero width. It has the distinct advantage of being the easiest to solve, for a given struck element. It depends on $\omega \ll 1/\omega_n$, ω_n = natural frequency of element = square root of stiffness divided by mass of element, for elastic action.
It is given numerically by $\frac{1}{2} P_t \cdot t_o$, as shown above.
- 2.2 TRIANGULAR INPUT -- This model uses a right triangular input with initial value of $P_t \psi$ and final value of zero at $t = t_o$, with a straight line decreasing pressure variation.
This is more precise than the impulse method, but less precise than the Friedlander equation following. It is more conservative, in general, than the Friedlander equation, if the same P_t and t_o are used.
- 2.3 FRIEDLANDER EQUATION
 $p = P_t \cdot (1 - t/t_o) \cdot e^{(-bt/t_o)}$ where p = pressure at any time t , b affects the duration of the negative phase and e = base of natural logarithms. $*$ denotes multiplication. t is measured from the start of the pulse.
Some authors (Reference 5.3) recommend using $b=1$ for each case. That approach will be examined later.

3. SDOF (SINGLE DEGREE OF FREEDOM) ANALYSIS

A SDOF model is used for first analyses of buildings, walls, columns, beams and slabs.
The basic equation of this analysis, in the elastic range, is:
 $m \cdot x'' + c \cdot x' + k \cdot x = f(t)$ where:
 m = mass in lbm (lbf/(in./sec²))
 c = viscous friction (lbf/(in./sec))
 k = stiffness (lbf/in.)
 x'' = acceleration (in./sec²)
 x' = velocity (in./sec)
 x = displacement (in.)
 $f(t)$ = applied force (lbf) vs. time

3.1 NUMERICAL SOLUTION METHODS

3.1.1 LAPLACE TRANSFORM METHOD

The Laplace transform method is used to solve linear differential equations, such as the one above. It substitutes algebraic equations in frequency for differential equations in time. Solutions for displacement, velocity, and acceleration at any point in time may be obtained. Nonlinear equations, such as elastic-plastic may only be solved by application of different differential equations at different points in time. The process is

expedited by tables of direct (time → frequency) and inverse (frequency → time) transform pairs.

A short table useful for the equations here is:

F(s)	f(t)
-----	-----
1/s	1
1/(s+a)	e ^{-at}
1/(s+a) ²	te ^{-at}
1/((s(s+a)))	(1/a)(1-e ^{-at})
1/(s ² (s+a))	(1/a ²)(at-1+e ^{-at})
1/(s(s+a) ²)	(1/a ²)(1-e ^{-at})-ate ^{-at})
1/(s ² +2ζwns+wn ²)	(1/z)e ^{-ζwnt} sin(zt)
	z=wn*sqrt(1-ζζ)
1/(s(s ² +2ζwns+wn ²))	1/wn ² -(1/wnz)e ^{-ζwnt} * sin(zt+cos ⁻¹ (ζ))
1/(s ² (s ² +2ζwns+wn ²))	t/wn ² -2ζ/wn ³ +(1/(wn ² *z))* e ^{-ζwnt} * sin(zt+cos ⁻¹ (2ζζ-1))
1/(s(s+a)(s+b))	(1/ab)(1-(b/(b-a))e ^{-at} +(a/(b-a))e ^{-bt})
1/(s(s+a)(s+b) ²)	(1/ab ²)-(1/(a(b-a) ²)e ^{-at}) +((2b-a)/(b ² (b-a) ²)e ^{-bt}) +(t/(b(b-a)))e ^{-bt})

Inversion methods include partial fraction expansion and the convolution integral.

3.1.2 THE RUNGE--KUTTA METHOD

The Runge--Kutta method is iterative, requiring the six (6)solutions, in sequence shown, at each time interval, where

$$u''=f(u,u',t) :$$

1. k1 = h*f(un,un',tn)
2. k2 = h*f(un+h*un'/2+h*k1/8,un'+k1/2,tn+h/2)
3. k3 = h*f(un+h*un'/2+h*k1/8,un'+k2/2,tn+h/2)
4. k4 = h*f(un+h*un' +h*k3/2,un'+k3, tn+h)
5. u(n+1) = un+h*(un'+(k1+k2+k3)/6)
6. u(n+1)' = un'+(k1+2*(k2+k3)+k1)/6

where :

- h = time interval, seconds
- * = multiplication
- un = displacement at nth interval
- un' = velocity at nth interval
- u(n+1) = displacement at (n+1)th interval
- u(n+1)' = velocity at (n+1)th interval

Starting conditions are generally u(0) = u(0)' = 0

3.2 LINEAR (ELASTIC) RANGE

3.2.1 IMPULSE INPUT -- ELASTIC ANALYSIS

This case is the most straightforward, as well as being, in general, conservative. The general equation of motion is:

$\mu'' + c\mu' + k\mu = \frac{1}{2} P t A \delta(t)$ where $\delta(t)$ is the Dirac delta function, an infinitely sharp peak with zero duration and unit area. Dividing the above equation by m and taking Laplace transforms, with $u(0) = u'(0) = 0$ and

$$k/m = \omega_n^2, \quad c = 2\zeta\omega_n m$$

$$s^2 U(s) + 2\zeta\omega_n s + \omega_n^2 = P t A \delta(t) / 2m$$

The inverse transform, i.e., the time displacement of the SDOF is :

$$u(t) = (P t A \delta(t) / (2m)) * (1 / (\omega_n (1 - \zeta^2)^{1/2})) * e^{(-\zeta\omega_n t)} * \sin(\omega_n (1 - \zeta^2)^{1/2} t)$$

The maximum occurs when $u'(t) = 0$. It can be shown that this occurs when :

$$t_{max} = \theta / (\omega_n (1 - \zeta^2)^{1/2}) \quad \text{and} \quad \theta = \tan^{-1}((1 - \zeta^2)^{1/2} / \zeta)$$

A program implementing this response is `dirac.c`.

3.2.2 RIGHT TRIANGLE INPUT

A new function is given here, namely $1(\text{expression})$. This equals zero when expression evaluates to $<$ zero, and equals one when expression evaluates to $> =$ zero.

$$f(t) = \text{applied force} = P t A * 1(t) * (1 - t/t_0) + 1(t - t_0) * (-1 + t/t_0)$$

Notice how the delayed function cancels the input after $t = t_0$. Taking Laplace transforms,

$$U(s) = (P t A / m) * (1 / (s * (s^2 + 2\zeta\omega_n s + \omega_n^2))) - (1/t_0) * (1 / (s^2 * (s^2 + 2\zeta\omega_n s + \omega_n^2))) + (1/t_0) * (1 / (s^2 * (s^2 + 2\zeta\omega_n s + \omega_n^2))) * e^{-t_0 s}$$

After a fair amount of work, this transform is inverted to :

$$u(t) = (P t A / m) * (1/z) * (e^{(-\zeta\omega_n t)} * \sin(z t + \cos^{-1}(\zeta)) - (1/t_0) * (t/\omega_n^2 - 2\zeta/\omega_n^3 + (1/z) * (\sin(z t + \cos^{-1}(\zeta)) * (2\zeta\zeta - 1))) + (1/t_0) * 1(t - t_0) * ((t - t_0)/\omega_n^2 - 2\zeta/\omega_n^3 + (1/z) * \sin(z * (t - t_0) + \cos^{-1}(2\zeta\zeta - 1))))$$

This function is evaluated in `triangle1.c`

A second method of solution is iteration of the equation for acceleration. If carried out to small enough intervals, it should match the transform method to any desired number of places. In general, it is much easier to implement than the transform method, and its comparison with the transform method helps cut down on arithmetic errors.

$$x''(t) = -\omega_n^2 u_n(t) - 2\zeta\omega_n u_n'(t) + f(t)/m$$

$$\text{where } f(t) = P t A * (1 - t/t_0) \text{ for } t < t_0 \\ = 0 \text{ for } t \geq t_0$$

The approach is outlined in section 3.1.2 above and in program `triangle2.c`.

3.2.3 FRIEDLANDER EQUATION

First examine solution by the transform method.

The basic equation may be written as:

$$u'' + (c/m)u' + (k/m)u = f(t)/m = (1/m)pA$$

Using the Friedlander equation,

$$u'' + (c/m)u' + (k/m)u = (1/m)PtA(1-t/to)e^{(-bt/to)}$$

Let $\omega_n^2 = k/m$, square of natural frequency of element, and

$c/m = 2\zeta\omega_n$, and $\zeta =$ damping ratio (generally less than 1)

$$u'' + 2\zeta\omega_n u' + \omega_n^2 u = (1/m)PsoA(1-t/to)e^{(-bt/to)}$$

Taking Laplace transforms of each side,

$$s^2 U(s) + 2\zeta\omega_n s U(s) + \omega_n^2 U(s) = (1/m)PtA \left[\frac{1}{(s+b/to)} - \frac{1}{to} \left(\frac{1}{(s+b/to)^2} \right) \right]$$

$$U(s) = \frac{(1/m)PtA \left[\frac{1}{(s+b/to)} - \frac{1}{to} \left(\frac{1}{(s+b/to)^2} \right) \right]}{\frac{1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}} = \frac{F(s)}{G(s)}$$

This multiplication in the frequency domain ($F(s)G(s)$) corresponds to convolution in the time domain.

Let $f(u) = (1/m)PtA \left(e^{(-b*u/to)} - \frac{1}{to} u e^{(-b*u/to)} \right) =$ inverse transform of $F(s)$ and

$g(u) = \frac{1}{b1} e^{(-\zeta\omega_n u)} \sin(b1 u) =$ inverse transform of $G(s)$, where $b1 = \omega_n \sqrt{1 - \zeta^2}$.

The convolution theorem states

$$u(t) = \int_0^t f(t-u)g(u)du$$

After a fair amount of work we have :

$$u(t) = c0 * [e^{(-\zeta\omega_n t)} (c1 \sin(b1 t) + c2 \cos(b1 t)) + e^{(-b t/to)} (c3 (1-t/to) + c4)]$$

$$\text{where : } \begin{aligned} c0 &= PtA / (b1 m) \\ c1 &= (to * (a^2 + b1^2) * a - (a^2 - b1^2)) / (to * (a^2 + b1^2)^2) \\ a &= b/to - \zeta\omega_n \\ c2 &= (to * (a^2 + b1^2) * (-b1) + 2 * a * b1) / (to * (a^2 + b1^2)^2) \\ c3 &= b1 / (a^2 + b1^2) \\ c4 &= -2 * a * b1 / (to * (a^2 + b1^2)^2) \end{aligned}$$

$$u'(t) = c0 * [e^{(-\zeta\omega_n t)} (b1 c1 \cos(b1 t) - b1 c2 \sin(b1 t)) - e^{(-\zeta\omega_n t)} \zeta\omega_n (c1 \sin(b1 t) + c2 \cos(b1 t)) + e^{(-b1 t/to)} (-c3/to) - e^{(-b1 t/to)} (b1/to) (c3 (1-t/T) + c4)]$$

See program frdlndr1.c for source code for above, as region1.

The second method to solve this linear SDOF problem is an iterative method, here being the Runge-Kutta method. It is advisable to solve each problem by both methods, to reduce the chance of error.

The iterative equations, solved in sequence shown, are :

- (1) $k1 = \Delta t * (-\omega_n^2 u_n - 2\zeta\omega_n u_n' + PsoA/m) * (1-t/to) * e^{(-b t/to)}$
- (2) $k2 = \Delta t * (-\omega_n^2 (u_n + \Delta t u_n' / 2 + \Delta t^2 k1 / 8) -$

$$\begin{aligned}
 & 2*\zeta*wn*(un0'+k1/2)+ \\
 & (Pso*A/m)*(1-(t+\delta/2)/to)*e^{((-b/to)*(t+\delta/2))} \\
 (3) \quad & k3=\delta*(-wn*wn*(un0+\delta*un0'/2+\delta*k1/8)- \\
 & 2*\zeta*wn*(un0'+k2/2)+ \\
 & (Pso*A/m)*(1-(t+\delta/2)/to)*e^{((-b/to)*(t+\delta/2))} \\
 (4) \quad & k4=\delta*(-wn*wn*(un0+\delta*un0'+\delta*k3/2) \\
 & -2*\zeta*wn*(un0'+k3)+ \\
 & +(Pso*A/m)*(1-(t+\delta)/to)*e^{((-b/to)*(t+\delta))} \\
 (5) \quad & un0 =un0+\delta*(un0'+(k1+k2+k3)/6) \\
 (6) \quad & un0' =un0'+(k1+2*(k2+k3)+k4)/6
 \end{aligned}$$

Iterative methods have the advantage of not requiring explicit solutions, but have the disadvantage of generally requiring a number of iterations for accuracy. It is recommended that delta be ≤ 0.001 times either to or $1/wn$, whichever is less. This source code for this program is `frdlndr2.c` as region 1.

3.3 NONLINEAR (ELASTIC-PLASTIC) RANGE -- FRIEDLANDER EQUATION

A simplified two part resistance function is used here to model elastic-plastic behavior, described as :

- (1) Linear response from initial conditions ($u=u'=0$) to elastic limit. The equation describing this region is: $u(t)''+2*\zeta*wn*u(t)'+wn^2*u(t)=p(t)/m$
 $R = \text{resistance} = k*u(t)$. This is called region 1. This region continues until $R = R_{max}$, if it occurs. The only addition to the previous linear equation is continuing calculation of u and u' and monitoring if u' goes to zero, which signals the end of this region (region 2).
- (2) Plastic region with $R=R_{max}$ and described by:
 $u(t)''+(c/m)*u(t)'+R_m=p(t)/m$.
 This continues until $u(t)'=0$, indicating maximum displacement.
- (3) The crossover time from linear to plastic regions is found by solving $u(t) = u_{max}$ for t as shown in program `crossover1.c`, where $u(t)$ is the unknown displacement. The program uses the Newton-Raphson iteration method to solve that equation. It is advisable to get a starting value for the iteration from either `frdlndr1` or `frdlndr2`, noting where $u(t)$ crosses u_{max} . If $u(t)$ does not cross u_{max} , then the solution is entirely linear.

The goals here are to first, determine if element reaction stays within the linear region, and secondly, If not, how deep into nonlinearity the element is forced.

3.3.1 TRANSFORM APPROACH TO REGION 2

$u(t)'' + (c/m)u(t)' + R_{max}/m =$
 $(PtA/m)(1-t_1/to)e^{-(b*t_1/to)}$, using Friedlander
 Equation, and $t_1 = t + \text{crossover time}$
 Now take Laplace transforms of each side of the
 equation.
 $U(s)(s^2 + sc/m) =$
 $u(0)/(s+c/m) + (u'(0) + (c/m)u(0))/(s(s+c/m)) -$
 $(R_m/m)/(s^2(s+c/m)) + d_0/(s(s+c/m)(s+b/to)) -$
 $d_1/(s(s+c/m)(s+b/to)^2)$
 where $d_0 = (PtA/m)(e^{-(b*tc_r/to)})(1-tc_r/to)$
 and $d_1 = (PtA/m)(e^{-(b*tc_r/to)})/to$, $tc_r = \text{crossover time}$

After a fair amount of work, the following equations are obtained :

$$\begin{aligned}
 t &= t - \text{cro}; \\
 d_0 &= (PtA/m) \exp(-b \cdot \text{cro}/to) (1 - \text{cro}/to); \\
 d_1 &= (PtA/m) \exp(-b \cdot \text{cro}/to) (1/to); \\
 e_1 &= \exp(-(c/m) \cdot t); \\
 e_2 &= \exp(-(b/to) \cdot t); \\
 f_1 &= 1/((b/to) - (c/m)); \\
 f_2 &= (b/to) / ((b/to) - (c/m)); \\
 f_3 &= 1/((b/to) * ((b/to) - (c/m))); \\
 f_4 &= (c/m) / (b/to - c/m); \\
 *un_0 &= *u_0 * e_1 + (*du_0 + (c/m) * (*u_0)) * (m/c) * (1 - e_1) - \\
 &\quad (R_m/m) * (m/c) * (m/c) * ((c/m) * t - 1 + e_1) + \\
 &\quad d_0 * (m * to / (b * c)) * (1 + f_4 * e_2 - f_2 * e_1) - \\
 &\quad d_1 * (1 / ((c/m) * (b/to) * (b/to))) - \\
 &\quad (m/c) * f_1 * f_1 * e_1 + \\
 &\quad (2 * b/to - c/m) * f_3 * f_3 * e_2 + \\
 &\quad (to/b) * f_3 * t * e_2; \\
 *dun_0 &= -(c/m) * (*u_0) * e_1 + (*du_0 + (c/m) * (*u_0)) * e_1 - \\
 &\quad (R_m/c) * (1 - e_1) + \\
 &\quad d_0 * (m * to / (b * c)) * (-f_4 * b * e_2 / to + f_2 * c * e_1 / m) - \\
 &\quad d_1 * (f_1 * f_1 * e_1 - (2 * b/to - c/m) * f_3 * f_3 * b * e_2 / to + \\
 &\quad f_3 * (e_2 + t * (-b/to) * e_2));
 \end{aligned}$$

3.3.2 ITERATIVE APPROACH TO PLASTIC REGION (REGION 2)

In this case,

$$u''(t) = (-c/m)u'(t) - R_{max}/m + (PtA/m)(1-t_1/to)e^{-(b/to)t_1}$$

where $t_1 = \text{local } t + \text{crossover time}$.

No reference is made to the displacement value $u(t)$ in the acceleration calculations (k_1 - k_4), and time resets to zero at $t = \text{cro}$, except for the Friedlander equation.

$$\begin{aligned}
 t &= t - \text{cro}; \\
 d_0 &= (PtA/m) \exp(-b \cdot \text{cro}/to) (1 - \text{cro}/to); \\
 d_1 &= (PtA/m) \exp(-b \cdot \text{cro}/to) (1/to); \\
 k_1 &= \text{delta} * (- (c/m) * (*dun_0) - R_m/m + d_0 * \exp(-b * t/to) - \\
 &\quad d_1 * t * \exp(-b * t/to)); \\
 k_2 &= \text{delta} * (- (c/m) * (*dun_0 + k_1/2) - R_m/m + d_0 * \exp(- \\
 &\quad b * (t + \text{delta}/2)/to) -
 \end{aligned}$$

```

d1*(t+delta/2)*exp(-b*(t+delta/2)/to));
k3 = delta*(-(c/m)*(*dun0+k2/2)-Rm/m+d0*
exp(-b*(t+delta/2)/to)-
d1*(t+delta/2)*exp(-b*(t+delta/2)/to));
k4 = delta*(-(c/m)*(*dun0+k3)-Rm/m+d0*exp(-b*(t+delta)/to)-
d1*(t+delta)*exp(-b*(t+delta)/to));
*un0 = *un0+delta*( *dun0+(k1+k2+k3)/6);
*dun0= *dun0+(k1+2*(k2+k3)+k4)/6;

```

- 3.4 Selection of Constant 'b' in the Friedlander Equation
 Choice of 'b' is difficult to get from the literature. Reference 5.3 gives a range from .1 to 10, with a recommended value of 1. Given no other information, a conservative approach is to use the value of b which maximizes the area of $t \leq t_0$ minus the area of $t > t_0$. This should provide for maximum deflection for a given P_t and t_0 .

The first area is $\int_0^{t_0} (1-t/t_0)e^{(-bt/t_0)} dt = t_0/b$

and the second $\int_{t_0}^{\infty} (1-t/t_0)e^{(-bt/t_0)} dt = t_0/b^2$

To maximize this function, find the first derivative and set equal to zero.

$$f(b) = 1/b - 1/b^2$$

$$f'(b) = -1/b^2 + 2/b^3 = 0 \rightarrow 2b^2 = b^3 \rightarrow b = 2$$

As a check on this result, calculate $f(b)$ from $b = .1$ to 10.

b	f(b)
--	-----
.1	-490.110
.5	-11.049
1	0.079
1.5	1.230
2	1.384
2.5	1.328
3	1.229
4	1.037
5	.885
6	.768
7	.678
8	.605
9	.547
10	.498

It is seen that the maximum occurs at $b=2$, which will be used in further calculations.

4. Example

4.1 GIVEN

Concrete Beam with:
 span = 20'
 tributary height = 10'
 pinned end connections

$A_s = A_s' = 3.60 \text{ in.}^2 \text{ (6\#7)}$
 $f_c' = 4000 \text{ psi}$
 $d_y = 60000 \text{ psi}$
 $b = 18''$
 $d' = 1.5625'' \text{ (}\frac{3}{4}\text{'' cover)}$
 $h = 12''$
 $d = 10.4375''$
 weak axis bending, remainder
 top + bottom bars not listed

4.2 FIND

Standoff distances for $W = 100, 1000, \text{ and } 10\,000$ pounds of TNT for moment M_n .

4.3 SOLUTION

4.3.1 Concrete Beam Strength Parameters

$a = \beta_c$, depth of stress block, in.
 A_s = tension steel area, in.^2
 A_s' = compression steel area, in.^2
 A_{s1} = balances force of compression concrete, in.^2
 A_{s2} = balances force of compression steel, in.^2
 b = width of section, in.
 c = distance from outermost compression fiber to neutral axis, in.
 d = distance from outermost compression fiber to centroid of A_s , in.
 d' = distance from outermost compression fiber to centroid of comp. steel, in.
 d_1 = distance from outermost compression fiber to centroid of compression concrete, in.
 E_s = steel modulus of elasticity, psi
 f_c' = specified compressive strength of concrete
 f_s' = compression steel stress, psi
 f_y = steel yield stress, psi
 M_n = nominal flexural strength of section, lbf-in.
 M_{n1} = nominal flexural strength due to A_{s1} and compression concrete, lbf-in.
 M_{n2} = nominal flexural strength due to A_{s2} and compression steel, lbf-in.

In this case, since $A_s = A_s'$, f_s' must be less than f_y , and the following equations hold :

- (1) $.003(c-d') = f_s'/E_s$ equality of strains
- (2) $A_{s2} = A_s' f_s' / f_y$ A_{s2} balances A_s' force
- (3) $.85 f_c' (ab - A_s') = f_y (A_s - A_{s2})$
 A_{s1} balances compression concrete
- (4) $a = \beta_c$ stress block assumption

Equations 1-4 (unknowns in a , A_{s2} , c , f_s') may be solved for a as:

$$\begin{aligned}
 & (.85fc'b)a^2 + (.003As'Es - fyAs - .85fc'As')a \\
 & - .003As'Es\beta d' = 0 \\
 & 3400(18)a^2 + 23600(3.6)a - 73950(3.6)1.5625 = 0 \\
 & \text{This solves to } a = 2.003784 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 d1 & = \text{depth of concrete stress block} \\
 d1 & = (ab(c/2) - As'd') / (ab - As') \\
 d1 & = .939733005 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 M1n & = .85fc'(ab - As')(d - d1) \\
 M1n & = 979725.4 \text{ lbf-in.} \\
 fs' & = (fyAs - .85fc'(ab - As')) / As' \\
 fs' & = 29335.7 \text{ psi} \\
 M2n & = 937274.6 \text{ lbf-in.} \\
 Mn & = Mn1 + Mn2 \\
 Mn & = 1917000 \text{ lbf-in.}
 \end{aligned}$$

Check:

$$\begin{aligned}
 FyAs1 & = .85fc'(ab - As') \rightarrow As1 = 1.83986 \text{ in.}^2 \\
 FyAs2 & = fs'As' \rightarrow As2 = 1.76014 \text{ in.}^2 \\
 As & = As1 + As2 = 3.60000 \text{ in.}^2, \text{ Q.E.D.}
 \end{aligned}$$

4.3.2 Cracking Moment of Inertia

The following terms are defined to specify the cracked moment of inertia :

$$\begin{aligned}
 Icr & = \text{cracked moment of inertia, in.}^4 \\
 n & = Es/Ec, \text{ dimensionless} \\
 ycr & = \text{distance from outermost compression fiber to} \\
 & \text{neutral axis}
 \end{aligned}$$

$$\begin{aligned}
 Ec & = 57000(fc')^{1/2} \\
 Ec & = 3604996 \text{ psi} \\
 Es & = 29000000 \text{ psi} \\
 n & = 8.044391
 \end{aligned}$$

Find ycr by summing first moments of area about neutral axis, and equating sum to zero.

$$\begin{aligned}
 (\frac{1}{2})bycr^2 + (n-1)As'(ycr - d') - nAs(d - ycr) & = 0 \\
 ycr & = 3.844818 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 Icr & = Icomp.conc. + Icomp.steel + Itensionsteel \\
 Icr & = bycr^3/3 + (n-1)As'(ycr - d')^2 + nAs(d - ycr)^2 \\
 Icr & = 227.346 + 132.099 + 1258.693 \\
 Icr & = 1618.138 \text{ in.}^4
 \end{aligned}$$

4.3.3 Program Parameters

$$\begin{aligned}
 m = \text{mass} & = (20 \times 12 \times 18 \times 12 / 1728) \times 150 = 4500 \text{ lbm} \\
 umax & = \text{displacement at } Mn, \text{ assumed end of elastic} \\
 & \text{range}
 \end{aligned}$$

find w (distributed load in lbf/in.) for m = Mn

$$Mn = 1/8 wL^2$$

$$w = 8 Mn/L^2$$

$w = 1917000/240^2 = 266.5 \text{ lbf/in.}$
 $\text{total load} = 266.5 \times 240 = 63900 \text{ lbf}$
 $u_{\text{max}} = 5wL^4/384EI_{\text{cr}}$
 $u_{\text{max}} = 5(266.5)240^4/384(3604997)1618.138$
 $u_{\text{max}} = 1.971755 \text{ in.}$
 $k = \text{stiffness} = \text{total load}/\Delta$
 $k = 32407.674 \text{ lbf/in.}$
 $\omega_n = \text{natural frequency} = (k/m)^{1/2} = 2.683599 \text{ rad/sec}$
 $A = 20(10) = 200 \text{ ft}^2 = 28800 \text{ in.}^2$
 $2\zeta\omega_n = c/m \rightarrow c = 2\zeta(\omega_n)m$
 $\text{let } \zeta = 0.1$
 $C = 2(.1)2.683599(4500)$
 $C = 2415.239 \text{ lbf/in.}$

4.3.4 Calculation of Standoff Parameters

Figure 2-5 on page 90 of UFC 3-340-02 gives P_t and $t_o/W^{1/3}$ verse $Z/W^{1/3}$, the normalized standoff distance. We use interpolation to seek the value of Z which gives the closest displacement to $u_{\text{max}} = 1.971755$ inches.

input form	W (lbf)	Z (ft/lbf ^{1/3})	Pt (psi)	t _o (sec)	standoff (ft)
impulse	100	4.14	348.452	.005491	19.216
	1000	6.09	114.564	.016733	60.900
	10000	9.45	37.052	.051821	203.594
triangle	100	4.14	348.452	.005491	19.216
	1000	6.09	114.564	.016733	60.900
	10000	9.45	37.052	.051821	203.594
Friedlander	100	2.92	946.189	.004050	13.553
	1000	4.29	314.049	.012208	42.900
	10000	6.34	102.465	.037374	136.591

The following conclusions are drawn from this example.

1. There is little difference between the impulse and triangle models.
2. The product $\frac{1}{2}P_t t_o = .9585 \pm .0015$ for impulse and triangle, with the last case being $1.916 \pm .001$, units being $\text{psi-sec} = \text{lbf*sec/in.}^2$

5. REFERENCES

5.1 "The Physics and Mechanisms of Primary Blast Injury", Stuhmiller,etal., Chapter 7 "Conventional Warfare Ballistics, Blast and Burn Injuries". To access this paper, go to www.google.com and type in the title.

5.2 "Structures to Resist the effects of Accidental Explosions", UFC 3-340-02, 12-05-08 www.hnd.usace.Mil/technology/engpubs.htm

5.3 "Elastic-Plastic Response Spectra for Exponential Blast Loading",Gantes, etal., International Journal of Impact Engineering, 2004, www.elsevier.com/locate/ijimpeng


```

    for(i=0;i<=7;i++)
    {
        for(j=0;j<=6;j++)
        {
            jay = j;
            *(pwr_mtrx+8*i+j) = pow(*(x_vek+i),7-jay);
        }
    }
    invert(pwr_mtrx);
    for(i=0;i<=7;i++)
    {
        *(unk1_vek+i) = 0;
        *(unk2_vek+i) = 0;
    }
    for(i=0;i<=7;i++)
    {
        for(j=0;j<=7;j++)
        {
            *(unk1_vek+i) += *(pwr_mtrx+8*i+j)**(y_vek+j);
            *(unk2_vek+i) += *(pwr_mtrx+8*i+j)**(z_vek+j);
        }
    }
    for(i=0;i<=7;i++)
    {
        fprintf(out,"%14.6f\n",*(unk1_vek+i));
    }
    fprintf(out,"\n\n");
    for(i=0;i<=7;i++)
    {
        fprintf(out,"%14.6f\n",*(unk2_vek+i));
    }
    fclose(out);
    printf("enter W,Z\n");
    scanf("%lf %lf",&W,&Z);
    logZ = log10(Z);
    for(i=0;i<=6;i++)
    {
        logPt += *(unk1_vek+i)*pow(logZ,7-i);
        logto += *(unk2_vek+i)*pow(logZ,7-i);
    }
    logPt += *(unk1_vek+7);
    logto += *(unk2_vek+7);
    Pt = pow(10,logPt);
    to = pow(10,logto);
    to = to*pow(W,frac);
    printf("Pt= ");printf("%14.6f",Pt);printf(" psi \n");
    printf("to= ");printf("%14.6f",to);printf(" msec\n");
    return 0;
}
void invert(double *mtrx)
{
    int i,j,k;
    double *old_kol,*old_row,pivco;
    old_kol=calloc(8,sizeof(double));
    old_row=calloc(8,sizeof(double));
    for(i=0;i<=7;i++)
    {
        for(j=0;j<=7;j++)
        {
            *(old_kol+j)=*(mtrx+8*j+i);
            *(old_row+j)=*(mtrx+8*i+j);
        }
        pivco=*(mtrx+8*i+i);
        *(mtrx+8*i+i)=1/pivco;
        for(j=0;j<=7;j++)
        {
            if(j!=i)
            {
                *(mtrx+8*j+i)+=(mtrx+8*j+i)/pivco;*(mtrx+8*i+j)-=(mtrx+8*i+j)/pivco;
            }
        }
    }
}

```

```

    }
    for(j=0;j<=7;j++)
    {
        for(k=0;k<=7;k++)
        {
            if((j!=i)&&(k!=i))
            {
                *(mtrx+8*j+k)=
                *(mtrx+8*j+k)-*(old_kol+j)*(*(old_row+k))/pivco;
            }
        }
    }
}

/* dirac.c : solution of damped SDOF driven by impulse : 11-30-09 : ml
*/
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
    int i;
    double A,b1,b2,delta,du,m,Pt,t,theta,tmax,to,ttotal,u,wn,zilch;
    double arg1,arg2,arg3,*input_data,wrkf;
    FILE *inn;
    FILE *out;
    input_data=calloc(8,sizeof(double));
    inn=fopen("dirac.in","r");
    out=fopen("dirac.out","w+");
    for(i=0;i<=7;i++)
    {
        fscanf(inn,"%lf",&wrkf);
        *(input_data+i)=wrkf;
    }
    fprintf(out," DIRAC.OUT\n\n");
    A = *(input_data+0);
    delta = *(input_data+1);
    m = *(input_data+2);
    Pt = *(input_data+3);
    to = *(input_data+4);
    ttotal = *(input_data+5);
    wn = *(input_data+6);
    zilch = *(input_data+7);
    fclose(inn);
    fprintf(out," A = ");fprintf(out,"%14.6f",A );fprintf(out," in.^2\n");
    fprintf(out," delta = ");fprintf(out,"%14.6f",delta );fprintf(out," sec\n");
    fprintf(out," m = ");fprintf(out,"%14.6f",m );fprintf(out," lbm\n");
    fprintf(out," Pt = ");fprintf(out,"%14.6f",Pt ); fprintf(out," lbf/in.^2\n");
    fprintf(out," to = ");fprintf(out,"%14.6f",to );fprintf(out," sec\n");
    fprintf(out," ttotal = ");fprintf(out,"%14.6f",ttotal);fprintf(out," sec\n");
    fprintf(out," wn = ");fprintf(out,"%14.6f",wn );fprintf(out," rad/sec\n");
    fprintf(out," zilch = ");fprintf(out,"%14.6f",zilch );fprintf(out," \n\n");
    fprintf(out," sec u du \n");
    fprintf(out," ----- - - - - - \n");
    arg1 = 1-zilch*zilch;
    b1 = wn*sqrt(arg1);
    b2 = sqrt(arg1);
    for(t=0;t<=ttotal+delta/2;t+=delta)
    {
        arg2 = -zilch*wn*t;
        arg3 = b1*t;
        u = (Pt*A*to/(2*m))*(1/b1)*(exp(arg2))*sin(arg3);
        du = (Pt*A*to/(2*m))*(1/b1)*(exp(arg2))*
            (b1*cos(arg3)-zilch*wn*sin(arg3));
        fprintf(out,"%10.6f %10.6f %10.6f\n",t,u,du);
    }
    theta = atan(b2/zilch);
    tmax = theta/b1;
    fprintf(out," \n\n");
}

```



```

        fprintf(out," tmax = ");
        fprintf(out,"%10.6f",tmax);
        fprintf(out," sec\n\n");
        fclose(out);
        return 0;
    }

/* triangle1.c : solution of equations of motion with three point input
by Laplace transforms : 11-30-09 : points = (0,0),(0,Pso),and(to,0)
A      =      affected area, in.^2
delta  =      time step, seconds
du     =      velocity, in./sec
m      =      member mass, lbm
Pt     =      magnitude of driving pressure,Pso+Pr, lbf/in.^2
to     =      duration of positive driving force, seconds
ttotal =      total time of run, seconds
u      =      displacement, in.
wn     =      natural frequency of member, rad/sec =(k/m)^(1/2)
zilch  =      fraction of critical damping
*/
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
    int i;
    double A,delta,Pt,m,to,ttotal,wn,zilch;
    double t,u,du;
    double c1,c2,c3,c4,c5,c6,c7,v1,v2,v3,v4;
    double *input_data,wrkf;
    FILE *inn;
    FILE *out;
    input_data=calloc(8,sizeof(double));
    inn=fopen("triangle1.in","r");
    out=fopen("triangle1.out","w+");
    for(i=0;i<=7;i++)
    {
        fscanf(inn,"%lf",&wrkf);
        *(input_data+i)=wrkf;
    }
    fprintf(out," TRIANGLE1.OUT\n\n");
    A      =      *(input_data+0);
    delta  =      *(input_data+1);
    m      =      *(input_data+2);
    Pt     =      *(input_data+3);
    to     =      *(input_data+4);
    ttotal =      *(input_data+5);
    wn     =      *(input_data+6);
    zilch  =      *(input_data+7);
    fprintf(out," A      = ");fprintf(out,"%14.6f",A      );fprintf(out," in.^2\n");
    fprintf(out," delta = ");fprintf(out,"%14.6f",delta );fprintf(out," sec\n");
    fprintf(out," m      = ");fprintf(out,"%14.6f",m      );fprintf(out," lbm\n");
    fprintf(out," Pt     = ");fprintf(out,"%14.6f",Pt     );fprintf(out," lbf\n");
    fprintf(out," to     = ");fprintf(out,"%14.6f",to     );fprintf(out," sec\n");
    fprintf(out," ttotal= ");fprintf(out,"%14.6f",ttotal);fprintf(out," sec\n");
    fprintf(out," wn     = ");fprintf(out,"%14.6f",wn     );fprintf(out," rad/sec\n");
    fprintf(out," zilch = ");fprintf(out,"%14.6f",zilch );fprintf(out," \n\n");
    fprintf(out,"      sec      u      du      \n");
    fprintf(out,"-----  -----  -----\n");
    c1      =      Pt*A/m;
    c2      =      1/(wn*wn);
    c3      =      1/(wn*wn*sqrt(1-zilch*zilch));
    c4      =      1.470628906; //acos(zilch);
    c5      =      2.941257811; //acos(2*zilch*zilch-1);
    c6      =      wn*sqrt(1-zilch*zilch);

```

```

c7      =      1/(wn*wn*wn*sqrt(1-zilch*zilch));

for(t=0;t<=tttotal+delta/2;t+=delta)
{
    v1      =      exp(-zilch*wn*t);
    v2      =      wn*(sqrt(1-zilch*zilch))*t;
    v3      =      exp(-zilch*wn*(t-to));
    v4      =      wn*(sqrt(1-zilch*zilch))*(t-to);
    if(t<=to)
    {
        u      =      c1*(c2-c3*v1*sin(v2+c4))-
            (c1/to)*(c2*t-2*zilch/(wn*wn*wn)+
            c7*v1*sin(v2+c5));
        du     =      c1*(-c3*(v1*c6*cos(v2+c4)-
            zilch*wn*v1*sin(v2+c4)))-
            (c1/to)*(c2+c7*(v1*c6*cos(v2+c5)-
            zilch*wn*v1*sin(v2+c5)));
    }
    else
    {
        u      =      c1*(c2-c3*v1*sin(v2+c4))-
            (c1/to)*(c2*t-2*zilch/(wn*wn*wn)+
            c7*v1*sin(v2+c5))+
            (c1/to)*(c2*(t-to)-
            2*zilch/(wn*wn*wn)+
            c7*v3*sin(v4+c5));
        du     =      c1*(-c3*(v1*c6*cos(v2+c4)-
            zilch*wn*v1*sin(v2+c4)))-
            (c1/to)*(c2+c7*(v1*c6*cos(v2+c5)-
            zilch*wn*v1*sin(v2+c5)))+
            (c1/to)*(c2+c7*(c6*v3*cos(v4+c5)-
            zilch*wn*v3*sin(v4+c5)));
    }
    fprintf(out,"%10.6f %10.6f %10.6f \n",t,u,du);
}
return 0;
}

```

```

/* triangle2.c : solution of equations of motion with triangle input
by runge-kutta iteration :
12-01-09 : choice of iterations vs. printing by variable arg : ml
A      =      affected area, in.^2
arg    =      every 'argth' value printed
delta  =      time step, seconds
dun0   =      velocity of element, in.sec
m      =      element mass, lbm
Pt     =      magnitude of incident pressure,Pso+Pr, lbf/in.^2
t      =      elapsed time, seconds
to     =      duration of positive incident force, sec
ttotal =      total time of run, sec
un0    =      displacement of element, in.
wn     =      natural frequency of element, rad/sec =(k/m)^(1/2)
zilch  =      fraction of critical damping
*/
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
    int arg,i,j,wrki;
    double gA,gdelta,gdun0,gm,gPt,gt,gto,gttotal,gun0,gwn,gzilch;
    double wrkf;
    void region1(double,double,double *,double,double,double,double,
        double *,double,double);
    void region2(double,double,double *,double,double,
        double,double,double *,double,double);
    double *input_data;
    FILE *inn;

```

```

FILE *out;
input_data=calloc(14,sizeof(double));
inn=fopen("triangle2.in","r");
out=fopen("triangle2.out","w+");
for(i=0;i<=7;i++)
{
    fscanf(inn,"%lf",&wrkf);
    *(input_data+i)=wrkf;
}
fprintf(out," TRIANGLE2.OUT\n\n");
gA = *(input_data+0);
gdelta = *(input_data+1);
gm = *(input_data+2);
gPt = *(input_data+3);
gto = *(input_data+4);
gttotal = *(input_data+5);
gwn = *(input_data+6);
gzilch = *(input_data+7);
fscanf(inn,"%d",&wrki);
arg = wrki;
fclose(inn);
fprintf(out," A = ");fprintf(out,"%14.6f",gA );fprintf(out," in.^2\n");
fprintf(out," delta = ");fprintf(out,"%14.6f",gdelta );fprintf(out," sec\n");
fprintf(out," m = ");fprintf(out,"%14.6f",gm );fprintf(out," lbm\n");
fprintf(out," Pt = ");fprintf(out,"%14.6f",gPt );fprintf(out," psi\n");
fprintf(out," to = ");fprintf(out,"%14.6f",gto );fprintf(out," sec\n");
fprintf(out," tttotal= ");fprintf(out,"%14.6f",gttotal);fprintf(out," sec\n");
fprintf(out," wn = ");fprintf(out,"%14.6f",gwn );fprintf(out," rad/s\n");
fprintf(out," zilch = ");fprintf(out,"%14.6f",gzilch );fprintf(out," \n");
fprintf(out," arg = ");fprintf(out,"%14d" ,arg );fprintf(out," \n\n");
fprintf(out," sec u du \n");
fprintf(out," ----- \n");
gun0 = 0;
gdun0 = 0;
j = 0;
for(gt=0;gt<=gttotal+gdelta/2;gt+=gdelta)
{
    if(gt<=gto)
    {
        if(((j+arg)%arg)==0)
        {
            fprintf(out,"%10.6f %10.6f %10.6f\n",gt,
                gun0,gdun0);
        }
        else
        {
            ;
        }
        region1(gA,gdelta,&gdun0,gm,gPt,gt,gto,&gun0,gwn,gzilch);
    }
    else
    {
        if(((j+arg)%arg)==0)
        {
            fprintf(out,"%10.6f %10.6f %10.6f\n",gt,gun0,gdun0);
        }
        else
        {
            ;
        }
        region2(gA,gdelta,&gdun0,gm,gPt,gt,gto,&gun0,gwn,gzilch);
    }
    j++;
}
fclose(out);
return 0;
}
void region1(double A,double delta,double *dun0,double m,
    double Pt,double t,double to,double *un0,double wn,double zilch)
{
    double k1,k2,k3,k4;

```

```

k1 = delta*(-wn*wn*( *un0)-2*zilch*wn*( *dun0)+
(Pt*A/m)*(1-t/to));
k2 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
(*dun0+k1/2)+(Pt*A/m)*(1-(t+delta/2)/to));
k3 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
(*dun0+k2/2)+(Pt*A/m)*(1-(t+delta/2)/to));
k4 = delta*(-wn*wn*( *un0+delta*( *dun0)+delta*k3/2)-2*zilch*wn*
(*dun0+k3)+(Pt*A/m)*(1-(t+delta)/to));
*un0 = *un0+delta*( *dun0+(k1+k2+k3)/6);
*dun0= *dun0+(k1+2*(k2+k3)+k4)/6;
}
void region2(double A,double delta,double *dun0,
double m,double Pt,double t,double to,
double *un0,double wn,double zilch)
{
double k1,k2,k3,k4;
t = t-to;
k1 = delta*(-wn*wn*( *un0)-2*zilch*wn*( *dun0));
k2 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
(*dun0+k1/2));
k3 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
(*dun0+k2/2));
k4 = delta*(-wn*wn*( *un0+delta*( *dun0)+delta*k3/2)-2*zilch*wn*
(*dun0+k3));
*un0 = *un0+delta*( *dun0+(k1+k2+k3)/6);
*dun0= *dun0+(k1+2*(k2+k3)+k4)/6;
}

/* crossover1.c : calculation of crossover time from elastic to plastic
regions : 12-13-09 : ml
*/
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
int i,no_it;
double A,b,m,Pt,strt,to,umax,wn,zilch; /* external */
double a,b1,c0,c1,c2,c3,c4,f0,fp0,t; /* internal */
FILE *inn;
FILE *out;
inn=fopen("crossover1.in","r");
out=fopen("crossover1.out","w+");
fscanf(inn,"%lf",&A);
fscanf(inn,"%lf",&b);
fscanf(inn,"%lf",&m);
fscanf(inn,"%d",&no_it);
fscanf(inn,"%lf",&Pt);
fscanf(inn,"%lf",&strt);
fscanf(inn,"%lf",&to);
fscanf(inn,"%lf",&umax);
fscanf(inn,"%lf",&wn);
fscanf(inn,"%lf",&zilch);
fprintf(out,"crossover1.out\n\n");
fprintf(out,"A = ");fprintf(out,"%14.6f",A );fprintf(out," in.^2\n");
fprintf(out,"b = ");fprintf(out,"%14.6f",b );fprintf(out," \n");
fprintf(out,"m = ");fprintf(out,"%14.6f",m );fprintf(out," lbm\n");
fprintf(out,"no_it = ");fprintf(out,"%14d",no_it );fprintf(out," \n");
fprintf(out,"Pt = ");fprintf(out,"%14.6f",Pt );fprintf(out," psi\n");
fprintf(out,"strt = ");fprintf(out,"%14.6f",strt );fprintf(out," sec\n");
fprintf(out,"to = ");fprintf(out,"%14.6f",to );fprintf(out," sec\n");
fprintf(out,"umax = ");fprintf(out,"%14.6f",umax );fprintf(out," in.\n");
fprintf(out,"wn = ");fprintf(out,"%14.6f",wn );fprintf(out," rad/sec\n");
fprintf(out,"zilch = ");fprintf(out,"%14.6f",zilch );fprintf(out," \n\n");
fprintf(out," t f0 fp0 \n");
fprintf(out," -----");
a = b/to-zilch*wn;
b1 = wn*sqrt(1-zilch*zilch);

```

```

c0      =      Pt*A/(b1*m);
c1      =      (to*a*(a*a+b1*b1)-(a*a-b1*b1))/(to*(a*a+b1*b1)*(a*a+b1*b1));
c2      =      (to*(a*a+b1*b1)*(-b1)+2*a*b1)/(to*(a*a+b1*b1)*(a*a+b1*b1));
c3      =      b1/(a*a+b1*b1);
c4      =      -2*a*b1/(to*(a*a+b1*b1)*(a*a+b1*b1));
t       =      strt;
for(i=0;i<=no_it-1;i++)
{
    f0 = c0*(exp(-zilch*wn*t)*(c1*sin(b1*t)+c2*cos(b1*t))+
        exp(-b*t/to)*(c3*(1-t/to)+c4))-umax;
    fp0 = c0*(exp(-zilch*wn*t)*
        (b1*c1*cos(b1*t)-b1*c2*sin(b1*t)-
        zilch*wn*(c1*sin(b1*t)+c2*cos(b1*t)))+
        exp(-b*t/to)*(-c3/to-(b/to)*(c3*(1-t/to)+c4)));
    t = t-f0/fp0;
    fprintf(out,"%14.6f %14.6f %14.6f \n",t,f0,fp0);
}
return 0;
}
}

```

```

/* frdlndr1.c : solution of equations of motion with friedlander equation
by transform method : elastic range = region1 and plastic range =
region2 : 12-11-09 : set cro > ttotal for linear only : ml
A       =      affected area, in.^2
b       =      parameter containing rate of wave amplitude decay
c       =      multiplier of du/dt to obtain viscous force,
            lbf/(in./sec)
cro     =      crossover time, seconds
delta   =      time step, seconds
dun0    =      velocity of element, in.sec
k       =      multiplier of u to obtain deflection force, lbf/in.
m       =      element mass, lbm
Pt      =      magnitude of total pressure, Pso+Pr, lbf/in.^2
m       =      member mass, lbm
Rm      =      maximum resisting force, lbf
t       =      elapsed time, seconds
to      =      duration of positive incident force, seconds
ttotal  =      total time of run, seconds
um      =      maximum linear displacement of element, in.
un0     =      displacement of element, in.
wn      =      natural frequency of element, rad/sec =(k/m)^(1/2)
zilch   =      fraction of critical damping
*/

```

```

#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
    int i;
    double gA,gb,gc,gcro,gdelta,gdun0,gk,gm,gPt,gRm,gt,gto,gttotal,gum,gun0,
        gwn,gzilch;
    double gu0,gdu0,wrkf;
    void region1(double,double,double *,double,double,double,double,
        double *,double,double,double *,double *);
    void region2(double,double,double,double *,double,double,double,
        double,double,double *,double *,double *,double);
    double *input_data;
    FILE *inn;
    FILE *out;
    input_data=calloc(14,sizeof(double));
    inn=fopen("frdlndr1.in","r");
    out=fopen("frdlndr1.out","w+");
    for(i=0;i<=13;i++)
    {
        fscanf(inn,"%lf",&wrkf);
        *(input_data+i)=wrkf;
    }
    fprintf(out,"NONLINEAR1.OUT\n\n");
}

```

```

gA      =      *(input_data+0);
gb      =      *(input_data+1);
gc      =      *(input_data+2);
gcro    =      *(input_data+3);
gdelta  =      *(input_data+4);
gk      =      *(input_data+5);
gm      =      *(input_data+6);
gPt     =      *(input_data+7);
gRm     =      *(input_data+8);
gto     =      *(input_data+9);
gttotal =      *(input_data+10);
gum     =      *(input_data+11);
gwn     =      *(input_data+12);
gzilch  =      *(input_data+13);
fprintf(out,"A      = ");fprintf(out,"%14.6f",gA      );
fprintf(out,"in.^2\n");
fprintf(out,"b      = ");fprintf(out,"%14.6f",gb      );
fprintf(out,"      \n");
fprintf(out,"c      = ");fprintf(out,"%14.6f",gc      );fprintf(out,"lbf/ips\n");
fprintf(out,"cro    = ");fprintf(out,"%14.6f",gcro   );fprintf(out,"seconds\n");
fprintf(out,"delta  = ");fprintf(out,"%14.6f",gdelta );fprintf(out,"sec\n");
fprintf(out,"k      = ");fprintf(out,"%14.6f",gk      );fprintf(out,"lbf/in.\n");
fprintf(out,"m      = ");fprintf(out,"%14.6f",gm      );fprintf(out,"lbm\n");
fprintf(out,"Pt     = ");fprintf(out,"%14.6f",gPt     );fprintf(out,"lbf/in.^2\n");
fprintf(out,"Rm     = ");fprintf(out,"%14.6f",gRm     );fprintf(out,"lbf/in.\n");
fprintf(out,"to     = ");fprintf(out,"%14.6f",gto     );fprintf(out,"sec\n");
fprintf(out,"ttotal= ");fprintf(out,"%14.6f",gttotal);fprintf(out,"sec\n");
fprintf(out,"um     = ");fprintf(out,"%14.6f",gum     );fprintf(out,"in.\n");
fprintf(out,"wn     = ");fprintf(out,"%14.6f",gwn     );fprintf(out,"rad/sec\n");
fprintf(out,"zilch = ");fprintf(out,"%14.6f",gzilch );fprintf(out,"\n\n");
fprintf(out," seconds      u      du      \n");
fprintf(out,"----- ----- ----- \n");
gun0    = 0;
gdun0   = 0;
region1(gA,gb,&gdun0,gm,gPt,gcro,gto,&gun0,gwn,gzilch,&gdu0,&gu0);
gu0 = gun0;
gdu0= gdun0;
for(gt=0;gt<=gttotal+gdelta/2;gt+=gdelta)
{
    if(gt<gcro)
    {
        region1(gA,gb,&gdun0,gm,gPt,gt,gto,&gun0,gwn,gzilch,&gdu0,&gu0);
        fprintf(out,"%7.3f %10.6f %10.6f\n",gt,gun0,gdun0);
    }
    else
    {
        region2(gA,gb,gc,&gdun0,gm,gPt,gRm,gt,gto,&gun0,&gdu0,&gu0,gcro);
        fprintf(out,"%7.3f %10.6f %10.6f\n",gt,gun0,gdun0);
    }
}
return 0;
}
void region1(double A,double b,double *dun0,double m,
double Pt,double t,double to,double *un0,
double wn,double zilch,double *du0,double *u0)
{
double      a,b1,c0,c1,c2,c3,c4;
a          =      b/to-zilch*wn;
b1         =      wn*sqrt(1-zilch*zilch);
c0         =      Pt*A/(b1*m);
c1         =      (to*a*(a+b1*b1)-(a*a-b1*b1))/(to*(a*a+b1*b1)*(a*a+b1*b1));
c2         =      (to*(a*a+b1*b1)*(-b1)+2*a*b1)/(to*(a*a+b1*b1)*(a*a+b1*b1));
c3         =      b1/(a*a+b1*b1);
c4         =      -2*a*b1/(to*(a*a+b1*b1)*(a*a+b1*b1));
*un0       =      c0*(exp(-zilch*wn*t)*(c1*sin(b1*t)+c2*cos(b1*t))+
exp(-b*t/to)*(c3*(1-t/to)+c4));
*dun0      =      c0*(exp(-zilch*wn*t)*
(b1*c1*cos(b1*t)-b1*c2*sin(b1*t)-
zilch*wn*(c1*sin(b1*t)+c2*cos(b1*t)))+
exp(-b*t/to)*(-c3/to-(b/to)*(c3*(1-t/to)+c4)));
}

```

```

void region2(double A,double b,double c,double *dun0,
             double m,double Pt,double Rm,double t,double to,
             double *un0,double *du0,double *u0,double cro)
{
    double      d0,d1,e1,e2,f1,f2,f3,f4;
    t           =      t-cro;
    d0          =      (Pt*A/m)*exp(-b*cro/to)*(1-cro/to);
    d1          =      (Pt*A/m)*exp(-b*cro/to)*(1/to);
    e1          =      exp(-(c/m)*t);
    e2          =      exp(-(b/to)*t);
    f1          =      1/((b/to)-(c/m));
    f2          =      (b/to)/((b/to)-(c/m));
    f3          =      1/((b/to)*((b/to)-(c/m)));
    f4          =      (c/m)/(b/to-c/m);

    *un0        =      *u0*e1+(*du0+(c/m)*(*u0))*(m/c)*(1-e1)-
                      (Rm/m)*(m/c)*(m/c)*((c/m)*t-1+e1)+
                      d0*(m*to/(b*c))*(1+f4*e2-f2*e1)-
                      d1*(1/((c/m)*(b/to)*(b/to))-
                      (m/c)*f1*f1*e1+
                      (2*b/to-c/m)*f3*f3*e2+
                      (to/b)*f3*t*e2);
    *dun0       =      -(c/m)*(*u0)*e1+(*du0+(c/m)*(*u0))*e1-
                      (Rm/c)*(1-e1)+
                      d0*(m*to/(b*c))*(-f4*b*e2/to+f2*c*e1/m)-
                      d1*(f1*f1*e1-(2*b/to-c/m)*f3*f3*b*e2/to+
                      f3*(e2+t*(-b/to)*e2));
}

/* frdlndr2.c : solution of equations of motion with fridlander equation
by runge-kutta iteration : elastic range and plastic range :
choice of iterations vs. printing by variable arg :
set cro > ttotal for linear only : ml : 12-11-09
A           =      affected area, in.^2
arg         =      every 'argth' value printed
b           =      parameter containing rate of wave amplitude decay
c           =      multiplier of du/dt to obtain viscous force, lbf/(in./sec)
cro         =      crossover ctime,sec
delta      =      time step, seconds
dun0       =      velocity of element, in.sec
k           =      multiplier of u to obtain deflection force, lbf/in.
m           =      element mass, lbm
Pt          =      magnitude of total pressure, Pso+Pr, lbf/in.^2
Rm          =      maximum resisting force, lbf
t           =      elapsed time, seconds
to         =      duration of positive incident force, sec
ttotal     =      total time of run, sec
um         =      maximum linear displacement of element, in.
un0        =      displacement of element, in.
wn         =      natural frequency of element, rad/sec =(k/m)^(1/2)
zilch      =      fraction of critical damping
*/
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
int main(void)
{
    int arg,i,j,wrki;
    double gA,gb,gc,gcro,gdelta,gdun0,gk,gm,gPt,gRm,gt,gto,gtttotal,gum,
           gun0,gwn,gzilch;
    double wrkf;
    void region1(double,double,double,double *,double,double,double,double,
                double *,double,double);
    void region2(double,double,double,double,double *,double,double,double,

```

```

        double,double,double *,double);
double *input_data;
FILE *inn;
FILE *out;
input_data=calloc(14,sizeof(double));
inn=fopen("nonlinear2.in","r");
out=fopen("nonlinear2.out","w+");
for(i=0;i<=13;i++)
{
    fscanf(inn,"%lf",&wrkf);
    *(input_data+i)=wrkf;
}
fprintf(out,"NONLINEAR2.OUT\n\n");
gA    =    *(input_data+0);
gB    =    *(input_data+1);
gC    =    *(input_data+2);
gCro  =    *(input_data+3);
gDelta =    *(input_data+4);
gK    =    *(input_data+5);
gM    =    *(input_data+6);
gPt   =    *(input_data+7);
gRm   =    *(input_data+8);
gTo   =    *(input_data+9);
gTtotal =    *(input_data+10);
gum   =    *(input_data+11);
gwn   =    *(input_data+12);
gzilch =    *(input_data+13);
fscanf(inn,"%d",&wrki);
arg    =    wrki;
fprintf(out,"A    = ");fprintf(out,"%14.6f",gA    );fprintf(out,"in.^2\n");
fprintf(out,"b    = ");fprintf(out,"%14.6f",gB    );fprintf(out,"    \n");
fprintf(out,"c    = ");fprintf(out,"%14.6f",gC    );fprintf(out,"lbf/ips\n");
fprintf(out,"cro  = ");fprintf(out,"%14.6f",gCro  );fprintf(out,"sec\n");
fprintf(out,"delta = ");fprintf(out,"%14.6f",gDelta);fprintf(out,"sec\n");
fprintf(out,"k    = ");fprintf(out,"%14.6f",gK    );fprintf(out,"lbf/in.\n");
fprintf(out,"m    = ");fprintf(out,"%14.6f",gM    );fprintf(out,"lbm\n");
fprintf(out,"Pt   = ");fprintf(out,"%14.6f",gPt   );fprintf(out,"lbf/in.^2\n");
fprintf(out,"Rm   = ");fprintf(out,"%14.6f",gRm   );fprintf(out,"lbf/in.\n");
fprintf(out,"to   = ");fprintf(out,"%14.6f",gTo   );fprintf(out,"sec\n");
fprintf(out,"ttotal= ");fprintf(out,"%14.6f",gTtotal);fprintf(out,"sec\n");
fprintf(out,"um   = ");fprintf(out,"%14.6f",gum   );fprintf(out,"in.\n");
fprintf(out,"wn   = ");fprintf(out,"%14.6f",gwn   );fprintf(out,"rad/sec\n");
fprintf(out,"zilch = ");fprintf(out,"%14.6f",gzilch);fprintf(out," \n");
fprintf(out,"arg  = ");fprintf(out,"%14d" ,arg  );fprintf(out," \n\n");
fprintf(out," seconds      u      du      \n");
fprintf(out,"-----  -----  ----- \n");
gun0   = 0;
gdun0  = 0;
j      = 0;
for(gt=0;gt<=gTtotal+gDelta/2;gt+=gDelta)
{
    if(gt<gCro)
    {
        if(((j+arg)%arg)==0)
        {
            fprintf(out,"%7.3f %10.6f %10.6f\n",gt,gun0,gdun0);
        }
        else
        {
            ;
        }
        region1(gA,gB,gDelta,&gdun0,gM,gPt,gt,gTo,&gun0,gwn,gzilch);
    }
    else
    {
        if(((j+arg)%arg)==0)
        {
            fprintf(out,"%7.3f %10.6f %10.6f\n",gt,
                gun0,gdun0);
        }
    }
}

```



```

        else
        {
            ;
        }
        region2(gA,gb,gc,gdelta,&gdun0,gm,gPt,gRm,gt,
                gto,&gun0,gcro);
    }
    j++;
}
return 0;
}

void region1(double A,double b,double delta,double *dun0,double m,
             double Pt,double t,double to,double *un0,double wn,double zilch)
{
    double k1,k2,k3,k4;
    k1 = delta*(-wn*wn*( *un0)-2*zilch*wn*( *dun0)+
               (Pt*A/m)*(1-t/to)*exp(-b*t/to));
    k2 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
               (*dun0+k1/2)+(Pt*A/m)*(1-(t+delta/2)/to)*
               exp((-b/to)*(t+delta/2)));
    k3 = delta*(-wn*wn*( *un0+delta*( *dun0)/2+delta*k1/8)-2*zilch*wn*
               (*dun0+k2/2)+(Pt*A/m)*(1-(t+delta/2)/to)*
               exp((-b/to)*(t+delta/2)));
    k4 = delta*(-wn*wn*( *un0+delta*( *dun0)+delta*k3/2)-2*zilch*wn*
               (*dun0+k3)+(Pt*A/m)*(1-(t+delta)/to)*exp(-(b/to)*(t+delta)));
    *un0 = *un0+delta*( *dun0+(k1+k2+k3)/6);
    *dun0= *dun0+(k1+2*(k2+k3)+k4)/6;
}

void region2(double A,double b,double c,double delta,double *dun0,
             double m,double Pt,double Rm,double t,double to,
             double *un0,double cro)
{
    double d0,d1,e1,e2,k1,k2,k3,k4;
    t = t-cro;
    d0 = (Pt*A/m)*exp(-b*cro/to)*(1-cro/to);
    d1 = (Pt*A/m)*exp(-b*cro/to)*(1/to);

    k1 = delta*(-(c/m)*( *dun0)-Rm/m+d0*exp(-b*t/to)-
               d1*t*exp(-b*t/to));
    k2 = delta*(-(c/m)*( *dun0+k1/2)-Rm/m+d0*exp(-b*(t+delta/2)/to)-
               d1*(t+delta/2)*exp(-b*(t+delta/2)/to));
    k3 = delta*(-(c/m)*( *dun0+k2/2)-Rm/m+d0*exp(-b*(t+delta/2)/to)-
               d1*(t+delta/2)*exp(-b*(t+delta/2)/to));
    k4 = delta*(-(c/m)*( *dun0+k3)-Rm/m+d0*exp(-b*(t+delta)/to)-
               d1*(t+delta)*exp(-b*(t+delta)/to));

    *un0 = *un0+delta*( *dun0+(k1+k2+k3)/6);
    *dun0= *dun0+(k1+2*(k2+k3)+k4)/6;
}

```

7. TYPICAL OUTPUTS

DIRAC.OUT

A	=	28800.000000 in.^2	delta	=	0.050000 sec
m	=	4500.000000 lbm	Pt	=	348.452000 lbf/in.^2
to	=	0.005491 sec	ttotal	=	1.000000 sec
wn	=	2.683599 rad/sec	zilch	=	0.100000
W	=	100.000000 lbf	Z	=	4.140000 ft/lbr^(1/3)

sec	u	du
0.000000	0.000000	6.122720
0.050000	0.301159	5.906536
0.100000	0.589002	5.591305
0.150000	0.858775	5.185241
0.200000	1.106172	4.697953

0.250000	1.327396	4.140235
0.300000	1.519217	3.523844
0.350000	1.679010	2.861268
0.400000	1.804790	2.165483
0.450000	1.895225	1.449710
0.500000	1.949648	0.727175
0.550000	1.968047	0.010873
0.600000	1.951049	-0.686655
0.650000	1.899893	-1.353533
0.700000	1.816391	-1.978743
0.750000	1.702880	-2.552299
0.800000	1.562169	-3.065395
0.850000	1.397474	-3.510530
0.900000	1.212351	-3.881607
0.950000	1.010625	-4.174006
1.000000	0.796315	-4.384624

tmax = 0.550767 sec

TRIANGLE1.OUT
TRANSFORM METHOD

A	=	28800.000000 in.^2	delta	=	0.050000 sec
m	=	4500.000000 lbm	Pt	=	114.564000 lbf
to	=	0.016733 sec	ttotal	=	1.000000 sec
wn	=	2.683599 rad/sec	zilch	=	0.100000
W	=	1000.000000 lbf	Z	=	6.090000 ft/lbf^(1/3)

sec	u	du
0.000000	-0.000001	-0.000000
0.050000	0.268604	5.946706
0.100000	0.558713	5.641588
0.150000	0.831229	5.244479
0.200000	1.081785	4.764857
0.250000	1.306517	4.213415
0.300000	1.502124	3.601836
0.350000	1.665906	2.942561
0.400000	1.795801	2.248548
0.450000	1.890402	1.533026
0.500000	1.948966	0.809257
0.550000	1.971408	0.090299
0.600000	1.958287	-0.611224
0.650000	1.910777	-1.283329
0.700000	1.830631	-1.914873
0.750000	1.720135	-2.495729
0.800000	1.582053	-3.016934
0.850000	1.419564	-3.470823
0.900000	1.236197	-3.851127
0.950000	1.035758	-4.153050
1.000000	0.822254	-4.373310

TRIANGLE2.OUT
ITERATIVE METHOD

A	=	28800.000000 in.^2	delta	=	0.000010 sec
m	=	4500.000000 lbm	Pt	=	114.564000 lbf/in.^2
to	=	0.016733 sec	ttotal	=	1.000000 sec
wn	=	2.683599 rad/sec	zilch	=	0.100000
W	=	1000.000000 lbf	Z	=	6.09 ft/lbf^(1/3)
arg	=	5000			

sec	u	du
0.000000	0.000000	0.000000
0.050000	0.268604	5.946705
0.100000	0.558713	5.641587

```

0.150000  0.831229  5.244478
0.200000  1.081785  4.764856
0.250000  1.306517  4.213414
0.300000  1.502123  3.601835
0.350000  1.665905  2.942560
0.400000  1.795800  2.248547
0.450000  1.890402  1.533026
0.500000  1.948966  0.809257
0.550000  1.971408  0.090299
0.600000  1.958286 -0.611224
0.650000  1.910776 -1.283329
0.700000  1.830630 -1.914873
0.750000  1.720134 -2.495728
0.800000  1.582052 -3.016933
0.850000  1.419564 -3.470822
0.900000  1.236197 -3.851127
0.950000  1.035758 -4.153049
1.000000  0.822254 -4.373309

```

crossover1.out

```

A      = 28800.000000 in.^2
b      = 2.000000
m      = 4500.000000 lbm
no_it  = 10
Pt     = 201.000000 psi
strt   = 0.300000 sec
to     = 0.030162 sec
umax   = 1.971755 in.
wn     = 2.683599 rad/sec
zilch  = 0.100000

```

t	f0	fp0
0.221383	0.439507	5.590506
0.229891	-0.060307	7.088084
0.229984	-0.000640	6.937082
0.229984	-0.000000	6.935428
0.229984	-0.000000	6.935427
0.229984	0.000000	6.935427
0.229984	-0.000000	6.935427
0.229984	0.000000	6.935427
0.229984	0.000000	6.935427
0.229984	-0.000000	6.935427

FRDLNDR1.OUT
ELASTIC-PLASTIC

```

A      = 28800.000000 in.^2      b      = 2.000000
c      = 2415.239000 lbf/ips     cro     = 0.229984 sec
delta  = 0.050000 sec           k      = 32407.674000 lbf/in.
m      = 4500.000000 lbm        Pt     = 201.000000 lbf/in.^2
Rm     = 63900.000000 lbf       to     = 0.030162 sec
ttotal= 1.000000 sec           um     = 1.971755 in.
wn     = 2.683599 rad/sec       zilch  = 0.100000

```

seconds	u	du
0.000	0.000000	-0.000000
0.050	0.460845	10.195211

0.100	0.934053	8.944595	
0.150	1.363328	8.231620	
0.200	1.755933	7.454185	
0.250	2.106998	6.578586	<-- crossover
0.300	2.413961	5.703824	
0.350	2.677767	4.852234	
0.400	2.899560	4.023193	
0.450	3.080452	3.216105	
0.500	3.221526	2.430387	
0.550	3.323837	1.665475	
0.600	3.388411	0.920817	
0.650	3.416248	0.195876	
0.700	3.408319	-0.509868	
0.750	3.365572	-1.196925	
0.800	3.288930	-1.865789	
0.850	3.179289	-2.516943	
0.900	3.037523	-3.150854	
0.950	2.864483	-3.767980	
1.000	2.660997	-4.368766	

FRDLNDR2.OUT
ELASTIC-PLASTIC

A	=	28800.000000 in.^2	b	=	2.000000
c	=	2415.239000 lbf/ips	cro	=	0.229984 sec
delta	=	0.000010 sec	k	=	32407.674000 lbf/in.
m	=	4500.000000 lbm	Pt	=	201.000000 lbf/in.^2
Rm	=	63900.000000 lbf	to	=	0.030162 sec
ttotal	=	1.000000 sec	um	=	1.971755 in.
wn	=	2.683599 rad/sec	zilch	=	0.100000
arg	=	5000			

seconds	u	du	
0.000	0.000000	0.000000	
0.050	0.460845	10.195211	
0.100	0.934053	8.944595	
0.150	1.363328	8.231620	
0.200	1.755933	7.454185	
0.250	2.106998	6.578586	<-- crossover
0.300	2.413961	5.703824	
0.350	2.677767	4.852234	
0.400	2.899560	4.023193	
0.450	3.080452	3.216105	
0.500	3.221526	2.430387	
0.550	3.323837	1.665475	
0.600	3.388411	0.920817	
0.650	3.416248	0.195876	
0.700	3.408319	-0.509868	
0.750	3.365572	-1.196925	
0.800	3.288930	-1.865789	
0.850	3.179289	-2.516943	
0.900	3.037523	-3.150854	
0.950	2.864483	-3.767980	
1.000	2.660997	-4.368766	

TABLE 7-1
BASIC PROPERTIES OF COMMON SINGLE-COMPOUND HIGH EXPLOSIVES

Explosive	Equivalent Weight Relative to TNT*	Density (g/cm ³)	Detonation Velocity (km/s)	Consistency
Ammonium Nitrate, AN	—	1.73	7.00	Solid
Nitroglycerin, NG	—	1.60	7.58	Liquid
Trinitrotoluene, TNT	1.00	1.65	6.90	Solid
Pentaerythritol Tetranitrate, PETN	1.27	1.70	7.98	Solid
Cyclotrimethylene trinitramine, RDX	1.19	1.80	8.75	Solid
Cyclotetramethylene tetranitramine, HMX	-1.30	1.90	9.10	Solid
Nitrocellulose, NC	—	1.2-1.7	7.30	Solid

*Based on the peak pressure produced compared to TNT

TABLE 7-2
BASIC PROPERTIES OF COMMON MIXED-COMPOUND EXPLOSIVES

Explosive*	Equivalent Weight Relative to TNT**	Density g/cm ³	Detonation Velocity km/s	Consistency
ANFO 94 AN/6 FG	0.82	-0.8	-4.7	Powder
Composition B 40 TNT/60 RDX	1.11	1.70	7.9	Solid
Octol 25 TNT/75 HMX	1.06	1.82	8.4	Solid
Pentolite 50 TNT/50 PETN	1.42	1.67	7.4	Solid
Dynamite 50 NG/0.2 NC/34 SN/15.8 C	0.90	1.40	-5.8	Solid
Bonded Mixtures				
Composition C4 91 RDX/ 2.1 rubber/ 1.6 oil/ 5.3 plasticizer	1.37	1.0-1.60	8.0	Plastic
Sheet Explosive 60-85 PETN/0-8 NC rubber and plasticizer	-1.27	-1.50	-7.0	Rubberlike sheets

*FO: Fuel oil; SN: Sodium Nitrate; C: Combustibles and chalk
**Equivalent weights based on peak overpressure produced compared to TNT

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Figure 2-15 Positive Phase Shock Wave Parameters for a Hemispherical TNT Explosion on the Surface at Sea Level

