PDHonline Course G513 (2 PDH)

Motor Vehicle Accident Reconstruction - Skid to Rest and Skid Mark Analysis

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2020

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Discussion Areas

- Derivation of the Skid to Rest formula
- Fundamental Assumptions of the Skid to Rest formula
- Extrapolations of the Skid to Rest formula
- Examples of Skid Marks
- Visual Analysis of Skid Marks and Application to Skid to Rest formula

Skid to Rest Formula

The Skid to Rest formula is classically known as:

$$S = \sqrt{30Df}$$

Where,

S = Speed in miles per hour

30 = A conversion factor

D = Skid distance in feet

f = Drag factor, dimensionless, of the road

The Drag factor is affected by the incline and decline angles of the road. The Drag factor was modified as follows:

$f = f_{baseline} + \alpha$

Where,

$f_{baseline}$ = Drag factor for a level surface

α = percentage of grade in decimals (which is equivalent to the angle converted to radians). Positive number indicates an incline, while negative number indicates a decline.

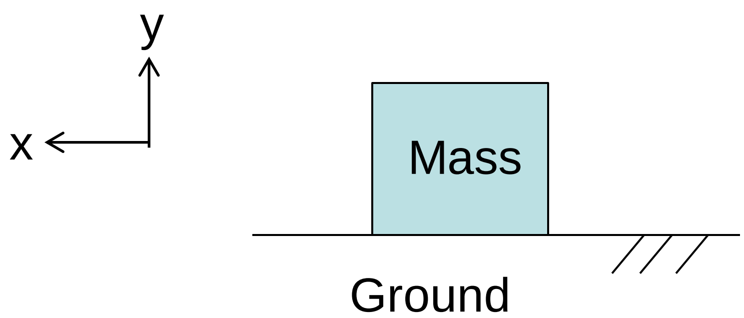
Importance to Understand Derivation

- At deposition or trial, a forensic engineer or motor vehicle accident reconstructionist may be asked to derive the equations.
- Inability to derive the skid to rest formula has been classically captured in the following video:
- http://youtu.be/sYqXlRQrBN4
- You may skip the above video if you have already viewed it in the past.

Skid to Rest Derivation

Newton's 2nd Law

- Let's imagine the car as a single lumped mass.
- For braking, we will imagine the mass sliding on a flat surface with a friction factor, f.

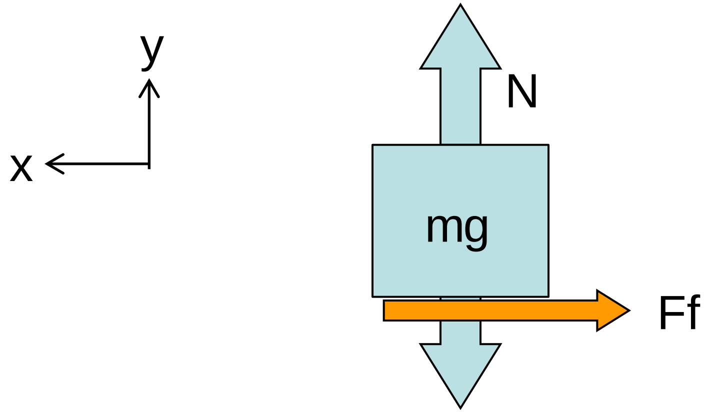


x = horizontal sliding direction
y = vertical direction

Skid to Rest Derivation

Newton's 2nd Law

- Apply Newton's 2nd Law in the vertical and horizontal direction.
- Free body diagram:



m = mass
g = gravitational acceleration
N = normal force
Ff = friction force

Newton's 2nd Law

Sum of forces in the y-direction:

$Fy = N – mg = ma_y = 0$

Or

$N = mg$

Sum of forces in the x-direction:

$Fx = Ff= -f\ N =ma_x$

$-fmg = ma_x$

$-fg = a_x$

Equations of Motion

We know from equations of motion and by definition:

$a_x = (Sf-Si)/t$

$d = (Sf+Si)t/2$

Where,

Sf = final speed (mph)

Si = initial speed (mph)

t = time (seconds)

d = distance traveled in miles

With Skid to Rest, the final speed = 0 and therefore equations become:

$a_x = (-Si)/t$ and $d = (Si)t/2$

Substituting Eqn. of Motion into Newton's 2nd Law

- Newton's 2nd Law: $-fg = a_x$
- Eqns. Of Motion: $a_x = (-Si)/t$ and $d = (Si)t/2$

Therefore,

$-fg = (-Si)/t$

We can solve for t in $d = (Si)t/2$

Or $t = 2d/Si$ and substitute back:

$fg = (Si)/t = Si^2/2d$

Si or for simplification, S, $S = \sqrt{2\,fgd}$

Solve for Skid to Rest formula

Si or for simplification, S, $S = \sqrt{2fgd}$

Gravitation constant, g, is 32.2 ft/sec^2

And distance, D, in feet, is related to d, in miles: D = d 5280 ft/mile

Therefore with unit conversions:

$$S = \sqrt{fD(32.2\,ft/\sec^2)(1mi/5280\,ft)^2(3600\sec/hour)^2}$$

$$S = \sqrt{29.9\,fD}$$

Or as simplified:

$$S = \sqrt{30Df}$$

Skills Check 1

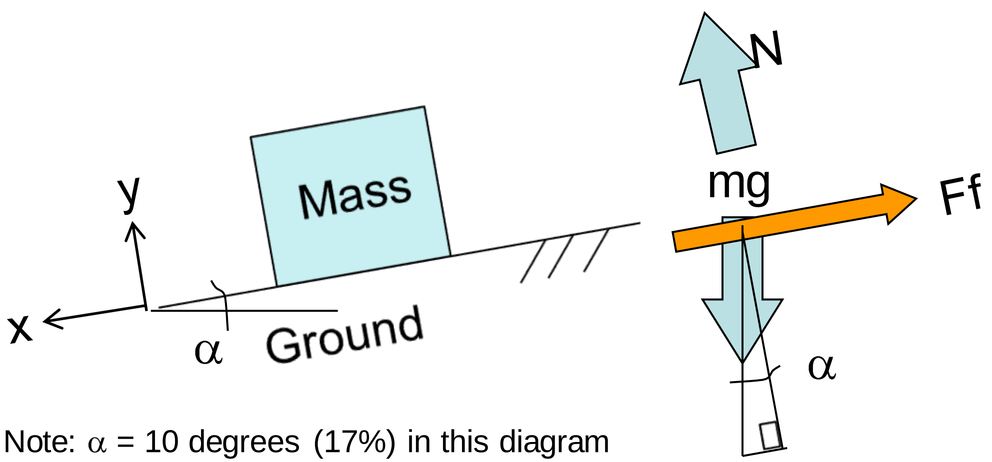
- At this point in time, please take a break and attempt to derive the skid to rest formula for yourself.
- Your sign convention, or your variable names or letters may be different. That is okay.

Underlying Assumptions of Skid to Rest

It is key to understand some of the underlying assumptions of the Skid to Rest formula as there may be modifications required for its use.

Assumption 1: Flat Road

The most common modification to the use of Skid to Rest is to account for roads that are not flat. The physics of a incline or decline can be analyzed with the same procedure as Skid to Rest but with the free body diagram as follows:



Note: α = 10 degrees (17%) in this diagram

α = angle of incline/decline in radians

Newton's 2^{nd} Law for Incline/Decline Equations

Newton's Laws Change:

Sum of forces in the y-direction:

$F_y = N - mg\cos(\alpha) = ma_y = 0$

Or

$N = mg\cos(\alpha)$

Sum of forces in the x-direction:

$F_x = F_f = -f N + mg\sin(\alpha) = ma_x$

$-f mg\cos(\alpha) + mg\sin(\alpha) = ma_x$

$-fg\cos(\alpha) + g\sin(\alpha) = a_x$

Equations of Motion Unchanged:

$a_x = (-S)/t$ and $d = (S)t/2$

Incline/Decline Simplification

- For small angles, $\cos(\alpha) = 1$, and $\sin(\alpha) = \alpha$

Angle (deg)	Angle (rad)	Cos(Angle)	Sin(Angle)
2	0.0349	0.9994	0.0349
4	0.0698	0.9976	0.0698
6	0.1047	0.9945	0.1045
8	0.1396	0.9903	0.1392
10	0.1745	0.9848	0.1736
12	0.2094	0.9781	0.2079
14	0.2443	0.9703	0.2419
16	0.2793	0.9613	0.2756
18	0.3142	0.9511	0.3090
20	0.3491	0.9397	0.3420

- Therefore, the Newton's Law simplifies to:

$-fg\cos(\alpha) + g\sin(\alpha) \Rightarrow -fg + (\alpha)g = a_x$

Or $-(f-\alpha)g = a_x$.

- The remaining derivation is now identical to the regular Skid to Rest, except the drag factor, f, is now simply modified by an adder (or subtractor).
- For inclines/declines, drag factor, f, is now: $f = f\text{baseline} + \alpha$, where α is positive for inclines and α is negative for declines.

How to Handle Larger Incline/Decline

- Rarely will we encounter a road or surface beyond 20 degrees (34%). However, there is a closed form solution to the problem.
- If we know the angle of the road, α, then from the free body diagram:

$-fg\cos(\alpha) + g\sin(\alpha) = a_x$ = a constant

- As long as we convert a_x to the units of miles/hr^2 from ft/sec^2 ($3600^2/5280$) then from Equations of Motion:

$a_x = (-Si)/t$ and $d = (Si)t/2$,

$t = 2d/Si$ and substitute back:

$a_x = (Si)/t = Si^2/2d$

Si or $S = \sqrt{\dfrac{2Da_x}{5280}}$

Where, D is the skid mark distance in feet.

San Francisco

Skills Check 2

- At this point in time, please take a break and attempt to derive the skid to rest formula for an incline/decline.
- Your sign convention, or your variable names or letters may be different. That is okay.

Assumption 2: Constant Drag

- Typically accident reconstruction methodology uses constant drag factors for cars, typically:
  - 0.65 to 0.9 for dry road (R. Limpert, Motor Vehicle Accident Reconstruction and Cause Analysis)
  - 0.3 for ice
- For motorcycles, drag factors are published as:
  - 0.35 to 1.1 (Obenski, Motorcycle Accident Reconstruction and Analysis
- For heavy truck, some people use a lower factor.
- In reality, to use a full constant drag factor, we are assuming all wheels locked (not rotating), brake efficiency at maximum capability, and the ability for the driver and vehicle brake system to maintain constant brake force.

Assumption 2: Constant Drag (cont.)

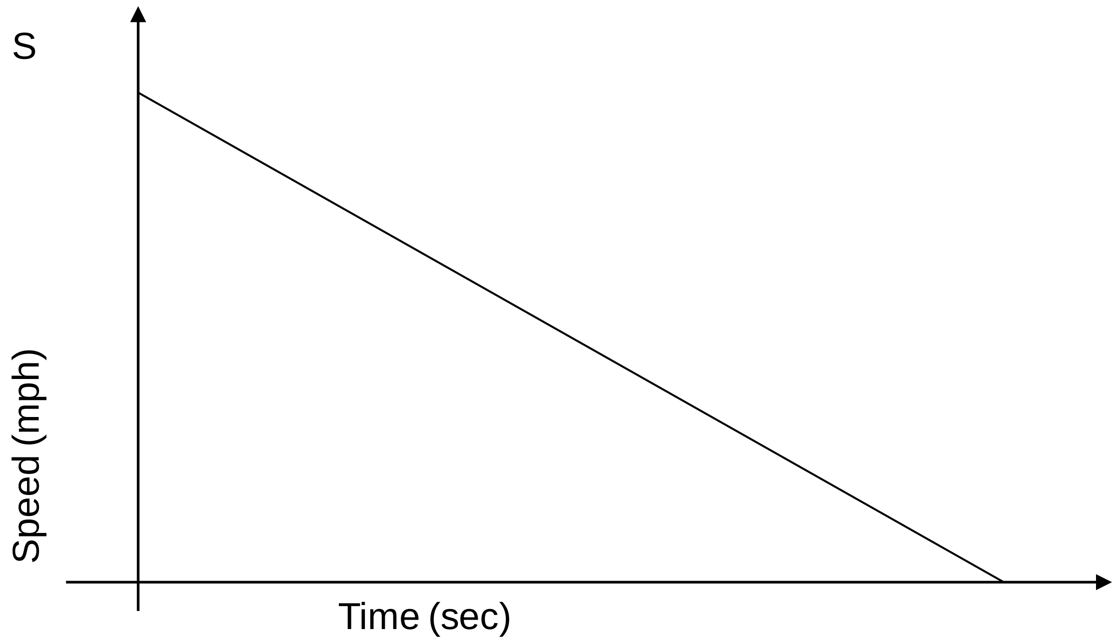
- Typically, the drag factor, f, is reduced if one or more wheels were not involved in skiding.
- Typically, an efficiency factor, n, may be added for cases where brakes were not applied at 100%. This data is sometimes recorded in Electronic Data Recorders in airbag control systems and may be downloaded with the CDR system. Skid to Rest becomes:

$$S = \sqrt{30Dfn}$$

Where n is the % efficiency of braking.

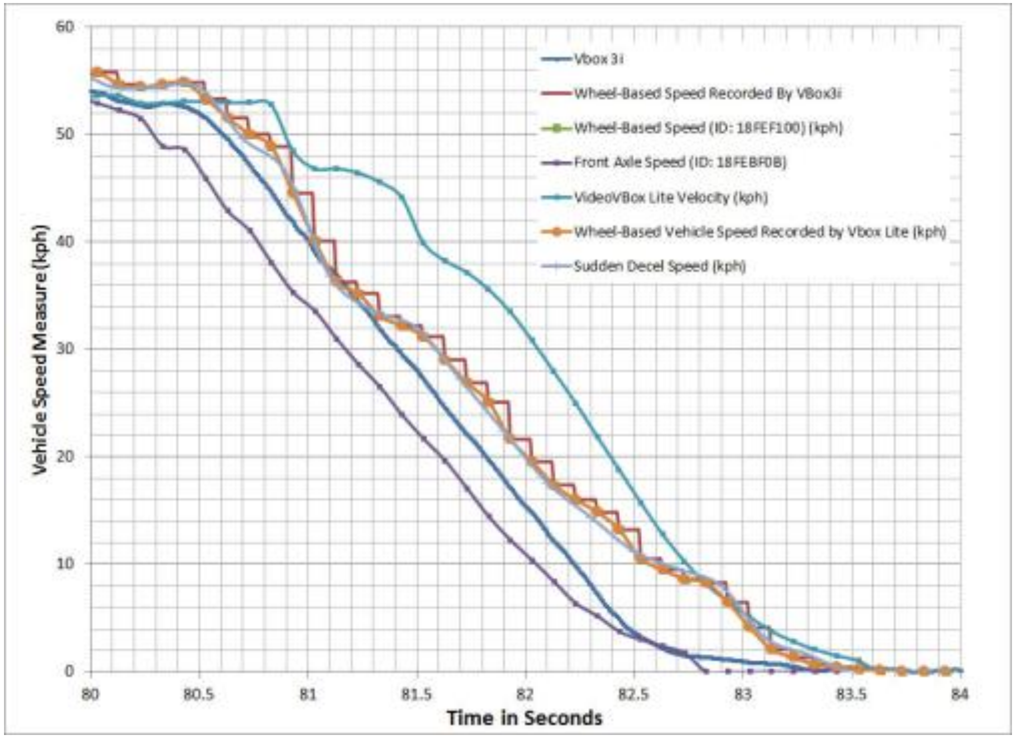
Assumption 3: Constant Deceleration

- The Skid to Rest formula assumes a constant speed decay from an initial speed to a final speed or to stop.



Assumption 3: Constant Deceleration (cont).

- In reality measured deceleration may not be constant especially during start of braking and the prior to stop.



(see http://tucrrc.utulsa.edu/J1939.html)

Assumption 3: How to Handle

- In most cases, the 0.25 to 0.5 second start and end of braking do not make a significant difference to the overall skid to rest analysis results.
- If complex braking is revealed in EDR data, a more detailed and iterative deceleration, time, distance, and velocity can be performed. The method prescribed would be to use a spreadsheet to integrate under the area of the curves for tabular data of speed.

Assumption 4: Uninterrupted Path

- The Skid to Rest formula assumes a straight line path from start of skid to point of rest or complete stop.
- Sometimes path may not be entirely on the road. Vehicles may skid off of the pavement, over curbs, and onto grass or dirt. A path involving differing drag factors can be managed using Combined Speed formula to determine the vehicle speed at the start of braking:

$$S = \sqrt{(S_1)^2 + (S_2)^2 + \dots (S_n)^2}$$

Where, n = number of segments with different drag factors.

Note: a common mistake made in accident reconstruction is to add different segments of skid to rest which results in speed calculations that are too high.

Note 2: the Combined Speed formula is derived from the Conservation of Kinetic Energy or KE = $\frac{1}{2} mSi^2 = \frac{1}{2} mS_1^2 + \frac{1}{2} mS_2^2 + \dots \frac{1}{2} mS_n^2$

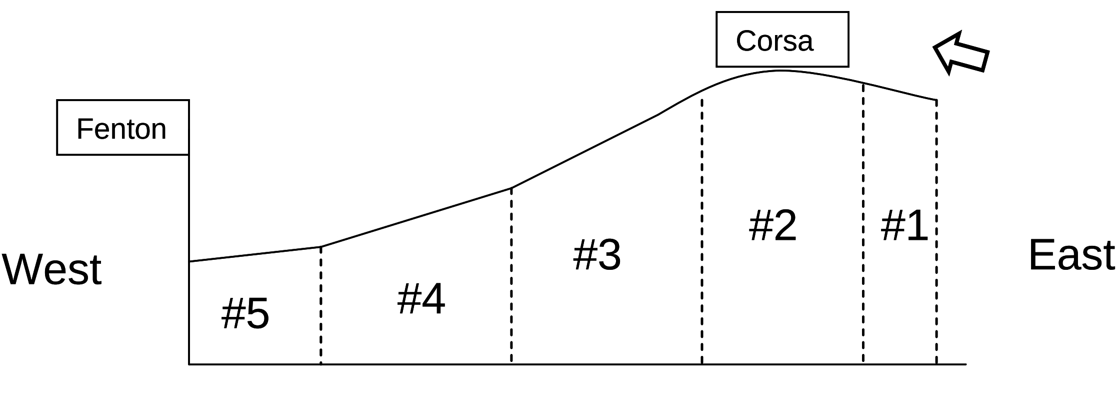
Example: Changing Path

The Police Report provided the following information:

- Accident occurred on XX/XX/2006, Saturday, at 5:00 am.
- Vehicle 1, 98 International Truck Model 4900 with a Thermo King refrigeration unit, operated by D. JONES was double parked.
- Vehicle 2, 93 Nissan 4 door sedan, operated by K. SMITH was “traveling westbound on BXXXXX Road, BXXXX, NY, traveling at a high rate of speed slammed into the back of his (sic, Mr. JONES’s) vehicle causing damage to the rear.”

The Site Inspection revealed that:

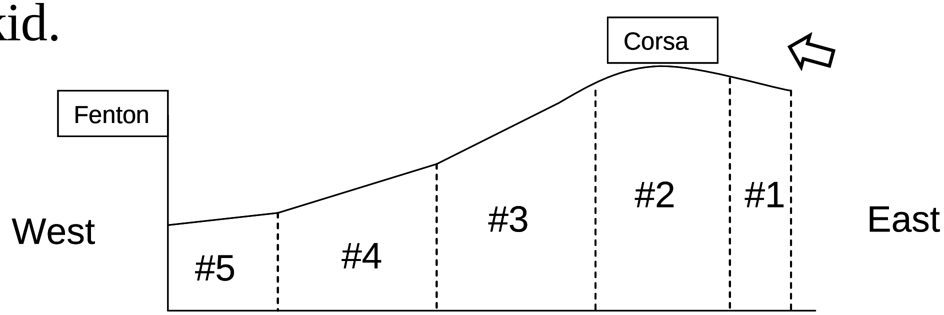
- The start of the skid marks was on a slight upgrade going through an intersection at the top of the hill, and then down a decline to the point of impact.



Not Drawn to Scale

Example: Changing Path (cont).

- We can analyze each segment of the path with the Skid to Rest formula making adjustments to the drag factor, assumed for this case to be 0.75.
- Then, with the combined speed formula, we can determine the initial speed before skid.



Date:	10/20/2008	5280 ft/mile
Engineer:	P. Chen	3600 sec/hour
Data Source:	10/17/2008 Inspection	

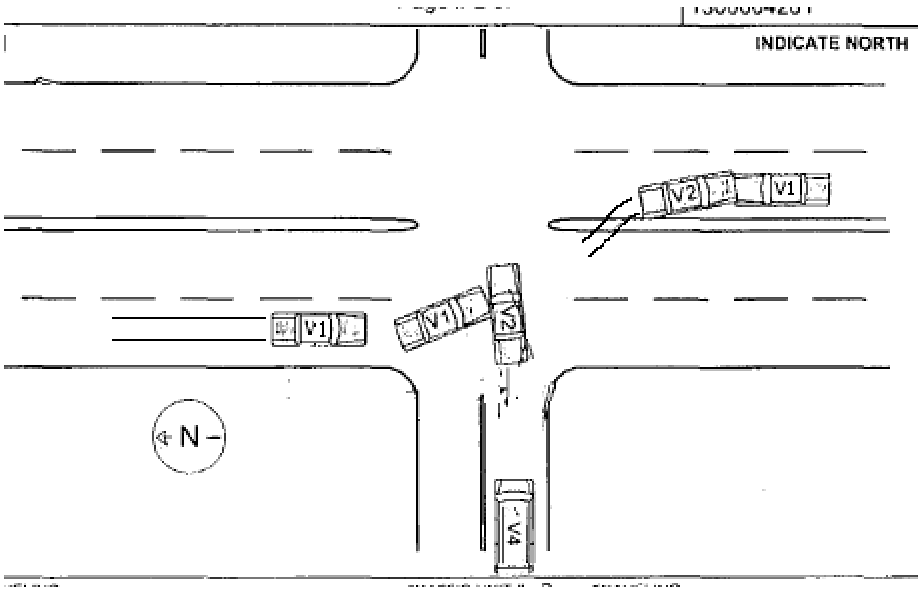
g, ft/s^2	drag coefficient	
32.17	0.75	Baseline

Segment	drag coefficient	Incline Ang (deg)	Ang (rad)	Segment Distance (ft)	Speed (mph)
#1	0.78	1.5	0.02618	22.5	22.89
#2	0.75	0	0	40	30.00
#3	0.69	-3.4	-0.059341	52.75	33.06
#4	0.71	-2.4	-0.041888	50	32.59
#5	0.72	-1.7	-0.029671	40.83	29.70

Combined Speed 67 mph

Assumption 4: Uninterrupted Path (cont)

- The Skid to Rest formula assumes the path isn't interrupted by collisions with other vehicles, trees, or other stationary objects.
- With collisions involving vehicles of similar weight, Conservation of Momentum principals are typically used for classic motor vehicle collision analysis in 2 dimensions.



Assumption 5: Straight Line Path

- The Skid to Rest formula assumes a straight line path.
- Some angle to the skid marks or slight steering won't invalidate the Skid to Rest formula. Also a sideways skid won't invalidate the Skid to Rest formula.
- Spin outs with complete or multiple rotations or roller overs would invalidate the Skid to Rest formula with regards to measuring and using the entire length of skid and using the entire length to determine speed.
- Skid to Rest may be used for spin outs or roll overs with regards to analyzing the path of the CG of the vehicle with a generalized drag factor, and/or with additional energy for rotation.

Extrapolation of Skid to Rest

Close Inspection of Skid to Rest reveals that:

- Equation is unaffected by mass of the vehicles.
  - So why are there published data for cars, motorcycles, and heavy trucks showing different drag factors?
  - Empirical testing is capturing the effects of different human perception reaction time, the braking speed of different systems (hydraulic for cars/motorcycles, pneumatic for heavy trucks), and differing braking efficiencies.
- Equation is not affected by Temperature.
  - Caveat: unless Temperature affects the physical properties or characteristics of the road, tires, or brakes.
  - Example: trucker losing braking power when brakes heat up when going down a steep mountain.

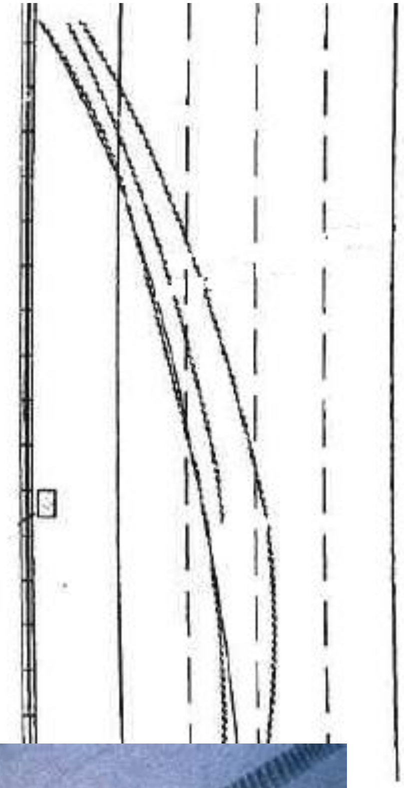
Assumption 6: Skid from Braking

The Skid to Rest formula assumes skid marks are the result of locked wheel brake skidding.

- Skid to Rest can not be applied to tire marks made as the result of critical yaw.
- Skid to Rest can not be applied to tire marks made as the result of acceleration.
- Skid to Rest can not be applied to tire marks made as a result of side swiping or other path discontinuity.

Critical Speed Yaw Marks

- Critical Speed Yaw Marks are the result of a vehicle attempting to turn at a speed beyond the capability of the tires and wheels to prevent side slipping.
- Critical Speed Yaw Marks may appear as thin tire marks indicating the tires are riding on their edges
- Critical Speed Yaw Marks may also appear as side slip tire tread marks.

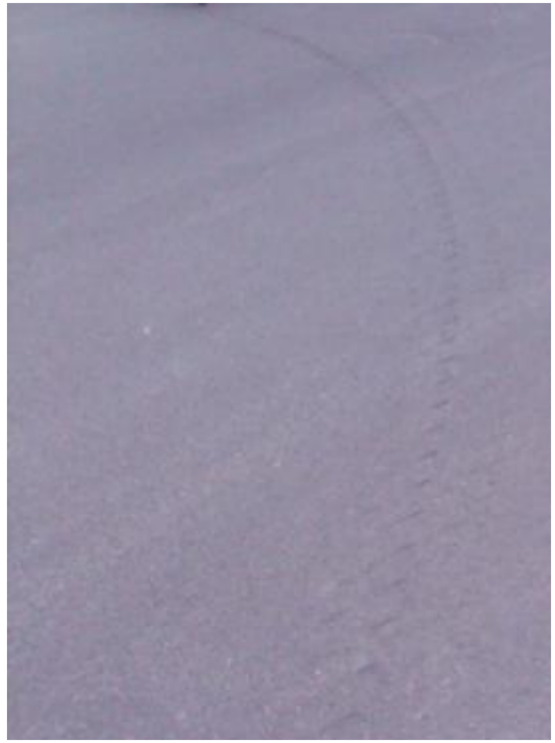


- Critical Speed Yaw Marks are analyzed with the formula:

$$S = 3.87\sqrt{Rf}$$

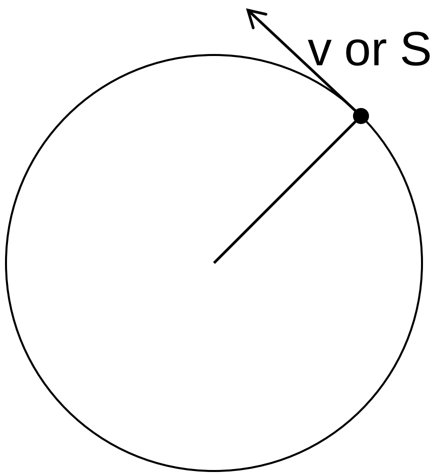
Where R = Radius of marks determined measuring a chord, c, and middle ordinate, m.

$$R = \frac{c^2}{8m} + \frac{m}{2}$$



Derivation of Critical Speed Yaw Formula

- Critical Speed Yaw formula is the result of Newton's Second Law:



Sum of forces in the y-direction:

$F_y = N - mg = ma_y = 0$

Or

$N = mg$

Sum of forces in the x-direction:

$F_x = F_f = f\,N = ma_x = mv^2/R$

Where for an object traveling at a constant speed about an arc,

$a_x = v^2/R$

Therefore:

$fmg = ma_x$

$fg = v^2/R$

$$v \text{ or } \quad S = \sqrt{Rfg} = \sqrt{\frac{fR(\,ft)}{(5280\,ft\,/\,mi)^2} 32.2\,ft\,/\sec^2 (3600\sec/hr)^2}$$

$$S = 3.87\sqrt{Rf}$$

Skills Check 3

- At this point in time, please take a break and attempt to derive the Critical Yaw Speed formula.
- Your sign convention, or your variable names or letters may be different. That is okay.

Acceleration Tire Marks

Acceleration tire marks are different that brake skidding tire marks. In general, acceleration tire marks are:

- Heaviest and darkest at the start of launch tapering off to the end.
- Could have stationary dark spin mark at the beginning with worn off pieces of tread.
- May have side slipping at the beginning of the mark.
- Path can zig-zag (fishtail) side to side in too short a distance to be accounted for during a normal skid to rest.

- The following drag racing videos may help you visualize the path and nature of the wheels during launch:
- http://youtu.be/mfJejgODr3E
- http://youtu.be/zv9Lw3Ll1to
- http://youtu.be/_FzhDBsbySs