PDHonline Course H140 (2 PDH)

Open Channel Hydraulics III – Sharp-crested Weirs

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Open Channel Hydraulics III - Sharp-crested Weirs

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COURSE CONTENT

1. Introduction

This course is intended to be taken after the course, H138, "Open Channel Hydraulics I – Uniform Flow." It will be assumed that anyone taking this course is familiar with the major classifications used for open channel flow (steady or unsteady state, laminar or turbulent flow, uniform or non-uniform flow, and critical, subcritical or supercritical flow) and with equations and calculations for uniform open channel flow.

A weir, widely used for measurement of open channel flow rate, consists of an obstruction in the path of flow. Water rises above the obstruction to flow over it, and the height of water above the obstruction can be correlated with the flow rate. The top of the weir, over which the liquid flows, is called the **crest** of the weir. Two commonly used types of weir are the sharp-crested weir and broad-crested weir. The sharp-crested weir will be covered in this course. The emphasis will be on calculations used for flow rate over the various types of sharp-crested weirs, but there will also be information on guidelines for installation and use of sharp-crested weirs.

2. General Information on Sharp-Crested Weirs

A sharp-crested weir is a vertical flat plate with a sharp edge at the top, over which the liquid must flow in order to drop into the pool below the weir. Figure 1, on the next page, shows a longitudinal section of flow over a sharp-crested weir.

Some of the terminology used in connection with sharp-crested weirs will be presented and discussed briefly here. As shown in Figure 1, **nappe** is a term used to refer to the sheet of water flowing over the weir. **Drawdown**, as shown in

Figure 1, occurs upstream of the weir plate due to the acceleration of the water as it approaches the weir. **Free flow** is said to occur with a sharp-crested weir, when

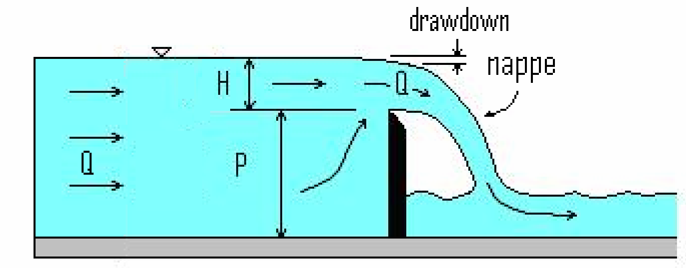


Figure 1. Longitudinal Section, Flow Over Sharp-crested Weir

there is free access of air under the **nappe.** The **Velocity of approach** is equal to the discharge, Q, divided by the cross-sectional area of flow at the head measuring station, which should be upstream far enough that it is not affected by the **drawdown**. The condition which occurs when downstream water rises above the weir crest elevation is called **submerged flow** or a **submerged weir**. The equations to be discussed for sharp-crested weirs all require **free flow** conditions. Accurate measurement of flow rate is not possible under **submerged flow** conditions.

The three common sharp-crested weir shapes shown in Figure 2 will be considered in some detail in this course. These shapes are **suppressed rectangular, V-notch**, and **contracted rectangular**.

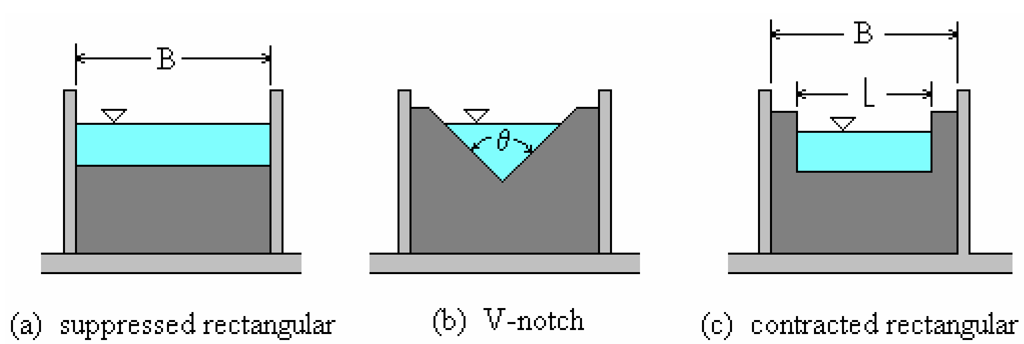


Figure 2. Common Sharp-crested Weir Shapes

The reference for the equations, graphs, etc to be used for calculating flow rate over a weir from measured values such as the head over the weir and various weir parameters, will be the 2001 revision, of the 1997 third edition, of the *Water Measurement Manual*, produced by the U.S. Dept. of the Interior, Bureau of Reclamation. *Water Measurement Manual* is available for on-line use or free download at: http://www.usbr.gov/pmts/hydraulics_lab/pubs/wmm/index.htm . This reference has very extensive coverage of water flow rate measurement. It is primarily oriented toward open channel flow, but has some discussion of pipe flow (closed conduit pressurized flow) also.

3. Suppressed Rectangular Weirs

The **suppressed rectangular,** sharp-crested weir, shown in Figure 2 (a), has the weir length equal to the width of the channel. The term suppressed is used to mean that the end contractions, as shown in Figure 2 (c), are suppressed (not present).

The Kindsvater-Carter method (from ref #3 for this course) is recommended in *Water Measurement Manual*, as a method with flexibility and a wide range of applicability. This method, for a sharp-crested rectangular weir, will be presented and discussed here. A more simplified equation, which works well if particular requirements are met, will then be presented and discussed as another alternative.

The Kindsvater-Carter equation for a rectangular, sharp-crested weir (suppressed or contracted) is:

$$Q = C_e L_e H_e^{3/2} \tag{1}$$

where:

Q = discharge, ft^3/s

e = a subscript denoting "effective"

C_e = effective coefficient of discharge, $ft^{1/2}/s$

$L_e = L + k_b$

$$H_e = H + k_h$$

In these relationships:

k_b = a correction factor to obtain effective weir length

L = measured length of weir crest

B = average width of approach channel, ft

H = head measured above the weir crest, ft

k_h = a correction factor with a value of 0.003 ft

The factor k_h has a constant value equal to 0.003 ft. The factor k_b varies with the ratio of crest length to average width of approach channel (L/B). Values of k_b for ratios of L/B from 0 to 1 are available from Figure 3 (as given in *Water Measurement Manual*).

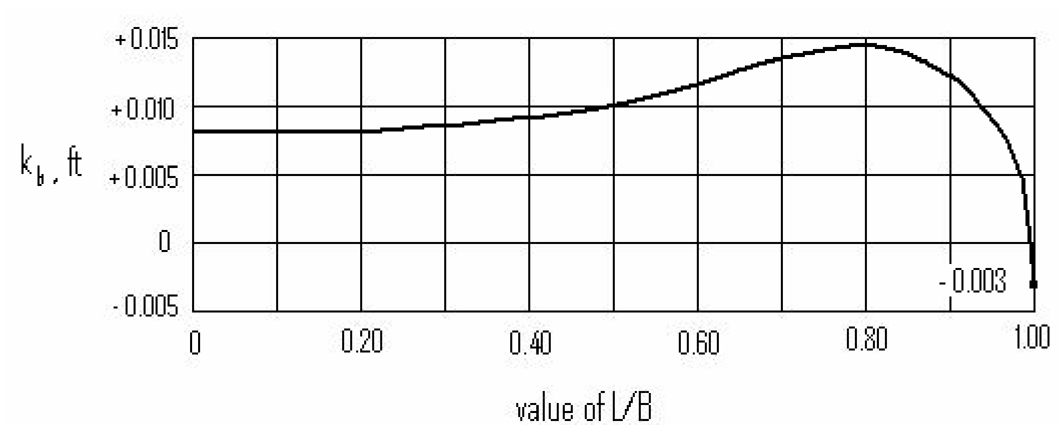


Figure 3. k_b as a function of L/B (as given in *Water Measurement Manual*)

The effective coefficient of discharge, C_e, includes effects of relative depth and relative width of the approach channel. Thus, C_e is a function of H/P and L/B. Values of C_e may be obtained from the family of curves given in Figure 4 (as given in *Water Measurement Manual*). Also, values of C_e may be calculated from the equations given after Figure 4.

As shown in Figure 1, P is the vertical distance to the weir crest from the approach pool invert. (NOTE: The term "invert" means the inside, upper surface of the channel.)

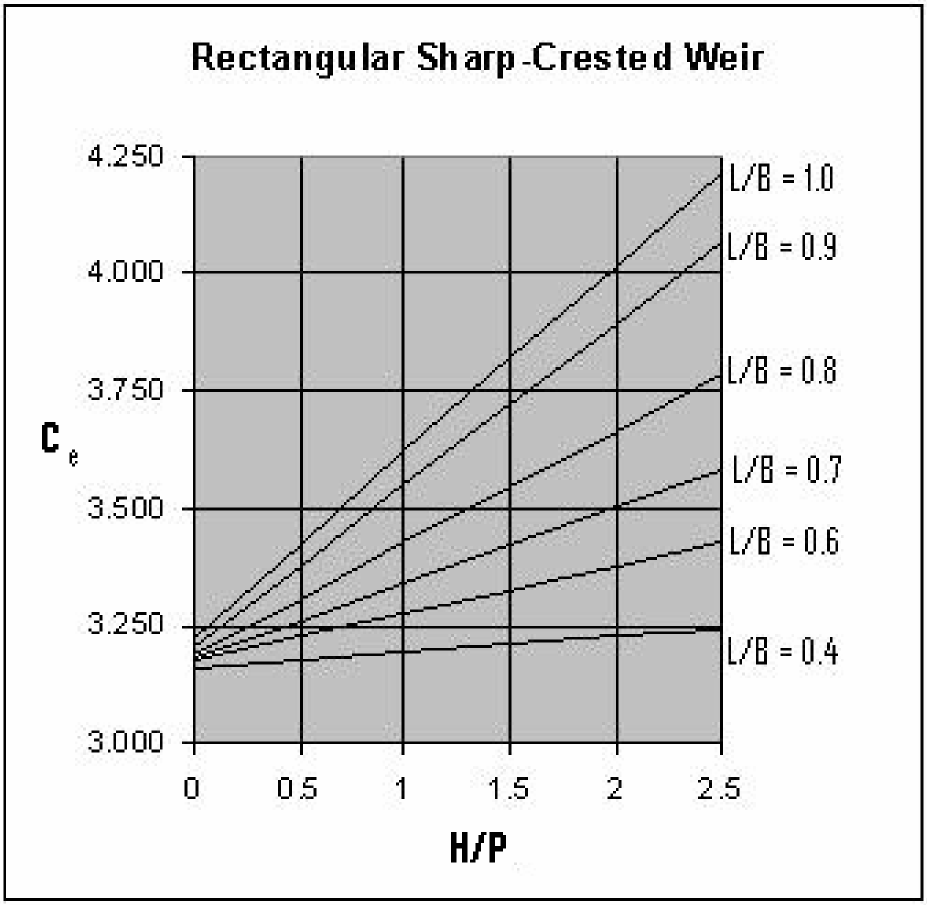


Figure 4. C_e as a function of H/P & L/B for Rect. Sharp-crested Weir

The straight lines in figure 4 have the equation form:

$$C_e = C_1\left(\frac{H}{P}\right) + C_2 \qquad (2)$$

Where the equation constants, C_1 and C_2, in equation (2) are functions of L/B as shown in Table 1, (from *Water Measurement Manual*) given below.

Table 1. Values of C_1 & C_2 for equation (7) (from *Water Measurement Manual*)

L/B	C_1	C_2
0.2	-0.0087	3.152
0.4	0.0317	3.164
0.5	0.0612	3.173
0.6	0.0995	3.178
0.7	0.1602	3.182
0.8	0.2376	3.189
0.9	0.3447	3.205
1.0	0.4000	3.220

Although the Kindsvater-Carter equation for a rectangular, sharp-crested weir, seems rather complicated as presented on the last three pages, it simplifies a great deal for a **suppressed rectangular** weir (L = B or L/B = 1.0). Equation (2) becomes:

$$C_e = 0.4000\left(\frac{H}{P}\right) + 3.220$$

From Figure 3 (L/B= 1): $k_b = -0.003$, so: $L_e = L + k_b = L - 0.003$

As given above: $k_h = 0.003$, so: $H_e = H + k_h = H + 0.003$

Substituting for C_e, L_e, & H_e into equation (1), gives the following Kindsvater-Carter equation for a **suppressed rectangular**, sharp-crested weir:

$$\mathbf{Q = (0.4000\left(\frac{H}{P}\right) + 3.220)(L - 0.003)(H + 0.003)^{3/2}} \qquad (3)$$

(Keep in mind that equation (3) is a dimensional equation with Q in cfs and H, P, & L in ft.)

The Bureau of Reclamation, in their *Water Measurement Manual*, gives equation (4) below, as an equation suitable for use with **suppressed rectangular**, sharp-crested weirs if the accompanying conditions are met:

(U.S. units: Q in cfs, B & H in ft): $$\mathbf{Q = 3.33\, B\, H^{3/2}} \qquad (4)$$

To be used only if:

$$\frac{H}{P} \leq 0.33 \quad \& \quad \frac{H}{B} \leq 0.33$$

Converting equation (4) to S.I. units gives the following:

(S.I. units: Q in m^3/s, B & H in m): $$\mathbf{Q = 1.84\, B\, H^{3/2}} \qquad (5)$$

To be used only if:

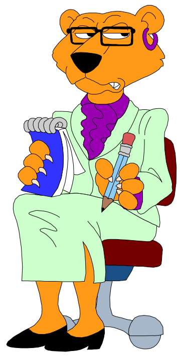
$$\frac{H}{P} \leq 0.33 \quad \& \quad \frac{H}{B} \leq 0.33$$

Example #1: There is a suppressed rectangular weir in a 2 ft wide rectangular channel. The weir crest is 1 ft above the channel invert. Calculate the flow rate for H = 0.2, 0.3, 0.4, 0.8, 1.0 ft, & 1.25 ft, using equations (3) & (4).

Solution: From the problem statement, L = B = 2 ft, and P = 1 ft, so it is necessary simply to substitute values into the two equations and calculate Q for each value of H. The calculations were made using an Excel spreadsheet. The results are shown in the table below, where Q_3 is calculated from equation (3) and Q_4 is calculated from equation (4).

H, ft:	0.2	0.3	0.4	0.8	1.0	1.25 (= H/P)
Q_3, cfs:	**0.603**	**1.112**	**1.727**	**5.087**	**7.262**	**10.42**
Q_4, cfs:	**0.596**	**1.094**	**1.685**	**4.766**	**6.660**	**9.308**
H/B:	0.1	0.15	0.2	0.4	0.5	0.625

Discussion of results: Since P = 1 ft in this example, the values for H in the table of results are equal to H/P. The values for H/B are shown in the table also.



Wow, is this all

really discussion

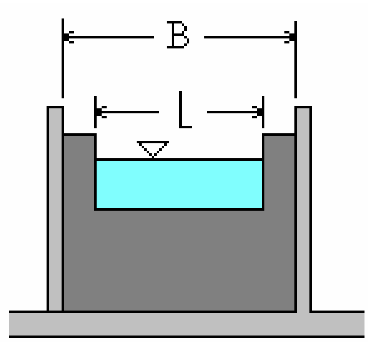
of results ?

Note that the results for equation (3), the Kindsvater-Carter equation, and equation (4) the simpler equation agree quite closely for the three smaller values of H. They are the same to two significant figures for those three H values. The two conditions for the use of equation (4) are met only for the two smallest H values. For H = 0.4 ft, H/P is just above the allowable maximum of 0.33. For the three largest values of H, both H/P and H/B are larger than the allowable values for use of equation (4), and the equation (4) results are quite a bit different from those for equation (5).

Conclusions: For a **suppressed rectangular**, sharp-crested weir, equation (4), $\mathbf{Q = 3.33BH^{3/2}}$, may be used if $H/P \leq 0.33$ & $H/B \leq 0.33$. For H/P > 0.33 or H/B > 0.33, the Kindsvater-Carter equation [equation (4)] should be used.

4. Contracted Rectangular Weirs

The **contracted rectangular** sharp-crested weir, shown in Figure 2c and in the figure below, has weir length, L, less than the width of the channel, B. This is sometimes called an unsuppressed rectangular weir.



contracted rectangular weir

For a **contracted rectangular,** sharp-crested weir (L/B < 1), the Kindsvater-Carter equation becomes:

$$\mathbf{Q = (C_1\left(\frac{H}{P}\right) + C_2)(L - 0.003)(H + k_b)^{3/2}} \qquad (6)$$

Where: C_1 and C_2 come from Table 1 and k_b comes from Figure 3, for a known value of L/B. (Table 1 and Figure 3 are on pages 4 & 5.)

The Bureau of Reclamation, in its *Water Measurement Manual*, gives equation (7) below as a commonly used, simpler Equation for a **fully contracted rectangular weir**, subject to the accompanying conditions:

(U.S. units: Q in cfs, L & H in ft): $\mathbf{Q = 3.33(L - 0.2\,H)(H^{3/2})}$ (7)

The equivalent equation for S.I. units is:

(S.I. units: Q in m^3/s, L & H in m): $\mathbf{Q = 1.84(L - 0.2\,H)(H^{3/2})}$ (8)

For both equation (7) and equation (8), the following conditions must be met:

i) Weir is fully contracted, i.e.: $\mathbf{B - L \geq 4\,H_{max}}$ and $\mathbf{P \geq 2\,H_{max}}$

ii) $\mathbf{H/L \leq 0.33}$

Example #2: Consider a contracted rectangular weir in a rectangular channel with B, L, H, & P having each of the following sets of values.

a) Determine whether the conditions for use of equation (7) are met for each set of values.

b) Calculate the flow rate, Q, using equations (6) & (7) for each set of values.

i) $B = 4$ ft, $L = 2$ ft, $H = 0.5$ ft, $P = 1$ ft

ii) $B = 10$ ft, $L = 6$ ft, $H = 0.8$ ft, $P = 2$ ft

iii) $B = 10$ ft, $L = 4$ ft, $H = 1$ ft, $P = 2.4$ ft

iv) $B = 10$ ft, $L = 4$ ft, $H = 2$ ft, $P = 2$ ft

v) $B = 10$ ft, $L = 8$ ft, $H = 2$ ft, $P = 1.5$ ft

Solution: a) The required conditions for use of equation (7) are:

1) $B - L \geq 4 H_{max}$, 2) $P \geq 2 H_{max}$ & 3) $H/L \leq 0.33$

i) $B - L = 2$ ft & $4H = 2$ ft, so **condition 1 is barely met**

$P = 2$ ft & $2H = 2$ ft, so **condition 2 is barely met**

$H/L = 0.5/2 = 0.25 < 0.33$, so **condition 3 is met**

ii) $B - L = 4$ ft & $4H = 3.2$ ft, so **condition 1 is met**

$P = 2$ ft & $2H = 1.6$ ft, so **condition 2 is met**

$H/L = 0.8/6 = 0.13 < 0.33$, so **condition 3 is met**

iii) $B - L = 6$ ft & $4H = 4$ ft, so **condition 1 is met**

$P = 2.4$ ft & $2H = 2$ ft, so **condition 2 is met**

$H/L = 1/4 = 0.25 < 0.33$, so **condition 3 is met**

iv) $B - L = 6$ ft & $4H = 8$ ft, so **condition 1 is not met**

$P = 2$ ft & $2H = 4$ ft, so **condition 2 is not met**

$H/L = 2/4 = 0.5 > 0.33$, so **condition 3 is not met**

v) $B - L = 2$ ft & $4H = 8$ ft, so **condition 1 is not met**

$P = 1.5$ ft & $2H = 4$ ft, so **condition 2 is not met**

$H/L = 2/8 = 0.25 < 0.33$, so **condition 3 is met**

Hey, This is a pretty long solution isn't it ?

b) The calculations for part b) were done with an Excel spreadsheet. The results are summarized in the table below. The flow rates from equation (7) was calculated using the given values for L, P & H. In order to use equation (6), values were needed for C_1, C_2, & k_b. All three are functions of L/B. Values for C_1, & C_2 were obtained from Table 1, and values for k_b were obtained from Figure 3. Q_6 & Q_7, are the flow rates from equations (6) & (7) respectively.

	i)	ii)	iii)	iv)	v)
eqn (7) ok?	yes	yes	yes	no	no
L/B	0.5	0.6	0.4	0.4	0.8
C_1	0.612	0.0995	0.0317	0.0317	0.2376
C_2	3.173	3.178	3.164	3.164	3.189
k_b	0.010	0.012	0.009	0.009	0.014
Q_6, cfs	**2.330**	**14.12**	**12.87**	**36.37**	**80.13**
Q_7, cfs	**2.237**	**13.92**	**12.65**	**33.91**	**71.58**

Discussion of Results: Both equations give similar results for cases i), ii), & iii), where the criteria for use of the simpler Equation (7) were met ($B - L \geq 4 H_{max}$, $P \geq 2 H_{max}$, & $H/L \leq 0.33$). The results for the two equations differ considerably for cases iv) and v), where the criteria were not met.

Conclusions: For a **contracted rectangular**, sharp-crested weir, the simpler equation [$\mathbf{Q = 3.33(L - 0.2\,H)(H^{3/2})}$], appears to be adequate when the three criteria mentioned above are met. This equation is also acceptable to the U.S. Bureau of Land Reclamation for use when those criteria are met. The U. S. Bureau of Land Reclamation recommends the use of the slightly more complicated Kindsvater-Carter equation [equation (6)] for use when any of the three criteria given above for use of the simpler Equation (7) are not met.

5. V-Notch Weirs

The **V-notch,** sharp-crested weir (also called a triangular weir), shown in Figure 2 (b) and in Figure 5, below, measures low flow rates better than the rectangular weir, because the flow area decreases as H decreases and reasonable heads are developed even at small flowrates.

The Bureau of Reclamation, in their *Water Measurement Manual*, gives equation (9) (see below) as an equation suitable for use with a fully contracted, **90^o V-notch**, sharp-crested weir if it meets the indicated conditions.

(U.S. units: Q in cfs, H in ft) $$\mathbf{Q = 2.49\,H^{2.48}} \qquad (9)$$

Subject to: $\mathbf{P \geq 2\,H_{max}}$, $\mathbf{S \geq 2\,H_{max}}$, $\mathbf{0.2\text{ ft} \leq H \leq 1.25\text{ ft}}$

Where: H_{max} = the maximum head expected over the weir

P = the height of the V-notch vertex above the channel invert

S = the distance from the channel wall to the to the V-notch edge at the top of the overflow

See Figure 5, below, for additional clarification of the parameters S, P, & H.

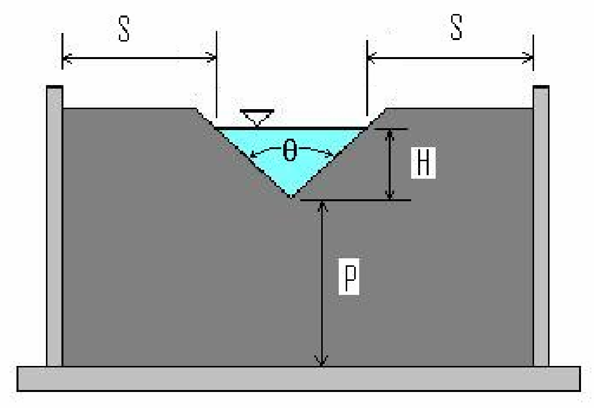


Figure 5. Fully Contracted V-notch Weir

A 90° V-notch weir is better than a rectangular weir for measuring relatively low flow rates and a rectangular weir is better for measuring higher flow rates. This will be illustrated with the next couple of examples.

Example #3: Calculate the minimum and maximum flow rates covered by the recommended range of 0.2 ft to 1.25 ft for the head over a fully contracted 90° V-notch weir. (Note: in order to be fully contracted, $P = 2\ H_{max} = 2.5$ ft.)

Solution: Substituting values of H into equation (9) gives:

$$Q_{min} = (2.49)(0.2^{2.48}) = \underline{\mathbf{0.046\ cfs}} = \mathbf{Q_{min}}$$

$$Q_{max} = (2.49)(1.25^{2.48}) = \underline{\mathbf{4.33\ cfs}} = \mathbf{Q_{max}}$$

Example #4: Calculate the minimum and maximum flow rates covered by the range of 0.2 ft to 1.25 ft for the head over a suppressed rectangular weir in a 2 ft wide rectangular channel. Assume that P = 2.5 ft, as with the V-notch example, so that the suppressed rectangular weir is fully contracted from the channel bottom.

Solution: Equation (4) can be used for H = 0.2 ft, because H/P < 0.33 and H/B < 0.33, however for H = 1.25 ft, H/B & H/P are both greater than 0.33, so equation (3) must be used. The calculations are shown below:

$$Q_{min} = (3.33)(2)(0.2^{3/2}) = \underline{\mathbf{0.596\ cfs}} = \mathbf{Q_{min}}$$

$Q_{max} = [3.32 + 0.40(1.25/2.5)](2 - 0.003)[(2 + 0.003)^{3/2}] = \underline{\mathbf{10.42\ cfs}} = \mathbf{Q_{max}}$

Comments: As shown by **Example #3** and **Example #4**, the 2 ft rectangular weir can carry more than twice the flow rate that the V-notch weir can carry for the same head above the channel bottom, however the V-notch weir can measure a much smaller flow rate than is possible with the rectangular weir.

Notch angles other than 90° in a V-notch**,** sharp-crested weir require the use of the Kindsvater-Carter equation, as given in *Water Measurement Manual*:

$$Q = 4.28\, Ce\, \mathrm{Tan}\left(\frac{\theta}{2}\right) (H + k)^{5/2} \qquad (10)$$

Where: Q = discharge over weir, cfs

Ce = effective discharge coefficient

H = head on weir in ft

k = head correction factor

θ = angle of V-notch

Ce, the effective weir coefficient, and k, the head correction factor, are both functions of θ only if the V-notch weir is fully contracted ($\mathbf{P \geq 2\, H_{max}}$ **&** $\mathbf{S \geq 2\, H_{max}}$). See Figure 5 on page 13, for clarification of the parameters P and H. Ce can be obtained from figure 6* or equation (11)*, and k, can be obtained from figure 7* or equation (12)*.

*from LMNO Engineering, Research and Software, Ltd website, at: http://www.lmnoeng.com/Weirs

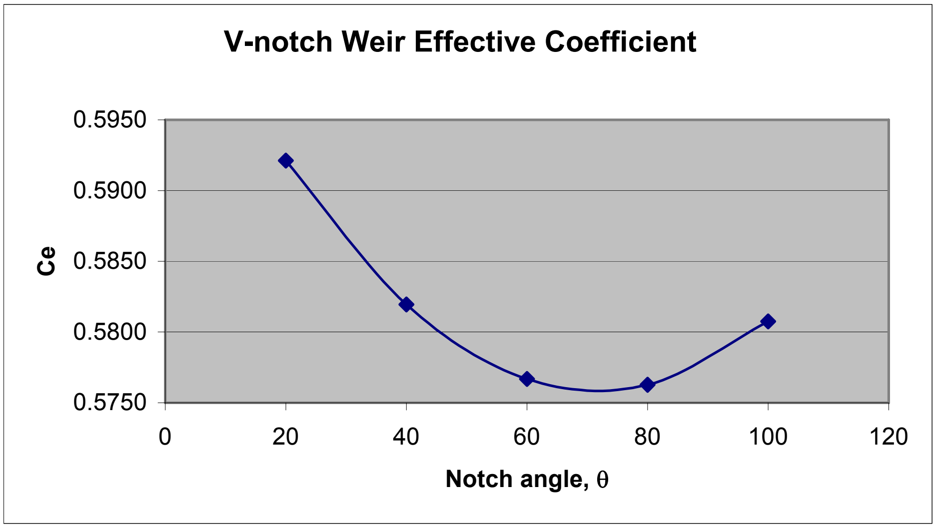


Figure 6. Effective V-notch Weir Coefficient, Ce

$$Ce = 0.607165052 - (0.000874466963)\theta + (6.10393334 \times 10^{-6})\theta^2 \qquad (11)$$

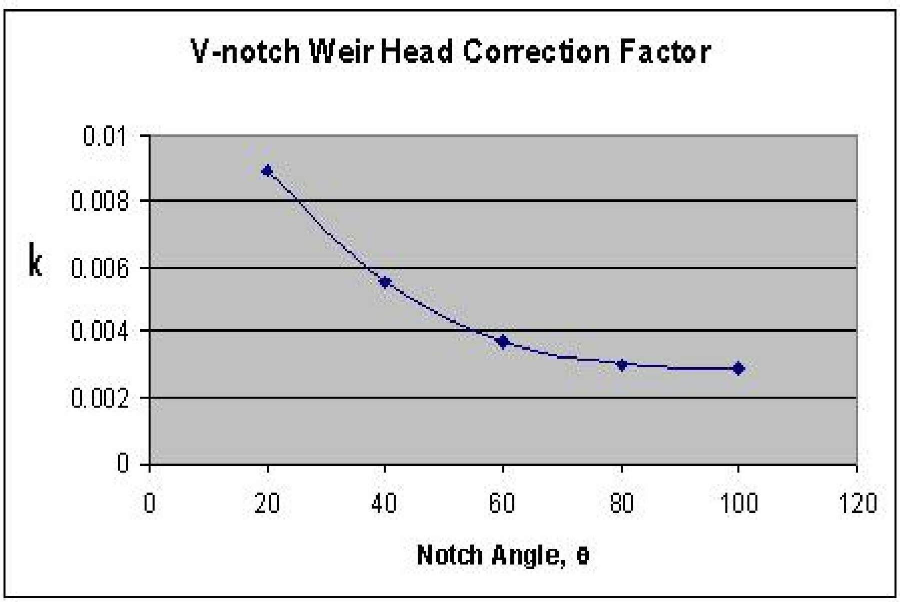


Figure 7. V-notch Weir, Head Correction Factor, k

$$k = 0.0144902648 - (0.00033955535)\theta$$

$$+ (3.29819003 \times 10^{-6})\theta^2 - (1.06215442 \times 10^{-8})\theta^3 \qquad (12)$$

Example #5: Estimate the flow rate through a fully contracted V-notch weir for a head of 0.2 feet and for a head of 1.25 feet.

a) with a notch angle of 60^o

b) with a notch angle of 40^o

Solution: Part a) From equation (11) & (12), with $\theta = 60^o$:

$$Ce = 0.5767 \quad \& \quad k = 0.0037$$

Substituting into equation (10), with H = 0.2 ft, gives: **($\theta = 60^o$, H = 0.2 ft)**

$$Q = (4.28)(0.5767)[\tan(30^o)]\,[(0.2 + 0.0037)^{5/2}] = \underline{\mathbf{0.0267\ cfs}}$$

Substituting into equation (10), with H = 1.25 ft, gives: **($\theta = 60^o$, H = 1.25 ft)**

$$Q = (4.28)(0.5767)[\tan(30^o)]\,[(1.25 + 0.0037)^{5/2}] = \underline{\mathbf{2.508\ cfs}}$$

Part b) From equation (11) & (12), with $\theta = 40^o$: Ce = 0.5820 & k = 0.0051

Substituting into equation (10), with H = 0.2 ft, gives: **($\theta = 40^o$, H = 0.2 ft)**

$$Q = (4.28)(0.5820)[\tan(20^o)]\,[(0.2 + 0.0051)^{5/2}] = \underline{\mathbf{0.0173\ cfs}}$$

Substituting into equation (10), with H = 1.25 ft, gives: **($\theta = 40^o$, H = 1.25 ft)**

$$Q = (4.28)(0.5820)[\tan(20^o)]\,[(1.25 + 0.0051)^{5/2}] = \underline{\mathbf{1.600\ cfs}}$$

Comment: This example is intended to simply illustrate the use of equations (10), (11), & (12) for V-notch weirs with notch angles other than 90^o.

6. Installation & Use Guidelines for Sharp-Crested Weirs

A summary of installation and measurement guidelines for sharp-crested weirs extracted from *Water Measurement Manual*, is given in this section.

a) The upstream face of the weir plates and bulkhead should be plumb, smooth, and normal to the axis of the channel.

b) The entire crest should be level for rectangular and trapezoidal shaped weir openings, and the bisector of V-notch angles should be plumb.

c) The top thickness of the crest and side plates should be between 0.03 and 0.08 inch.

d) The upstream edges of the weir opening plates must be straight and sharp. Edges of plates require machining or filing perpendicular to the upstream face to remove burrs or scratches and should not be smoothed off with abrasive cloth or paper. Avoid knife edges because they are a safety hazard and damage easily

e) The overflow sheet or nappe should touch only the upstream faces of the crest and side plates.

f) Maximum downstream water surface level should be at least 0.2 ft below crest elevation. However, when measuring close to the crest, frequent observations are necessary to verify that the nappe is continually ventilated without waves periodically filling the under nappe cavity.

g) The measurement of head on the weir is the difference in elevation between the crest and the water surface at a point located upstream from the weir a distance of at least four times the maximum head on the crest.

h) The approach to the weir crest must be kept free of sediment deposits.

i) If the weir crest length is greater than 50% of the approach channel width, then ten average approach flow widths of straight, unobstructed approach are required.

j) If the weir crest length is less than 50% of the approach channel width, then twenty average approach flow widths of straight, unobstructed approach are required.

k) If upstream flow is below critical depth, a jump should be forced to occur. In this case, thirty measuring heads of straight, unobstructed approach after the jump should be provided.

l) The minimum head over the weir should be 0.2 ft.

For Rectangular, Sharp-crested Weirs:

All of the general conditions, a) through l) above, apply. Also:

a) The crest length, L, should be at least 6 inches.

b) The crest height, P, should be at least 4 inches.

c) Values of H/P should be less than 2.4.

For V-notch Weirs:

All of the general conditions, a) through l) above, apply. Also:

a) For a fully contracted V-notch weir, the maximum measuring head should be less than 1.25 ft.

b) For a fully contracted V-notch weir, H/B should be less than 0.2.

c) The average width of the approach channel, B, should always be greater than 3 ft for a fully contracted V-notch.

d) The V-notch of the weir should always be at least 1.5 ft above the invert of the weir pool for a fully contracted V-notch.

7. Summary

Sharp-crested weirs are commonly used for flow rate measurement in open channels. Three types of sharp-crested weirs: suppressed rectangular, contracted rectangular, and V-notch, are covered in detail in this course. Emphasis is on calculation of flow rate over a weir for given head over the weir and weir/channel dimensions. For each of the three types of sharp-crested weir, a general equation with a wide range of applicability is presented and discussed along with equations and/or graphs as needed for use with the main equation. Also, for each of the three types of sharp-crested weir, a simpler equation is presented along with a set of conditions under which the simpler equation can be used. Several worked examples are included covering all three types of weirs. Practical installation and use guidelines for sharp-crested weirs are presented.