PDHonline Course H142 (4 PDH)

Hydrologic Probability and Statistics

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Overview

- Hydrologic Statistics
  - Probabilistic Treatment of Hydrologic Data
  - Frequency and Probability Functions
  - Statistical Parameters
  - Probability Distributions for Hydrologic Variables
- Probability/Frequency Analysis
  - Return Period
  - Extreme Value Distributions
  - Frequency Analysis Using Frequency Factors
  - Probability Plotting
  - Confidence Limits
- Application to Stream Flow
- USGS PEAK-FQ
- Application to Rainfall

References

- Reference: Chow, et.al., *Applied Hydrology*, McGraw-Hill Publishing, 1988
- Streamflow Data: http://waterdata.usgs.gov/nwis
- Rainfall Data: http://hdsc.nws.noaa.gov/hdsc/pfds/

Hydrologic Statistics

- Extreme hydrologic processes can be considered as **random** with little or no correlation to adjacent processes (i.e. time and space independent). Thus, the output from a hydrologic process can be treated as **stochastic** (i.e. non-deterministic process comprised of predictable and random actions)

- Probabilistic and statistical methods are used to analyze stochastic processes and involve varying degrees of uncertainty.

- The focus of probability and statistical methods is on the observations and not the physical process.

- We will focus on two aspects of hydrology where the stochastic approach can be applied: rainfall and streamflow.

Probabilistic Treatment of Hydrologic Data

- A random variable (X) can be described by a probability distribution, which specifies that the chance an observed value of “x” will fall within the range of X.

- For example, if X is annual precipitation at a specified location, then the probability distribution of X specifies the chance that the observed annual precipitation will lie within a defined range, such as less than 30”, 30” – 40”, etc.

Probabilistic Treatment of Hydrologic Data

- The probability of an event A = P(A). P(A) can be estimated using an observed set of data. If a sample of "n" observations has "n_A" values in the range of event A, then P(A) is estimated to be n_A/n. As "n" approaches ∞, P(A) becomes more accurate.

- **Example:** The following rainfall depths were observed in the month of May over the past 10 years at the Philadelphia rain gage. Based on the sample of data below, estimate the probability that May's total rainfall will not exceed 4" in any given year.

Month, Year	Observed Rainfall (in)
May, 1999	**3.2**
May, 2000	4.8
May, 2001	4.2
May, 2002	5.9
May, 2003	**2.0**
May, 2004	**3.1**
May, 2005	**3.8**
May, 2006	4.5
May, 2007	**2.1**
May, 2008	5.2

$$A = 4''$$

$$n = 10$$

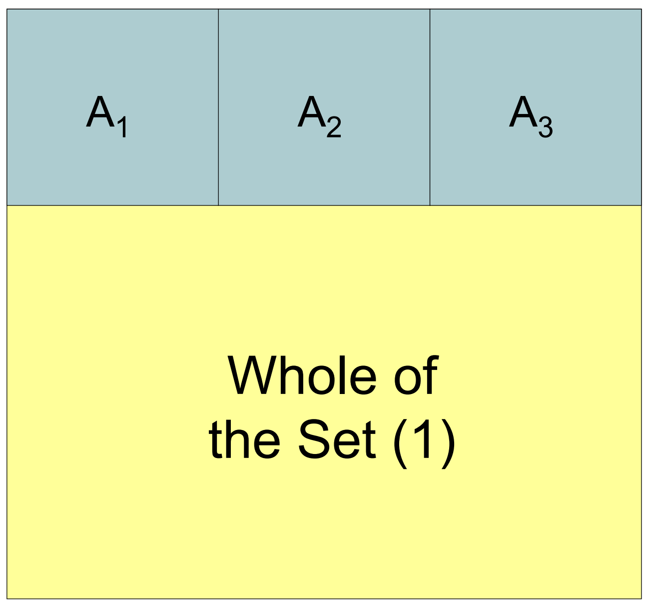
$$n_A = 5$$

$$P(A) = \frac{5}{10} = 0.5 => 50\%$$

Probabilistic Treatment of Hydrologic Data

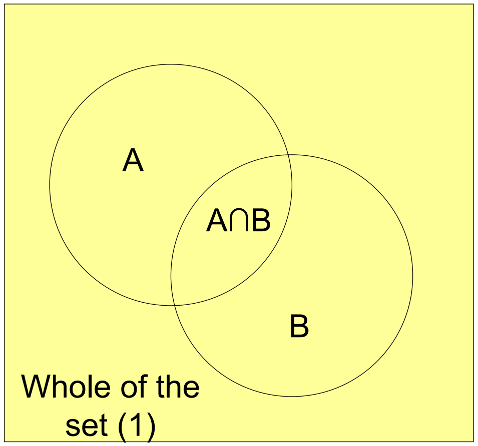
- The probability of a events obey certain principles:
  - **Total Probability**: If the sample space (1 represents the whole space) is completely divided into non-overlapping events (i.e. A_1 **or** A_2 **or** A_3, etc.), then,

$$P(A_1)+P(A_2)+P(A_3)+....+P(A_m)=1$$



Probabilistic Treatment of Hydrologic Data

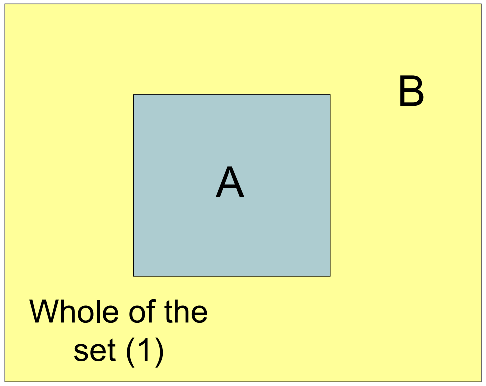
- The probability principles (con't):
  - **Conditional Probability**: Two events, A and B. The overlap is the event that both occur (A∩B or A **intersects** with B). P(B|A) is the *conditional probability* that event B will occur given that event A has already occurred. Therefore, the **joint probability** that **both** will occur (A **and** B) is:



$$P(A \& B) = P(A \cap B) = P(B | A)P(A)$$

  - If A and B are independent events, then,

$$P(A \cap B) = P(A)P(B)$$



  - **Complementary**: If B is the complement to A, then,

$$P(B) = 1 - P(A)$$

Probabilistic Treatment of Hydrologic Data

- **Example 1**: The values of annual precipitation in College Station, TX, from 1911 to 1979 are shown in Table 11.1.1 and plotted as a time series (below). What is the probability that the annual precipitation R in any given year will be less than 35 inches? Greater than 45 inches? Between 35 and 45 inches?

TABLE 11.1.1
Annual Precipitation in College Station, Texas, 1911–1979 (in)

Year	1910	1920	1930	1940	1950	1960	1970
0		48.7	44.8	49.3	31.2	46.0	33.9
1	39.9	44.1	34.0	44.2	27.0	44.3	31.7
2	31.0	42.8	45.6	41.7	37.0	37.8	31.5
3	42.3	48.4	37.3	30.8	46.8	29.6	59.6
4	42.1	34.2	43.7	53.6	26.9	35.1	50.5
5	41.1	32.4	41.8	34.5	25.4	49.7	38.6
6	28.7	46.4	41.1	50.3	23.0	36.6	43.4
7	16.8	38.9	31.2	43.8	56.5	32.5	28.7
8	34.1	37.3	35.2	21.6	43.4	61.7	32.0
9	56.4	50.6	35.1	47.1	41.3	47.4	51.8

Probabilistic Treatment of Hydrologic Data

- **Solution 1:** There are n=79-11+1=69 data. Let A be the event R<35.0 inches, B the event R>45.0 inches. The numbers of values in the previous table falling in these ranges are n_A=23 and n_B=19. Therefore,

$$P(A) = P(R < 35.0) = \frac{23}{69} = 0.33$$

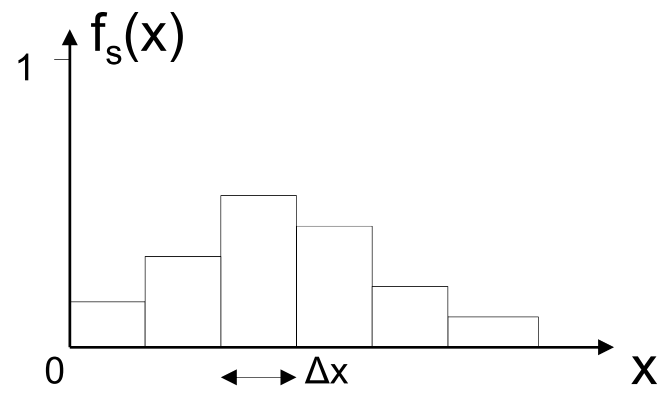
$$P(B) = P(R > 45.0) = \frac{19}{69} = 0.28$$

$$P(35.0 \le R \le 45.0) = 1 - P(A) - P(B) = 1 - 0.33 - 0.28 = 0.39$$

Frequency and Probability Functions

- **Probabilities** estimated **from sample data**, as in Example 1, are **approximate** because they depend on specific values of the observations in a sample of limited size. The sample data represents observations for a specific period of time and may not reflect long-term changes.

- If observations in a sample are identically distributed (i.e. each sample can be represented by the same probability distribution), the data can be arranged on a **frequency histogram** (below).
  - n = Total # of observations
  - n_i = # of observations in interval i
  - i = Interval #
  - Δx = Width of the interval
  - $(x_i - \Delta x, x_i)$ = Range of the interval



- For a **sample data** set,
  - Relative Frequency Function: $f_s(x_i) = \frac{n_i}{n}$
  - Cumulative Frequency Function: $F_s(x_i) = \sum_{j=1}^{i} f_s(x_j)$

Frequency and Probability Functions

- An alternative approach is to **fit a probability distribution function** to the data then determine the probabilities of events from this distribution function.

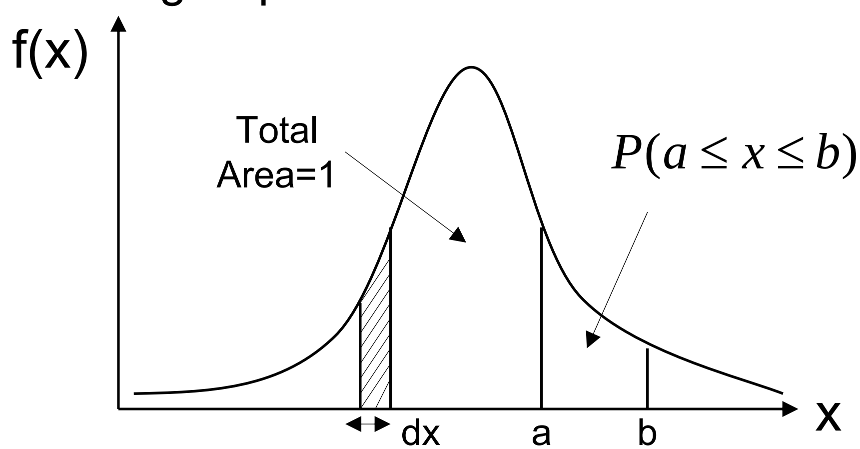
- The probability law that the continuous variable “x” follows is “f(x)” is typically represented by a function, called the **probability density function**. If “f(x)” is known, we can calculate the probability that “x” will take a value within an interval (a,b) by:

$$P(a \le x \le b) = \int_a^b f(x)dx$$

- For this definition to be valid, the following requirements must be met:

$$f(x) \ge 0$$

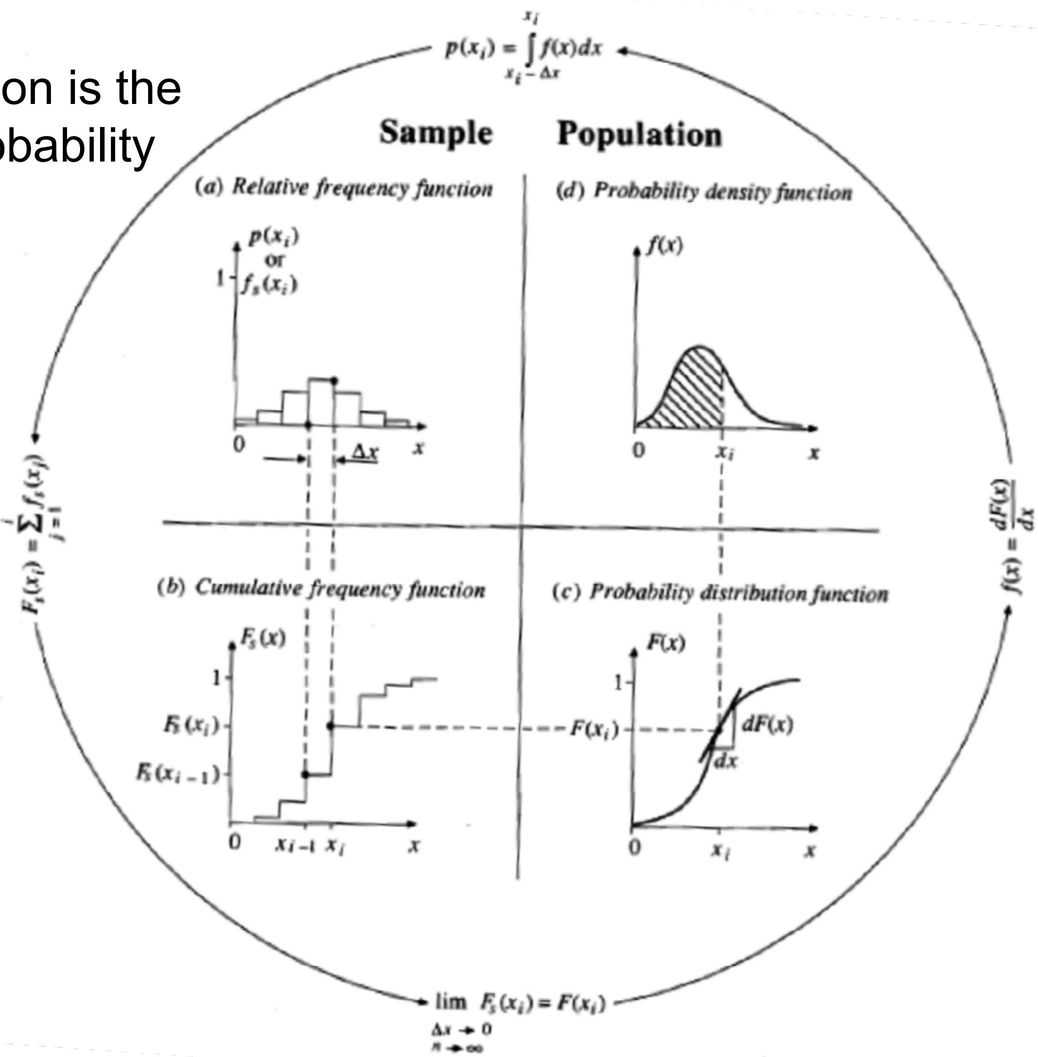
$$\int_{-\infty}^{\infty} f(x)dx = 1$$



Frequency and Probability Functions

- The Probability Density Function is the derivative (or slope) of the Probability Distribution Function.

$$f(x) = \frac{dF(x)}{dx}$$



Standard Normal Probability Distribution

- One of the most widely known probability density functions is the **Standard Normal Probability Distribution**:
  - Probability Density Function: $f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2}$ $z = \frac{x-\mu}{\sigma}$
  - Cumulative Probability Function: $F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$

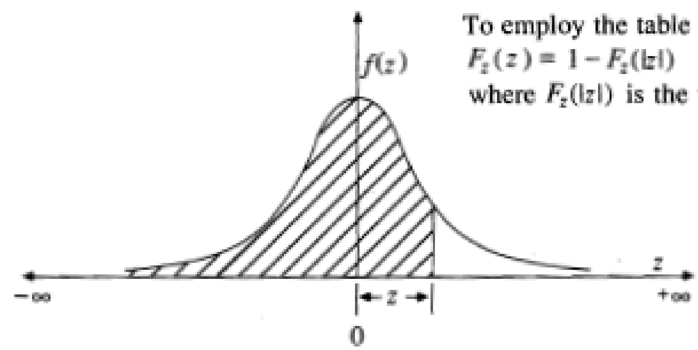
- The value of F(z) was approximated by a polynomial (Abramowitz and Stegun (1965)); results are in Table 11.2.1.

Standard Normal Probability Distribution

TABLE 11.2.1
Cumulative probability of the standard normal distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Source: Grant, E. L., and R. S. Leavenworth, *Statistical Quality and Control*, Table A, p.643, McGraw-Hill, New York, 1972. Used with permission.



To employ the table for $z < 0$, use $F_z(z) = 1 - F_z(|z|)$ where $F_z(|z|)$ is the tabulated value.

Probability Distribution for Hydrologic Variables

- Various probability distributions have been found to fit well with different hydrologic variables. The next few slides will summarize these findings.

- **Standard Normal Distribution**:
  - Hydrologic Application – Annual precipitation; sum of the effects of independent events
  - Limitations – Varies over a continuous range (-∞,∞) while most hydrologic variables are non-negative; Symmetrical about the mean while hydrologic data tend to be skewed.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$$-\infty \le x \le \infty$$

$$\mu = \bar{x}$$

$$\sigma = s_x$$

Probability Distribution for Hydrologic Variables

- **Lognormal Distribution**:
  - Hydrologic Application – Distribution of hydraulic conductivity in a porous medium, distribution of raindrop sizes in a storm, and other hydrologic variables.
  - Limitations – Has only 2 parameters and requires the logarithms of the data to be symmetric about the mean.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\left(-\frac{(y-\mu_y)^2}{2\sigma_y^{\,2}}\right)}$$

$$-\infty \le x \le \infty$$

$$y = \log x$$

$$x > 0$$

$$\mu_y = \overline{y}$$

$$\sigma_y = s_y$$

Probability Distribution for Hydrologic Variables

- **Extreme Value Distribution (Type I)**:
  - Hydrologic Application – Annual maximum discharges.
  - Limitations – Limited to sets of extreme data.

$$f(x) = \frac{1}{\alpha} e^{\left[-\frac{x-u}{\alpha} - e^{-\left(\frac{x-u}{\alpha}\right)} \right]}$$

$$-\infty \leq x \leq \infty$$

$$\alpha = \frac{\sqrt{6} s_x}{\pi}$$

$$u = \bar{x} - 0.5772\alpha$$

Probability Distribution for Hydrologic Variables

- **Pearson Type III Distribution**:
  - Hydrologic Application – A 3-parameter Gamma Distribution, applicable to annual maximum flood peaks.
  - Limitations – Few; has wide application.

$$f(x) = \frac{\lambda^{\beta}(x-\varepsilon)^{\beta-1}e^{\lambda(x-\varepsilon)}}{\Gamma(\beta)}$$

$$x \geq \varepsilon$$

$$\lambda = \frac{s_x}{\sqrt{\beta}}$$

$$\beta = \left(\frac{2}{C_s}\right)^2$$

$$\varepsilon = \bar{x} - s_x\sqrt{\beta}$$

Probability Distribution for Hydrologic Variables

- **Log-Pearson Type III Distribution:**
  - Hydrologic Application – Similar to the Pearson Type III, applicable to annual maximum flood peaks.
  - Limitations – Few; has wide application.

$$f(x) = \frac{\lambda^{\beta}(y-\varepsilon)^{\beta-1}e^{\lambda(y-\varepsilon)}}{\Gamma(\beta)}$$

$$y = \log x$$

$$\log x \geq \varepsilon$$

$$\lambda = \frac{s_y}{\sqrt{\beta}}$$

$$\Gamma(\beta) = (\beta-1)!$$

$$\beta = \left(\frac{2}{C_s(y)}\right)^2$$

$$\varepsilon = \overline{y} - s_y\sqrt{\beta}$$

$$C_s(y) > 0$$

Fitting and Testing a Probability Distribution

- A **probability distribution** can be fit to a particular situation based on sample data using one of two methods; **Method of Moments** and **Method of Maximum Likelihood**.
- Similarly, the goodness of the fit of a probably distribution can be evaluated using sample data.
- Reviewing the fitting and testing of probability distribution functions are beyond the scope of this class.

Statistical Parameter Estimation

- A statistical parameter is the **Expected Value (E)** of some function of a random variable.

- **Mid-Point**

	Mean		Median
$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$	Population Function	"x" such that F(x)=0.5	Population Function
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	Sample	50^{th}-percentile value of data	Sample

Statistical Parameter Estimation

- **Variability**

Variance

$$\sigma^2 = E\left[(x-\mu)^2\right]$$

Population Function

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Sample

Standard Deviation

$$\sigma = \left\{E\left[(x-\mu)^2\right]\right\}^{1/2}$$

Population Function

$$s = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{1/2}$$

Sample

- **Symmetry**

Coefficient of Skewness

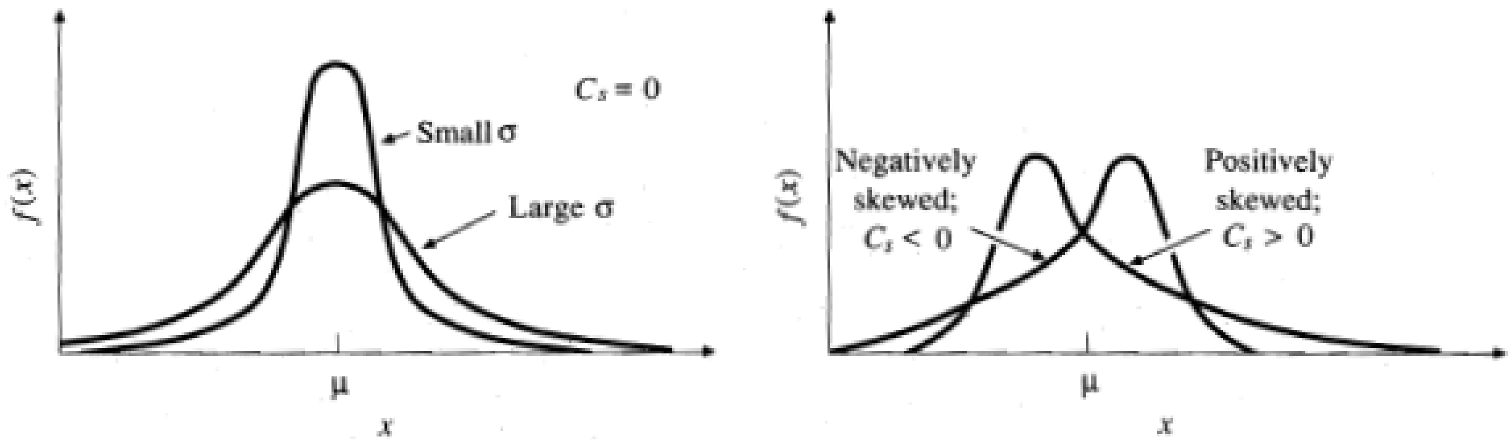
$$\gamma = \frac{E\left[(x-\mu)^3\right]}{\sigma^3}$$

Population Function

$$C_s = \frac{n\sum_{i=1}^{n}(x_i - \bar{x})^3}{(n-1)(n-2)s^3}$$

Sample

Statistical Parameter Estimation



(*a*) Standard deviation σ. (*b*) Coefficient of skewness C_s.

Hydrologic Statistics/Flood-Frequency

- **Annual Exceedance Probability:** % Chance of Being Equaled or Exceeded in Any Given Year (p)
- **Recurrence Interval or Return Period**: 1/Annual Exceedance Probability (T)
- **Annual Exceedance Probability**: 1/Return Period
- **Example**: 100-year Return Period = 1/100 or 1% Annual Exceedance Probability
- Extreme hydrologic events, including rainfall, flood discharge, stage, and droughts, can be expressed in the terms above to quantify flood risk and assess the economic benefit of a structural flood protection measure.

Hydrologic Statistics/Flood-Frequency

- What is the probability that a T-year return period event will occur at least once in N years. First, consider the situation where no T-year event occurs in 3 years resulting in N successive "failures":

$$P\left(X < x_T\_each\_year\_for\_N - years\right) = (1 - p)^N$$

- The complement to this situation is:

$$P\left(X \geq x_T\_at\_least\_once\_in\_N - years\right) = 1 - (1 - p)^N$$

- Since p=1/T:

$$P\left(X \geq x_T\_at\_least\_once\_in\_N - years\right) = 1 - \left(1 - \frac{1}{T}\right)^N$$

Hydrologic Statistics/Flood-Frequency

- **Example 2:** Estimate the probability that the annual maximum discharge (Q) on the Guadalupe River in Texas will exceed 50,000 cfs in any given year:

$$T = \frac{41}{8} = 5.1 years$$ (Return Period)

$$P(X \geq x_T) = \frac{1}{T} = 0.195$$ (19.5% Exceedance Probability in Any Given Year)

TABLE 12.1.1
Annual maximum discharges of the Guadalupe River near Victoria, Texas, 1935–1978, in cfs

Year	1930	1940	1950	1960	1970
0		55,900	13,300	23,700	9,190
1		58,000	12,300	55,800	9,740
2		56,000	28,400	10,800	58,500
3		7,710	11,600	4,100	33,100
4		12,300	8,560	5,720	25,200
5	38,500	22,000	4,950	15,000	30,200
6	179,000	17,900	1,730	9,790	14,100
7	17,200	46,000	25,300	70,000	54,500
8	25,400	6,970	58,300	44,300	12,700
9	4,940	20,600	10,100	15,200	

Hydrologic Statistics/Flood-Frequency

- **Example 1 (con't):** Also, estimate the probability that the annual maximum discharge (Q) on the Guadalupe River in Texas will exceed 50,000 cfs *at least once in the next three years*:

$$P(X \geq x_T) = \frac{1}{T} = 0.195$$

$$P(X \geq x_T\_at\_least\_once\_in\_N - years) = 1 - \left(1 - \frac{1}{T}\right)^N = 1 - (1 - 0.195)^3 = 0.48$$

Hydrologic Statistics/Flood-Frequency

- **Extreme Value Distribution:** Extreme value distributions are widely used in hydrology. Storm rainfalls are most commonly modeled by the **Extreme Value Type I distribution**. Below is the "cumulative" probability distribution version of the function.

$$F(x) = e^{-e^{\left[-\frac{x-u}{\alpha}\right]}} \_ -\infty \le x \le \infty$$

$$\alpha = \frac{\sqrt{6}s_x}{\pi}$$

$$u = \bar{x} - 0.5772\alpha$$

$$Let\_y = \frac{x-u}{\alpha}$$

- Correlating "y" with Return Period (T), the Extreme Value Distribution can be simplified to:

$$x_T = u + \alpha y_T$$

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

Hydrologic Statistics/Flood-Frequency

- **Extreme Value Distribution Example 3**: Annual maximum values of 10-minute-duration rainfall at Chicago, IL, from 1913 to 1947 are presented in the table below. Develop a model for storm frequency analysis using the Extreme Value Type I distribution and calculate the 5, 10-, and 50-year return period maximum values of the 10-minute rainfall.

- Sample moments calculated from the data are: $\bar{x} = 0.649\_and\_s = 0.177$

- Therefore: $\alpha = \frac{\sqrt{6}s}{\pi} = \frac{\sqrt{6}*0.177}{\pi} = 0.138$

  $u = \bar{x} - 0.5772\alpha = 0.649 - 0.5772*0.138 = 0.569$

- The probability model becomes:

$$F(x) = e^{-e^{\left[-\frac{x-0.569}{0.138}\right]}}$$

TABLE 12.2.1
Annual maximum 10-minute rainfall in inches at Chicago, Illinois, 1913–1947

Year	1910	1920	1930	1940
0		0.53	0.33	0.34
1		0.76	0.96	0.70
2		0.57	0.94	0.57
3	0.49	0.80	0.80	0.92
4	0.66	0.66	0.62	0.66
5	0.36	0.68	0.71	0.65
6	0.58	0.68	1.11	0.63
7	0.41	0.61	0.64	0.60
8	0.47	0.88	0.52	
9	0.74	0.49	0.64	

Mean = 0.649 in
Standard deviation = 0.177 in

Hydrologic Statistics/Flood-Frequency

- To determine the values of x_T for various values of return period T, it is convenient to use y_T. For T=5 years:

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right] = -\ln\left[\ln\left(\frac{5}{5-1}\right)\right] = 1.500$$

$$x_T = u + \alpha y_T = 0.569 + 0.138 * 1.500 = 0.78''$$

- Similar application of Extreme Value probability function can be used to compute the 10- and 50-year values (0.88” and 1.11”, respectively).

- The next slide provides a comparison of the exceedance probability using the sample data set and the Extreme Value probability function

Hydrologic Statistics/Flood-Frequency

EV(Type I) Probability Function

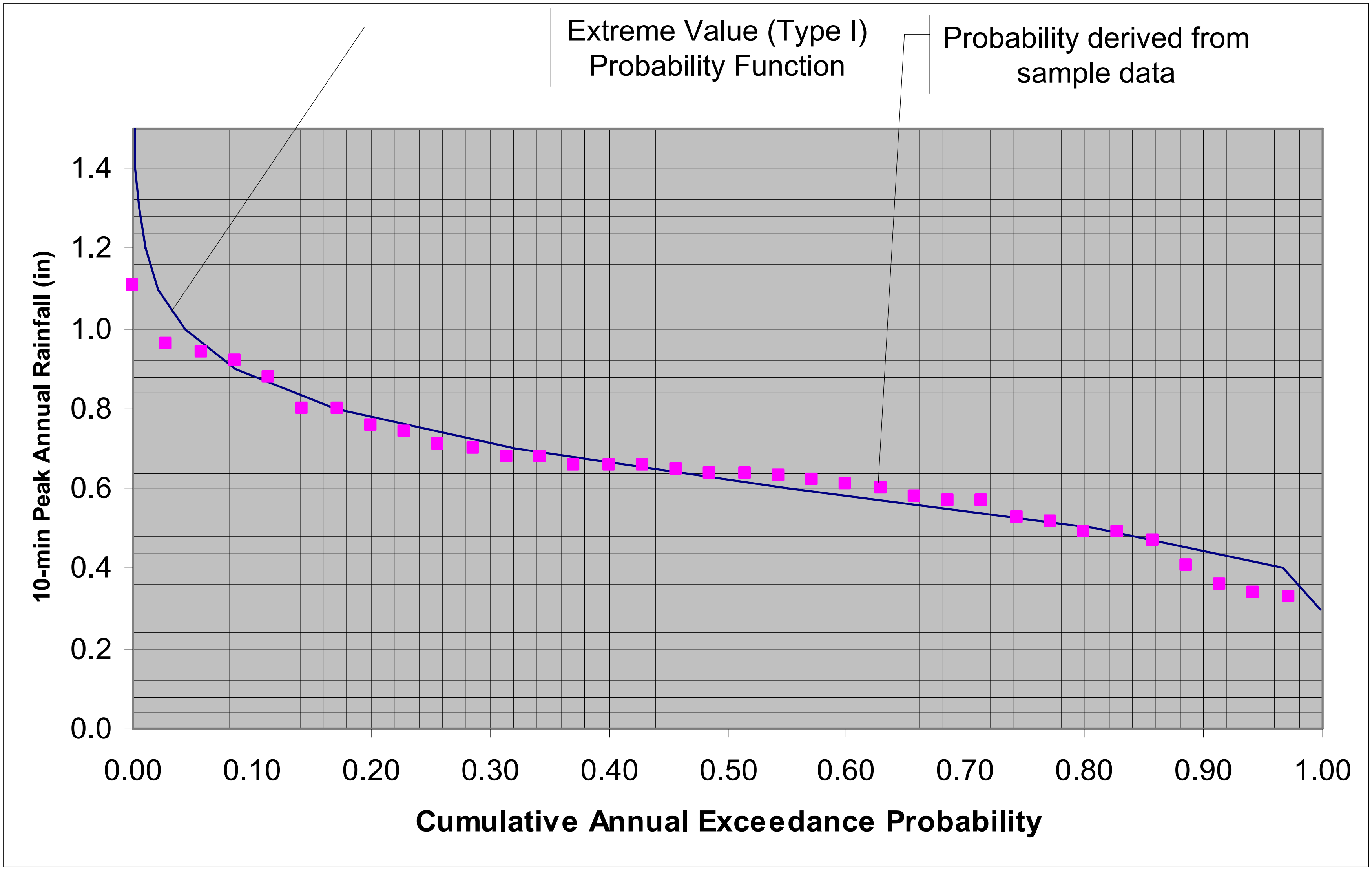
x	F(x)	1-F(x)	T
0.3	0.0009	0.9991	1.00
0.4	0.0333	0.9667	1.03
0.5	0.1923	0.8077	1.24
0.6	0.4499	0.5501	1.82
0.7	0.6791	0.3209	3.12
0.8	0.8290	0.1710	5.85
0.9	0.9132	0.0868	11.51
1.0	0.9569	0.0431	23.22
1.1	0.9789	0.0211	47.39
1.2	0.9897	0.0103	97.28
1.3	0.9950	0.0050	200.26
1.4	0.9976	0.0024	412.80
1.5	0.9988	0.0012	851.47

$$F(x) = e^{-e^{\left[-\frac{x-0.569}{0.138}\right]}}$$

Actual Data (Re-ordered)

Rank	Year	10-Min Rainfall (peak)	F(x) (Sample)	1-F(x) (Sample)
1	1930	0.33	0.03	0.97
2	1940	0.34	0.06	0.94
3	1915	0.36	0.09	0.91
4	1917	0.41	0.11	0.89
5	1918	0.47	0.14	0.86
6	1913	0.49	0.17	0.83
7	1929	0.49	0.20	0.80
8	1938	0.52	0.23	0.77
9	1920	0.53	0.26	0.74
10	1922	0.57	0.29	0.71
11	1942	0.57	0.31	0.69
12	1916	0.58	0.34	0.66
13	1947	0.60	0.37	0.63
14	1927	0.61	0.40	0.60
15	1934	0.62	0.43	0.57
16	1946	0.63	0.46	0.54
17	1937	0.64	0.49	0.51
18	1939	0.64	0.51	0.49
19	1945	0.65	0.54	0.46
20	1914	0.66	0.57	0.43
21	1924	0.66	0.60	0.40
22	1944	0.66	0.63	0.37
23	1925	0.68	0.66	0.34
24	1926	0.68	0.69	0.31
25	1941	0.70	0.71	0.29
26	1935	0.71	0.74	0.26
27	1919	0.74	0.77	0.23
28	1921	0.76	0.80	0.20
29	1923	0.80	0.83	0.17
30	1933	0.80	0.86	0.14
31	1928	0.88	0.89	0.11
32	1943	0.92	0.91	0.09
33	1932	0.94	0.94	0.06
34	1931	0.96	0.97	0.03
35	1936	1.11	1.00	0.00

Hydrologic Statistics/Flood-Frequency



Frequency Analysis using Frequency Factors

- Calculating the magnitudes of extreme events, as in the previous example, using Normal and Pearson Type III distributions is better done using a **frequency factor (K_T)**.

$$x_T = \mu + K_T \sigma$$

$$x_T = \bar{x} + K_T s$$

- **Normal Distribution**:

$$K_T = z = \frac{x_T - \mu}{\sigma}$$

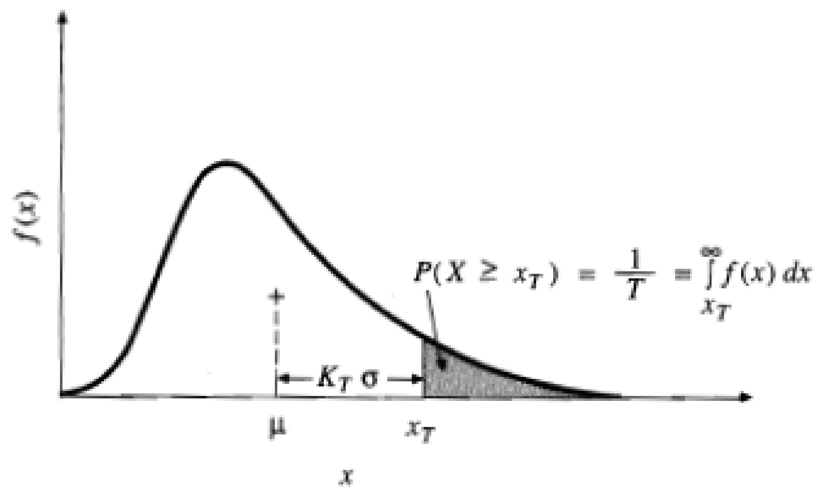
$$z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}$$

$$w = \left[\ln\left(\frac{1}{p^2}\right)\right]^{1/2}, (0 < p \le 0.5)$$

- **Extreme Value Distribution**:

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

$$T = \frac{1}{1 - e^{-e^{\left[-\left(\gamma + \frac{\pi K_T}{\sqrt{6}}\right)\right]}}}$$



Frequency Analysis using Frequency Factors

- **Log-Pearson Type III Distribution**:

$$K_T = z + (z^2 - 1)k + \frac{1}{3}(z^3 - 6z)k^2 - (z^2 - 1)k^3 + zk^4 + \frac{1}{3}k^5$$

$k = \frac{C_s}{6}$ and C_s = Coefficient of Skewness for $\log_{10}$ of the hydrologic data ($y=\log_{10}x$)

TABLE 12.3.1
K_T values for Pearson Type III distribution (positive skew)

Skew coefficient C_s or C_w	Return period in years: 2	5	10	25	50	100	200
	Exceedence probability: 0.50	0.20	0.10	0.04	0.02	0.01	0.005
3.0	−0.396	0.420	1.180	2.278	3.152	4.051	4.970
2.9	−0.390	0.440	1.195	2.277	3.134	4.013	4.909
2.8	−0.384	0.460	1.210	2.275	3.114	3.973	4.847
2.7	−0.376	0.479	1.224	2.272	3.093	3.932	4.783
2.6	−0.368	0.499	1.238	2.267	3.071	3.889	4.718
2.5	−0.360	0.518	1.250	2.262	3.048	3.845	4.652
2.4	−0.351	0.537	1.262	2.256	3.023	3.800	4.584
2.3	−0.341	0.555	1.274	2.248	2.997	3.753	4.515
2.2	−0.330	0.574	1.284	2.240	2.970	3.705	4.444
2.1	−0.319	0.592	1.294	2.230	2.942	3.656	4.372
2.0	−0.307	0.609	1.302	2.219	2.912	3.605	4.298
1.9	−0.294	0.627	1.310	2.207	2.881	3.553	4.223
1.8	−0.282	0.643	1.318	2.193	2.848	3.499	4.147
1.7	−0.268	0.660	1.324	2.179	2.815	3.444	4.069
1.6	−0.254	0.675	1.329	2.163	2.780	3.388	3.990
1.5	−0.240	0.690	1.333	2.146	2.743	3.330	3.910
1.4	−0.225	0.705	1.337	2.128	2.706	3.271	3.828
1.3	−0.210	0.719	1.339	2.108	2.666	3.211	3.745
1.2	−0.195	0.732	1.340	2.087	2.626	3.149	3.661
1.1	−0.180	0.745	1.341	2.066	2.585	3.087	3.575
1.0	−0.164	0.758	1.340	2.043	2.542	3.022	3.489
0.9	−0.148	0.769	1.339	2.018	2.498	2.957	3.401
0.8	−0.132	0.780	1.336	1.993	2.453	2.891	3.312
0.7	−0.116	0.790	1.333	1.967	2.407	2.824	3.223
0.6	−0.099	0.800	1.328	1.939	2.359	2.755	3.132
0.5	−0.083	0.808	1.323	1.910	2.311	2.686	3.041
0.4	−0.066	0.816	1.317	1.880	2.261	2.615	2.949
0.3	−0.050	0.824	1.309	1.849	2.211	2.544	2.856
0.2	−0.033	0.830	1.301	1.818	2.159	2.472	2.763
0.1	−0.017	0.836	1.292	1.785	2.107	2.400	2.670
0.0	0	0.842	1.282	1.751	2.054	2.326	2.576

TABLE 12.3.1 *(cont.)*
K_T values for Pearson Type III distribution (negative skew)

Skew coefficient C_s or C_w	Return period in years: 2	5	10	25	50	100	200
	Exceedence probability: 0.50	0.20	0.10	0.04	0.02	0.01	0.005
−0.1	0.017	0.846	1.270	1.716	2.000	2.252	2.482
−0.2	0.033	0.850	1.258	1.680	1.945	2.178	2.388
−0.3	0.050	0.853	1.245	1.643	1.890	2.104	2.294
−0.4	0.066	0.855	1.231	1.606	1.834	2.029	2.201
−0.5	0.083	0.856	1.216	1.567	1.777	1.955	2.108
−0.6	0.099	0.857	1.200	1.528	1.720	1.880	2.016
−0.7	0.116	0.857	1.183	1.488	1.663	1.806	1.926
−0.8	0.132	0.856	1.166	1.448	1.606	1.733	1.837
−0.9	0.148	0.854	1.147	1.407	1.549	1.660	1.749
−1.0	0.164	0.852	1.128	1.366	1.492	1.588	1.664
−1.1	0.180	0.848	1.107	1.324	1.435	1.518	1.581
−1.2	0.195	0.844	1.086	1.282	1.379	1.449	1.501
−1.3	0.210	0.838	1.064	1.240	1.324	1.383	1.424
−1.4	0.225	0.832	1.041	1.198	1.270	1.318	1.351
−1.5	0.240	0.825	1.018	1.157	1.217	1.256	1.282
−1.6	0.254	0.817	0.994	1.116	1.166	1.197	1.216
−1.7	0.268	0.808	0.970	1.075	1.116	1.140	1.155
−1.8	0.282	0.799	0.945	1.035	1.069	1.087	1.097
−1.9	0.294	0.788	0.920	0.996	1.023	1.037	1.044
−2.0	0.307	0.777	0.895	0.959	0.980	0.990	0.995
−2.1	0.319	0.765	0.869	0.923	0.939	0.946	0.949
−2.2	0.330	0.752	0.844	0.888	0.900	0.905	0.907
−2.3	0.341	0.739	0.819	0.855	0.864	0.867	0.869
−2.4	0.351	0.725	0.795	0.823	0.830	0.832	0.833
−2.5	0.360	0.711	0.771	0.793	0.798	0.799	0.800
−2.6	0.368	0.696	0.747	0.764	0.768	0.769	0.769
−2.7	0.376	0.681	0.724	0.738	0.740	0.740	0.741
−2.8	0.384	0.666	0.702	0.712	0.714	0.714	0.714
−2.9	0.390	0.651	0.681	0.683	0.689	0.690	0.690
−3.0	0.396	0.636	0.666	0.666	0.666	0.667	0.667

Source: U. S. Water Resources Council (1981).

Frequency Analysis using Frequency Factors

- **Log-Pearson Type III Distribution Example**: Calculate the 5- and 50-year return period annual maximum discharges of the Guadalupe River, Texas, using the Log-Normal and Log-Pearson Type III distributions.

TABLE 12.1.1
Annual maximum discharges of the Guadalupe River near Victoria, Texas, 1935–1978, in cfs

Year	1930	1940	1950	1960	1970
0		55,900	13,300	23,700	9,190
1		58,000	12,300	55,800	9,740
2		56,000	28,400	10,800	58,500
3		7,710	11,600	4,100	33,100
4		12,300	8,560	5,720	25,200
5	38,500	22,000	4,950	15,000	30,200
6	179,000	17,900	1,730	9,790	14,100
7	17,200	46,000	25,300	70,000	54,500
8	25,400	6,970	58,300	44,300	12,700
9	4,940	20,600	10,100	15,200	

Frequency Analysis using Frequency Factors

- **Solution**: The logarithms of the discharge values and the resulting statistics are computed:

$$\bar{y} = 4.2743$$

$$s_y = 0.4027$$

$$C_s = -0.0696$$

- Using C_s=-0.0696, the value of K_{50} is obtained by interpolation from Table 12.3.1 or from the Log Pearson equation for K_T presented previously. Interpolating on Table 12.3.1 at T=50 years:

$$K_{50} = 2.054 + \frac{2.00 - 2.054}{-0.1 - 0}(-0.0696 - 0) = 2.016$$

$$\therefore y_{50} = \bar{y} + K_{50}s_y = 4.2743 + 2.016 * 0.4027 = 5.0863$$

$$x_{50} = 10^{5.0863} = 121{,}990 cfs$$

- A similar calculation can be performed for the 5-year storm (T=5):

$$K_5 = 0.845$$

$$y_5 = 4.6146$$

$$x_5 = 41{,}170 cfs$$

Probability Plotting

- As a check that a probability distribution fits a set of hydrologic data, the data may be plotted on specially designed probability paper, or using a plotting scale that **linearizes** the distribution function. The data is fitted with a straight line to interpolation and extrapolation.

- The following is the Weibull equation for plotting hydrologic data:

$$P(X \geq x_m) = \frac{m}{n+1}$$

$$T = \frac{n+1}{m}$$

where,

n = Total number of values

m = Ranking

T = Return interval

Probability Plotting

- **Example**: Perform a probability plotting analysis of the annual maximum discharges for the Guadelupe River near Victoria, Texas, given previous and again below. Compare the plotted data with the fitting of a Log-Normal probability distribution.

- **Solution**:
  - Develop the results of the the Log-Normal probability distribution function for each discharge in the sample set. The frequency factor (K_T) can be obtained from the corresponding equation for the Log-Normal function (provided again below) or using Table 12.3.1 at a skewness coefficient of 0. The results are provided on the next page.

$$y_T = \mu + K_T \sigma$$

$$K_T = z = \frac{x_T - \mu}{\sigma}$$

$$z = w - \frac{2.515517 + 0.802853 w + 0.010328 w^2}{1 + 1.432788 w + 0.189269 w^2 + 0.001308 w^3}$$

$$w = \left[\ln \left(\frac{1}{p^2} \right) \right]^{1/2}, (0 < p \leq 0.5)$$

TABLE 12.1.1
Annual maximum discharges of the Guadalupe River near Victoria, Texas, 1935–1978, in cfs

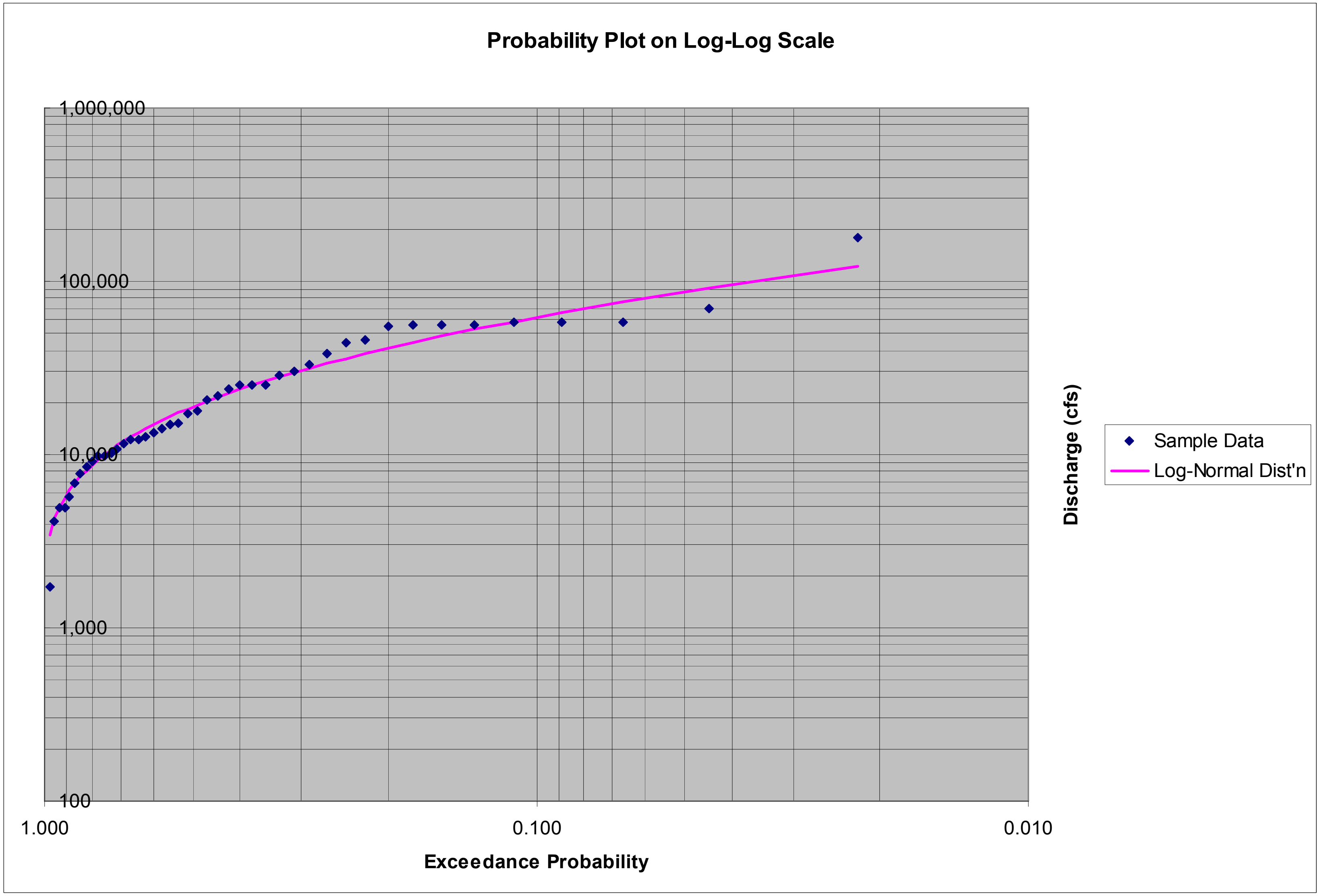
Year	1930	1940	1950	1960	1970
0		55,900	13,300	23,700	9,190
1		58,000	12,300	55,800	9,740
2		56,000	28,400	10,800	58,500
3		7,710	11,600	4,100	33,100
4		12,300	8,560	5,720	25,200
5	38,500	22,000	4,950	15,000	30,200
6	179,000	17,900	1,730	9,790	14,100
7	17,200	46,000	25,300	70,000	54,500
8	25,400	6,970	58,300	44,300	12,700
9	4,940	20,600	10,100	15,200	

Probability Plotting

Probability Plotting using the Log-Normal Distribution and Weibull's Formula for the Annual Maximum Discharges of the Guadelupe River near Victoria, TX

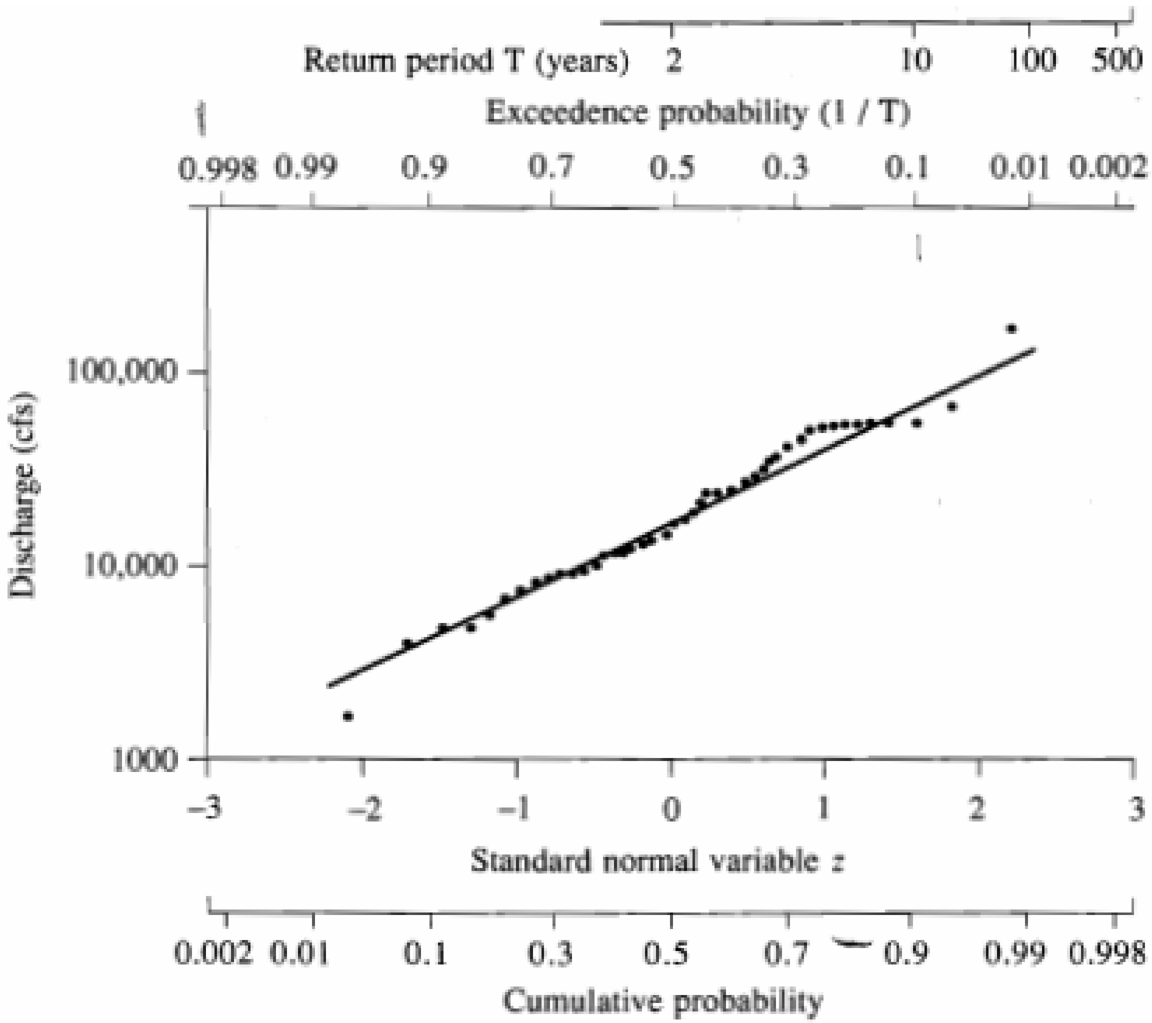
Year	1 Discharge *Q (cfs)*	2 Rank *m*	3 Exceedance Probability using Weibull Plotting Formula *m/(n+1)*	4 *w*	5 Standard Normal Variable *z=KT*	6 Log Q from Log-Normal Distribution *yT*	7 Log Q from Data	8 Q from Log-Normal Distribution
1936	179,000	1	0.022	2.759	2.010	5.084	5.253	121,368
1967	70,000	2	0.044	2.495	1.702	4.960	4.845	91,145
1972	58,500	3	0.067	2.327	1.501	4.879	4.767	75,688
1958	58,300	4	0.089	2.200	1.348	4.817	4.766	65,638
1941	58,000	5	0.111	2.096	1.221	4.766	4.763	58,338
1942	56,000	6	0.133	2.007	1.111	4.722	4.748	52,680
1940	55,900	7	0.156	1.929	1.013	4.682	4.747	48,103
1961	55,800	8	0.178	1.859	0.924	4.646	4.747	44,286
1977	54,500	9	0.200	1.794	0.841	4.613	4.736	41,030
1947	46,000	10	0.222	1.734	0.764	4.582	4.663	38,201
1968	44,300	11	0.244	1.679	0.692	4.553	4.646	35,710
1935	38,500	12	0.267	1.626	0.623	4.525	4.585	33,489
1973	33,100	13	0.289	1.576	0.556	4.498	4.520	31,490
1975	30,200	14	0.311	1.528	0.492	4.472	4.480	29,675
1952	28,400	15	0.333	1.482	0.430	4.447	4.453	28,016
1938	25,400	16	0.356	1.438	0.370	4.423	4.405	26,490
1957	25,300	17	0.378	1.395	0.311	4.399	4.403	25,078
1974	25,200	18	0.400	1.354	0.253	4.376	4.401	23,765
1960	23,700	19	0.422	1.313	0.196	4.353	4.375	22,539
1945	22,000	20	0.444	1.274	0.139	4.330	4.342	21,389
1949	20,600	21	0.467	1.235	0.083	4.308	4.314	20,307
1946	17,900	22	0.489	1.196	0.028	4.285	4.253	19,285
1937	17,200	23	0.511	1.159	-0.028	4.263	4.236	18,316
1969	15,200	24	0.533	1.121	-0.083	4.240	4.182	17,395
1965	15,000	25	0.556	1.084	-0.139	4.218	4.176	16,518
1976	14,100	26	0.578	1.047	-0.195	4.195	4.149	15,679
1950	13,300	27	0.600	1.011	-0.252	4.172	4.124	14,875
1978	12,700	28	0.622	0.974	-0.310	4.149	4.104	14,102
1944	12,300	29	0.644	0.937	-0.368	4.126	4.090	13,358
1951	12,300	30	0.667	0.901	-0.428	4.102	4.090	12,639
1953	11,600	31	0.689	0.863	-0.489	4.077	4.064	11,942
1962	10,800	32	0.711	0.826	-0.552	4.052	4.033	11,266
1959	10,100	33	0.733	0.788	-0.616	4.026	4.004	10,607
1966	9,790	34	0.756	0.749	-0.684	3.998	3.991	9,964
1971	9,740	35	0.778	0.709	-0.754	3.970	3.989	9,332
1970	9,190	36	0.800	0.668	-0.829	3.940	3.963	8,711
1954	8,560	37	0.822	0.626	-0.908	3.908	3.932	8,097
1943	7,710	38	0.844	0.582	-0.992	3.874	3.887	7,486
1948	6,790	39	0.867	0.535	-1.084	3.837	3.832	6,874
1964	5,720	40	0.889	0.485	-1.186	3.796	3.757	6,256
1955	4,950	41	0.911	0.431	-1.300	3.750	3.695	5,623
1939	4,940	42	0.933	0.371	-1.435	3.696	3.694	4,963
1963	4,100	43	0.956	0.302	-1.602	3.629	3.613	4,251
1956	1,730	44	0.978	0.212	-1.835	3.535	3.238	3,424
						μ=	**4.274**	
						σ=	**0.403**	

Probability Plotting

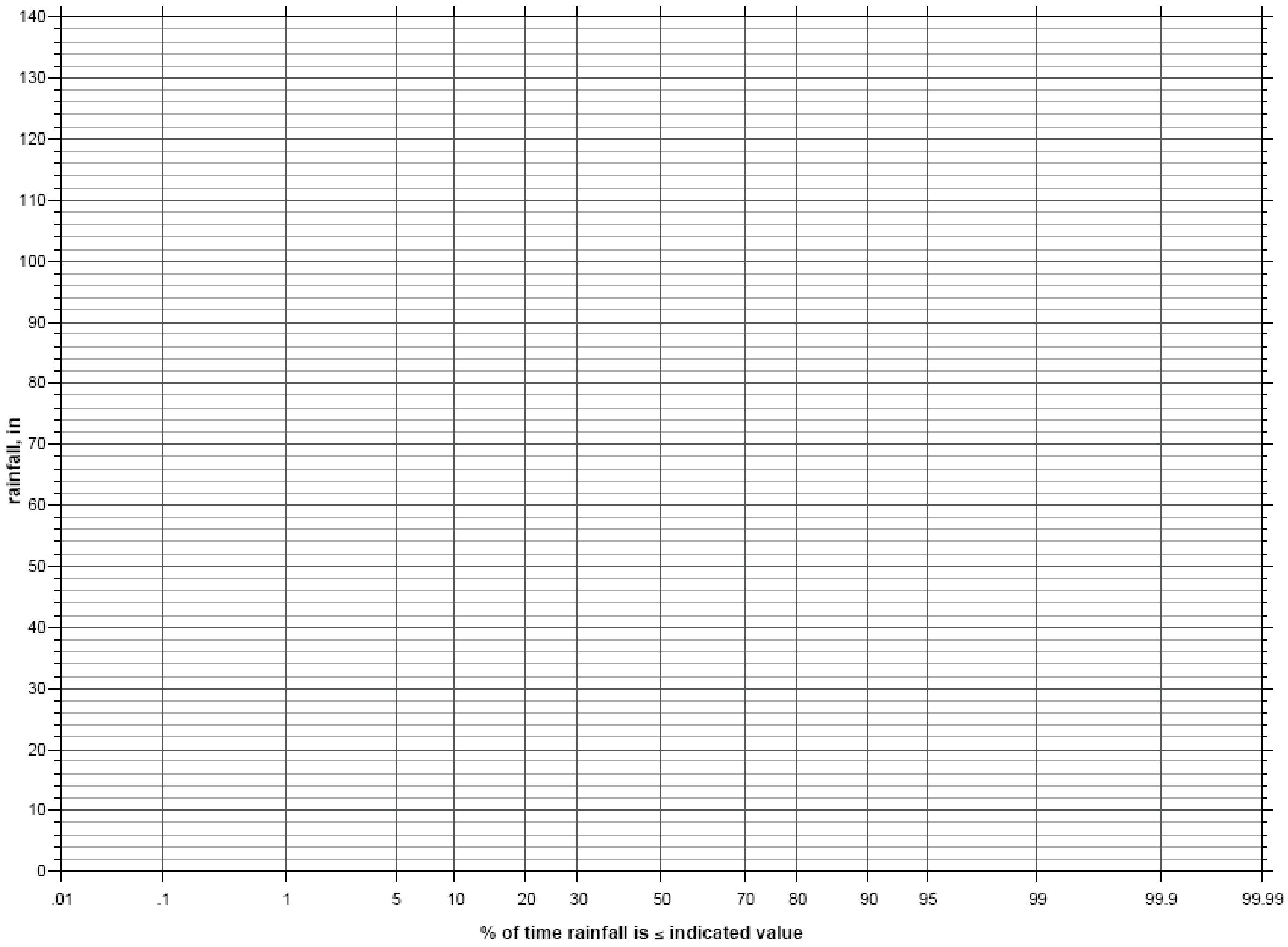


Probability Plotting

Probability Plotting Linearized on Plotting Paper



Probability Plotting Graphs



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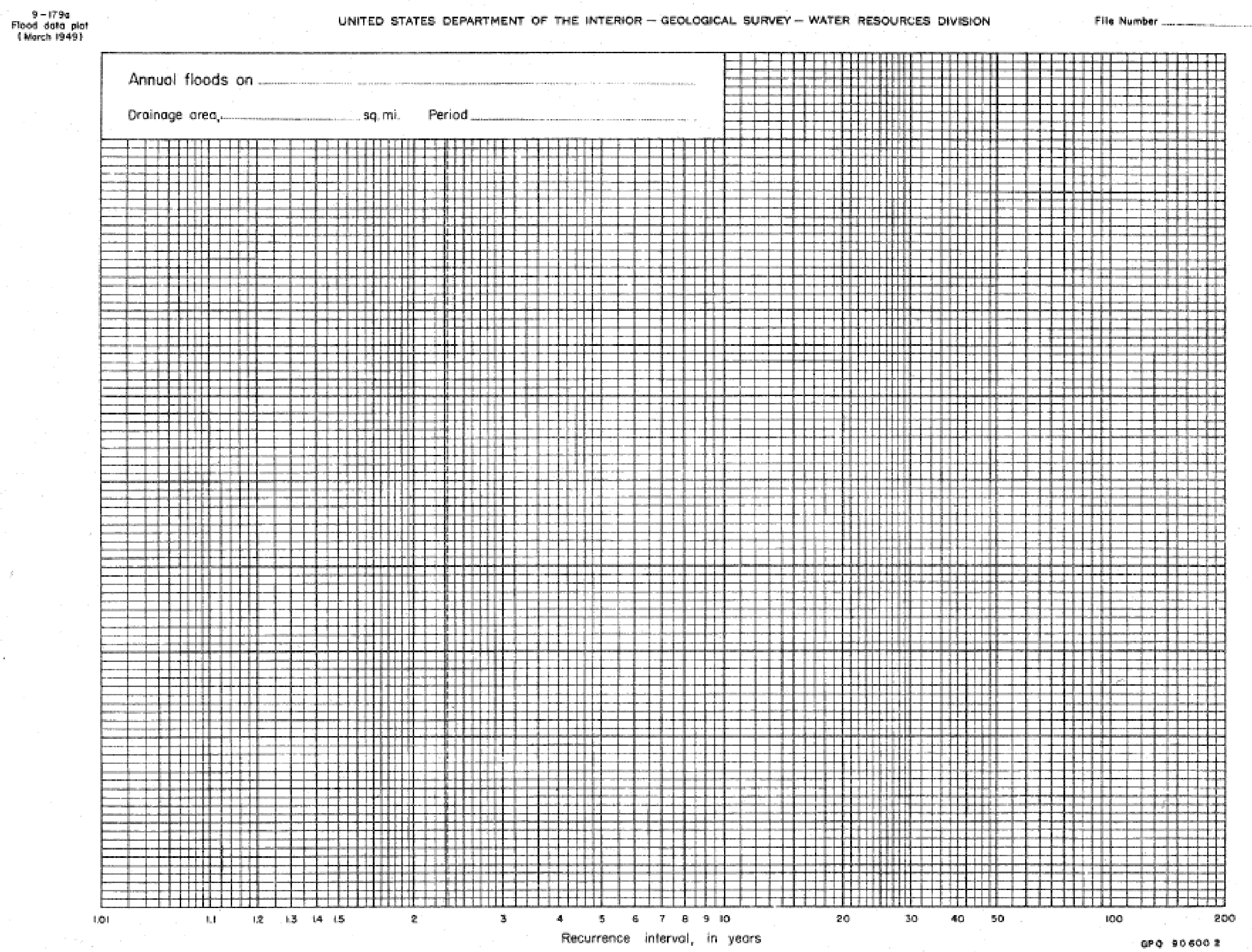
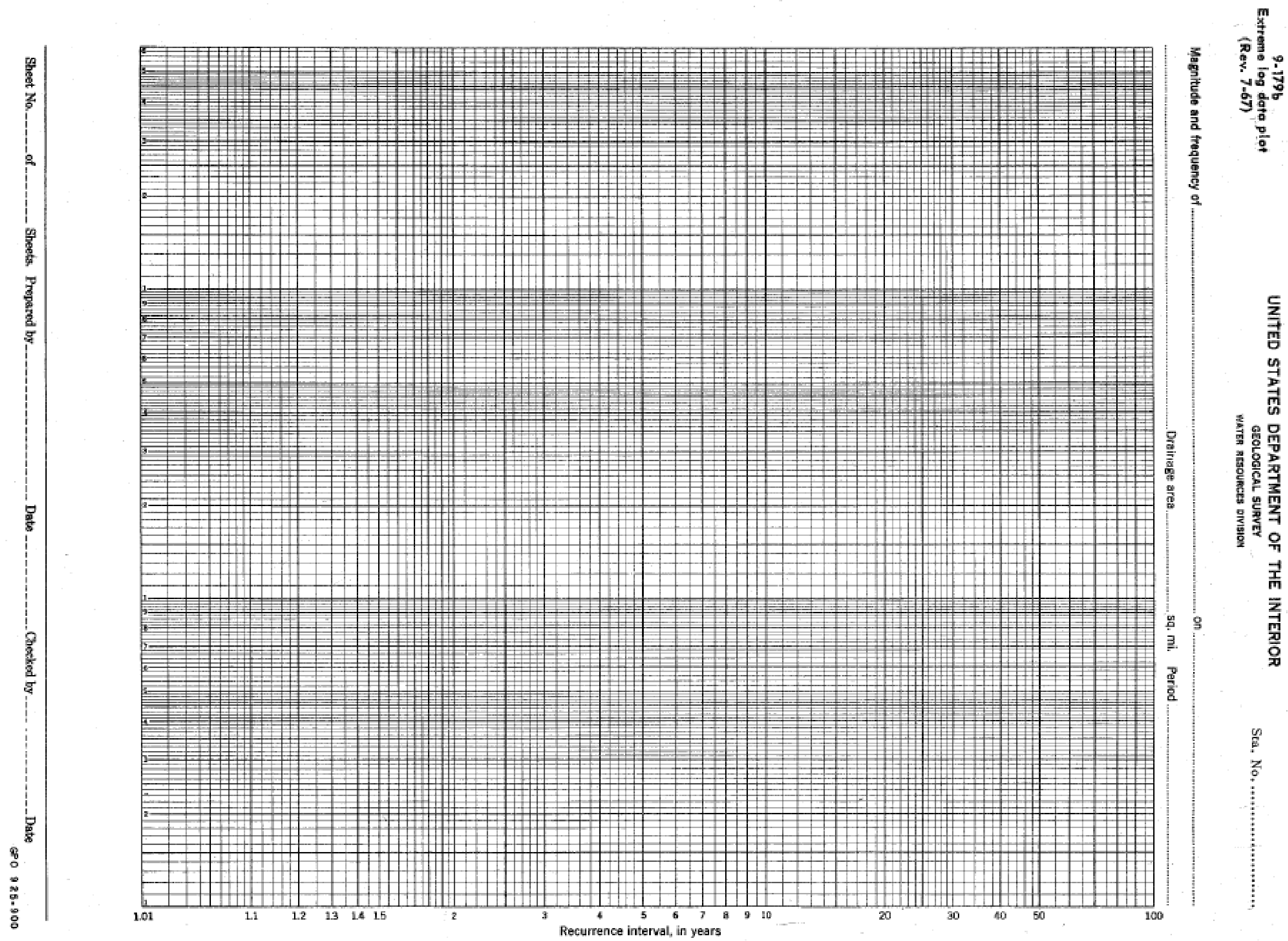
Probability Plotting Graphs



USGS (Gumbel) for Flood Frequency

Probability Plotting Graphs

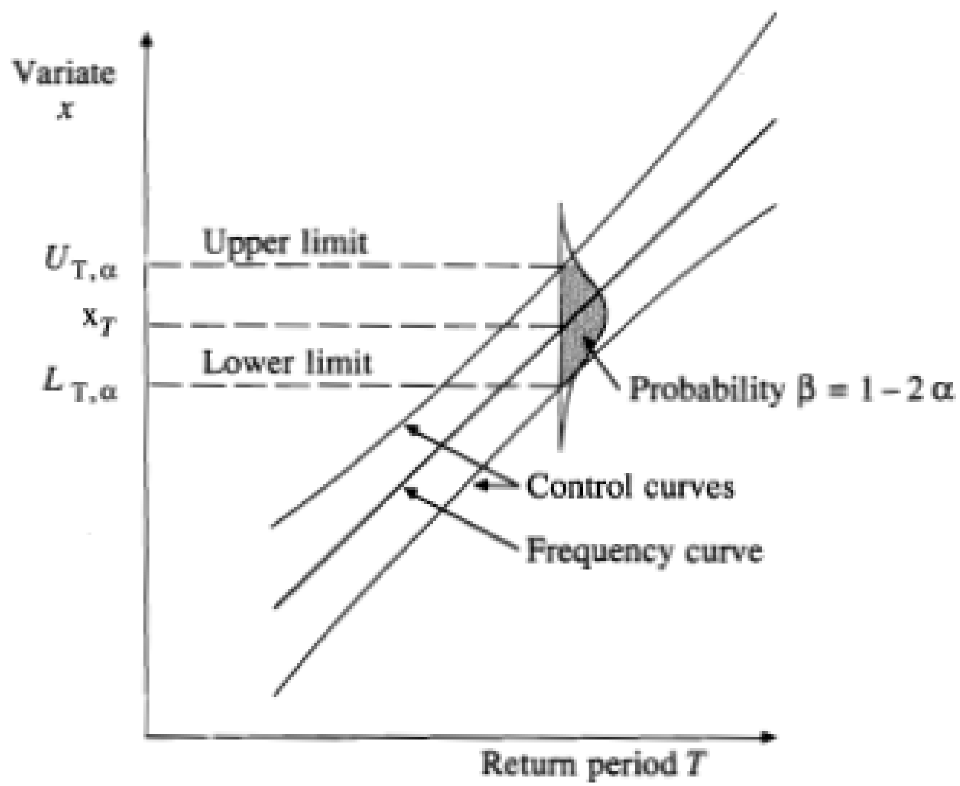


USGS (log-Gumbel) for Flood Frequency

Confidence Limits

- Statistical estimates generally include a range of possibilities that contain the true value. This range is referred to as the **Confidence Interval (β)**. The **Confidence Limits** represent the upper and lower bounds of the interval. The **Significance Level** is given by:

$$\alpha = \frac{1-\beta}{2}$$



Confidence Limits

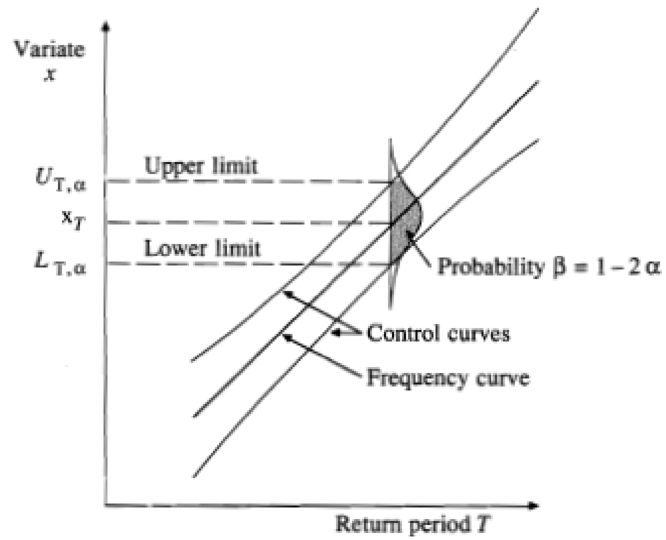
$$U_{T,\alpha} = \bar{y} + s_y K^U_{T,\alpha}$$

$$L_{T,\alpha} = \bar{y} + s_y K^L_{T,\alpha}$$

Upper and lower confidence limit factors, based on Normal and Pearson Type III distributions, were developed as follows:

$$K^U_{T,\alpha} = \frac{K_T + \sqrt{K_T^2 - ab}}{a}$$

$$K^L_{T,\alpha} = \frac{K_T - \sqrt{K_T^2 - ab}}{a}$$



$$a = 1 - \frac{z_\alpha^2}{2(n-1)}$$

$$b = K_T^2 - \frac{z_\alpha^2}{n}$$

The quantity z_α is the standard normal variable with exceedance probability "α" from Table 11.2.1. β represents the confidence interval and "α" represents the limits. For example, the 90% confidence interval would have a 95% upper confidence limit and 5% lower confidence limit.

USGS PEAK-FQ

```
                         DANVILLE ADJ PEAKFQ.PRT
1
  Program PeakFq          U. S. GEOLOGICAL SURVEY             Seq.000.000
  Ver. 5.2              Annual peak flow frequency analysis   Run Date / Time
  11/01/2007            following Bulletin 17-B Guidelines    05/08/2009 10:43

                           --- PROCESSING OPTIONS ---

                        Plot option          = Graphics device
                        Basin char output    = None
                        Print option         = Yes
                        Debug print          = No
                        Input peaks listing  = Long
                        Input peaks format   = WATSTORE peak file

                        Input files used:
                           peaks (ascii)  - C:\PROGRAM FILES\PKFQWIN\DATA\DANVILLE ADJ
PEAKFQ.TXT
                           specifications - PKFQWPSF.TMP

                        Output file(s):
                           main - C:\PROGRAM FILES\PKFQWIN\DATA\DANVILLE ADJ
PEAKFQ.PRT

1


  Program PeakFq          U. S. GEOLOGICAL SURVEY             Seq.001.001
  Ver. 5.2              Annual peak flow frequency analysis   Run Date / Time
  11/01/2007            following Bulletin 17-B Guidelines    05/08/2009 10:43

             Station - 01540500  Susquehanna River at Danville, PA


                     I N P U T   D A T A   S U M M A R Y

                  Number of peaks in record          =       109
                  Peaks not used in analysis         =         0
                  Systematic peaks in analysis       =       109
                  Historic peaks in analysis         =         0
                  Years of historic record           =         0
                  Generalized skew                   =     0.380
                       Standard error                =     0.550
                       Mean Square error             =     0.303
                  Skew option                        =    WEIGHTED
                  Gage base discharge                =       0.0
                  User supplied high outlier threshold =    --
                  User supplied low outlier criterion  =    --
                  Plotting position parameter        =      0.00


  *********  NOTICE  --  Preliminary machine computations.        *********
  *********  User responsible for assessment and interpretation.  *********

    WCF134I-NO SYSTEMATIC PEAKS WERE BELOW GAGE BASE.                    0.0
    WCF195I-NO LOW OUTLIERS WERE DETECTED BELOW CRITERION.           41245.8
    WCF163I-NO HIGH OUTLIERS OR HISTORIC PEAKS EXCEEDED HHBASE.     354546.4
1

  Program PeakFq          U. S. GEOLOGICAL SURVEY             Seq.001.002
  Ver. 5.2              Annual peak flow frequency analysis   Run Date / Time
  11/01/2007            following Bulletin 17-B Guidelines    05/08/2009 10:43
                                         Page 1
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                         DANVILLE ADJ PEAKFQ.PRT

           Station - 01540500  Susquehanna River at Danville, PA


         ANNUAL FREQUENCY CURVE PARAMETERS -- LOG-PEARSON TYPE III

                          FLOOD BASE                LOGARITHMIC
                     ---------------------  -------------------------------
                                EXCEEDANCE                STANDARD
                     DISCHARGE  PROBABILITY     MEAN      DEVIATION     SKEW
                     ------------------------------------------------------
SYSTEMATIC RECORD         0.0      1.0000      5.0825       0.1534      0.203
BULL.17B ESTIMATE         0.0      1.0000      5.0825       0.1534      0.231


       ANNUAL FREQUENCY CURVE -- DISCHARGES AT SELECTED EXCEEDANCE PROBABILITIES

     ANNUAL                                'EXPECTED    90-PCT CONFIDENCE LIMITS
   EXCEEDANCE    BULL.17B     SYSTEMATIC  PROBABILITY'  FOR BULL. 17B ESTIMATES
   PROBABILITY   ESTIMATE       RECORD      ESTIMATE       LOWER         UPPER

     0.9950      52560.0       52060.0      51690.0      47960.0       56870.0
     0.9900      56480.0       56050.0      55720.0      51840.0       60810.0
     0.9500      69280.0       69070.0      68810.0      64610.0       73660.0
     0.9000      77640.0       77540.0      77290.0      72980.0       82030.0
     0.8000      89530.0       89560.0      89320.0      84890.0       93990.0
     0.6667     102700.0      102900.0     102700.0      98020.0      107400.0
     0.5000     119300.0      119500.0     119300.0     114200.0      124600.0
     0.4292     127100.0      127300.0     127100.0     121700.0      132800.0
     0.2000     162000.0      162200.0     162500.0     154400.0      170800.0
     0.1000     191700.0      191500.0     192700.0     181300.0      204100.0
     0.0400     230700.0      229900.0     233000.0     215800.0      249000.0
     0.0200     260800.0      259500.0     264600.0     242100.0      284300.0
     0.0100     292000.0      289900.0     297700.0     269000.0      321300.0
     0.0050     324400.0      321300.0     332600.0     296600.0      360100.0
     0.0020     369300.0      364800.0     381900.0     334600.0      414700.0


 Program PeakFq          U. S. GEOLOGICAL SURVEY             Seq.001.003
 Ver. 5.2              Annual peak flow frequency analysis   Run Date / Time
 11/01/2007            following Bulletin 17-B Guidelines    05/08/2009 10:43

           Station - 01540500  Susquehanna River at Danville, PA


                    I N P U T   D A T A   L I S T I N G

    WATER YEAR    DISCHARGE    CODES      WATER YEAR    DISCHARGE    CODES

       1900         94885.0                  1955         81336.0
       1901        121995.0                  1956        165702.0
       1902        219592.0                  1957        107943.0
       1903        119284.0                  1958        160021.0
       1904        133743.0                  1959        106049.0
       1905        122899.0                  1960        187480.0
       1906         89915.0                  1961        158962.0
       1907         66329.0                  1962        129454.0
       1908        110248.0                  1963        123743.0
       1909        121092.0                  1964        248438.0
       1910        149106.0                  1965         42739.0
                                        Page 2
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USGS PEAK-FQ

DANVILLE ADJ PEAKFQ.PRT

1911	87927.0	1966	94140.0
1912	116573.0	1967	83289.0
1913	173505.0	1968	98994.0
1914	168083.0	1969	77768.0
1915	127417.0	1970	116128.0
1916	158142.0	1971	105658.0
1917	83951.0	1972	345529.0
1918	125610.0	1973	94806.0
1919	73017.0	1974	98043.0
1920	153624.0	1975	244631.0
1921	91271.0	1976	114224.0
1922	120188.0	1977	116128.0
1923	94885.0	1978	110417.0
1924	128321.0	1979	178952.0
1925	146394.0	1980	104000.0
1926	91271.0	1981	105000.0
1927	128321.0	1982	83300.0
1928	140972.0	1983	149000.0
1929	147298.0	1984	194000.0
1930	71119.0	1985	55300.0
1931	79975.0	1986	173000.0
1932	107537.0	1987	104000.0
1933	107537.0	1988	83500.0
1934	89102.0	1989	116000.0
1935	138261.0	1990	70900.0
1936	225917.0	1991	124000.0
1937	84403.0	1992	89200.0
1938	71751.0	1993	187000.0
1939	126083.0	1994	139000.0
1940	201369.0	1995	73700.0
1941	128804.0	1996	209000.0
1942	108248.0	1997	130000.0
1943	190367.0	1998	143000.0
1944	91077.0	1999	116000.0
1945	112914.0	2000	132000.0
1946	218362.0	2001	97800.0
1947	139975.0	2002	84700.0
1948	171703.0	2003	130000.0
1949	83612.0	2004	220000.0
1950	157814.0	2005	202000.0
1951	124040.0	2006	260000.0
1952	120252.0	2007	123000.0
1953	97528.0	2008	124000.0
1954	77738.0		

Explanation of peak discharge qualification codes

PeakFQ CODE	NWIS CODE	DEFINITION
D	3	Dam failure, non-recurrent flow anomaly
G	8	Discharge greater than stated value
X	3+8	Both of the above
L	4	Discharge less than stated value
K	6 OR C	Known effect of regulation or urbanization
H	7	Historic peak

- Minus-flagged discharge -- Not used in computation
  -8888.0 -- No discharge value given
- Minus-flagged water year -- Historic peak used in computation

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DANVILLE ADJ PEAKFQ.PRT

1

Program PeakFq U. S. GEOLOGICAL SURVEY Seq.001.004
Ver. 5.2 Annual peak flow frequency analysis Run Date / Time
11/01/2007 following Bulletin 17-B Guidelines 05/08/2009 10:43

Station - 01540500 Susquehanna River at Danville, PA

EMPIRICAL FREQUENCY CURVES -- WEIBULL PLOTTING POSITIONS

WATER YEAR	RANKED DISCHARGE	SYSTEMATIC RECORD	BULL.17B ESTIMATE
1972	345529.0	0.0091	0.0091
2006	260000.0	0.0182	0.0182
1964	248438.0	0.0273	0.0273
1975	244631.0	0.0364	0.0364
1936	225917.0	0.0455	0.0455
2004	220000.0	0.0545	0.0545
1902	219592.0	0.0636	0.0636
1946	218362.0	0.0727	0.0727
1996	209000.0	0.0818	0.0818
2005	202000.0	0.0909	0.0909
1940	201369.0	0.1000	0.1000
1984	194000.0	0.1091	0.1091
1943	190367.0	0.1182	0.1182
1960	187480.0	0.1273	0.1273
1993	187000.0	0.1364	0.1364
1979	178952.0	0.1455	0.1455
1913	173505.0	0.1545	0.1545
1986	173000.0	0.1636	0.1636
1948	171703.0	0.1727	0.1727
1914	168083.0	0.1818	0.1818
1956	165702.0	0.1909	0.1909
1958	160021.0	0.2000	0.2000
1961	158962.0	0.2091	0.2091
1916	158142.0	0.2182	0.2182
1950	157814.0	0.2273	0.2273
1920	153624.0	0.2364	0.2364
1910	149106.0	0.2455	0.2455
1983	149000.0	0.2545	0.2545
1929	147298.0	0.2636	0.2636
1925	146394.0	0.2727	0.2727
1998	143000.0	0.2818	0.2818
1928	140972.0	0.2909	0.2909
1947	139975.0	0.3000	0.3000
1994	139000.0	0.3091	0.3091
1935	138261.0	0.3182	0.3182
1904	133743.0	0.3273	0.3273
2000	132000.0	0.3364	0.3364
1997	130000.0	0.3455	0.3455
2003	130000.0	0.3545	0.3545
1962	129454.0	0.3636	0.3636
1941	128804.0	0.3727	0.3727
1924	128321.0	0.3818	0.3818
1927	128321.0	0.3909	0.3909
1915	127417.0	0.4000	0.4000
1939	126083.0	0.4091	0.4091
1918	125610.0	0.4182	0.4182
1951	124040.0	0.4273	0.4273

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USGS PEAK-FQ

DANVILLE ADJ PEAKFQ.PRT

1991	124000.0	0.4364	0.4364
2008	124000.0	0.4455	0.4455
1963	123743.0	0.4545	0.4545
2007	123000.0	0.4636	0.4636
1905	122899.0	0.4727	0.4727
1901	121995.0	0.4818	0.4818
1909	121092.0	0.4909	0.4909
1952	120252.0	0.5000	0.5000
1922	120188.0	0.5091	0.5091
1903	119284.0	0.5182	0.5182
1912	116573.0	0.5273	0.5273
1970	116128.0	0.5364	0.5364
1977	116128.0	0.5455	0.5455
1989	116000.0	0.5545	0.5545
1999	116000.0	0.5636	0.5636
1976	114224.0	0.5727	0.5727
1945	112914.0	0.5818	0.5818
1978	110417.0	0.5909	0.5909
1908	110248.0	0.6000	0.6000
1942	108248.0	0.6091	0.6091
1957	107943.0	0.6182	0.6182
1932	107537.0	0.6273	0.6273
1933	107537.0	0.6364	0.6364
1959	106049.0	0.6455	0.6455
1971	105658.0	0.6545	0.6545
1981	105000.0	0.6636	0.6636
1980	104000.0	0.6727	0.6727
1987	104000.0	0.6818	0.6818
1968	98994.0	0.6909	0.6909
1974	98043.0	0.7000	0.7000
2001	97800.0	0.7091	0.7091
1953	97528.0	0.7182	0.7182
1900	94885.0	0.7273	0.7273
1923	94885.0	0.7364	0.7364
1973	94806.0	0.7455	0.7455
1966	94140.0	0.7545	0.7545
1921	91271.0	0.7636	0.7636
1926	91271.0	0.7727	0.7727
1944	91077.0	0.7818	0.7818
1906	89915.0	0.7909	0.7909
1992	89200.0	0.8000	0.8000
1934	89102.0	0.8091	0.8091
1911	87927.0	0.8182	0.8182
2002	84700.0	0.8273	0.8273
1937	84403.0	0.8364	0.8364
1917	83951.0	0.8455	0.8455
1949	83612.0	0.8545	0.8545
1988	83500.0	0.8636	0.8636
1982	83300.0	0.8727	0.8727
1967	83289.0	0.8818	0.8818
1955	81336.0	0.8909	0.8909
1931	79975.0	0.9000	0.9000
1969	77768.0	0.9091	0.9091
1954	77738.0	0.9182	0.9182
1995	73700.0	0.9273	0.9273
1919	73017.0	0.9364	0.9364
1938	71751.0	0.9455	0.9455
1930	71119.0	0.9545	0.9545
1990	70900.0	0.9636	0.9636
1907	66329.0	0.9727	0.9727
1985	55300.0	0.9818	0.9818
1965	42739.0	0.9909	0.9909

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DANVILLE ADJ PEAKFQ.PRT

```
End PeakFQ analysis.
  Stations processed :      1
  Number of errors   :      0
  Stations skipped   :      0
  Station years      :    109

Data records may have been ignored for the stations listed below.
(Card type must be Y, Z, N, H, I, 2, 3, 4,  or *.)
(2, 4, and * records are ignored.)

 For the station below, the following records were ignored:

 FINISHED PROCESSING STATION:  01540500     USGS Susquehanna River at Danville


 For the station below, the following records were ignored:

 FINISHED PROCESSING STATION:
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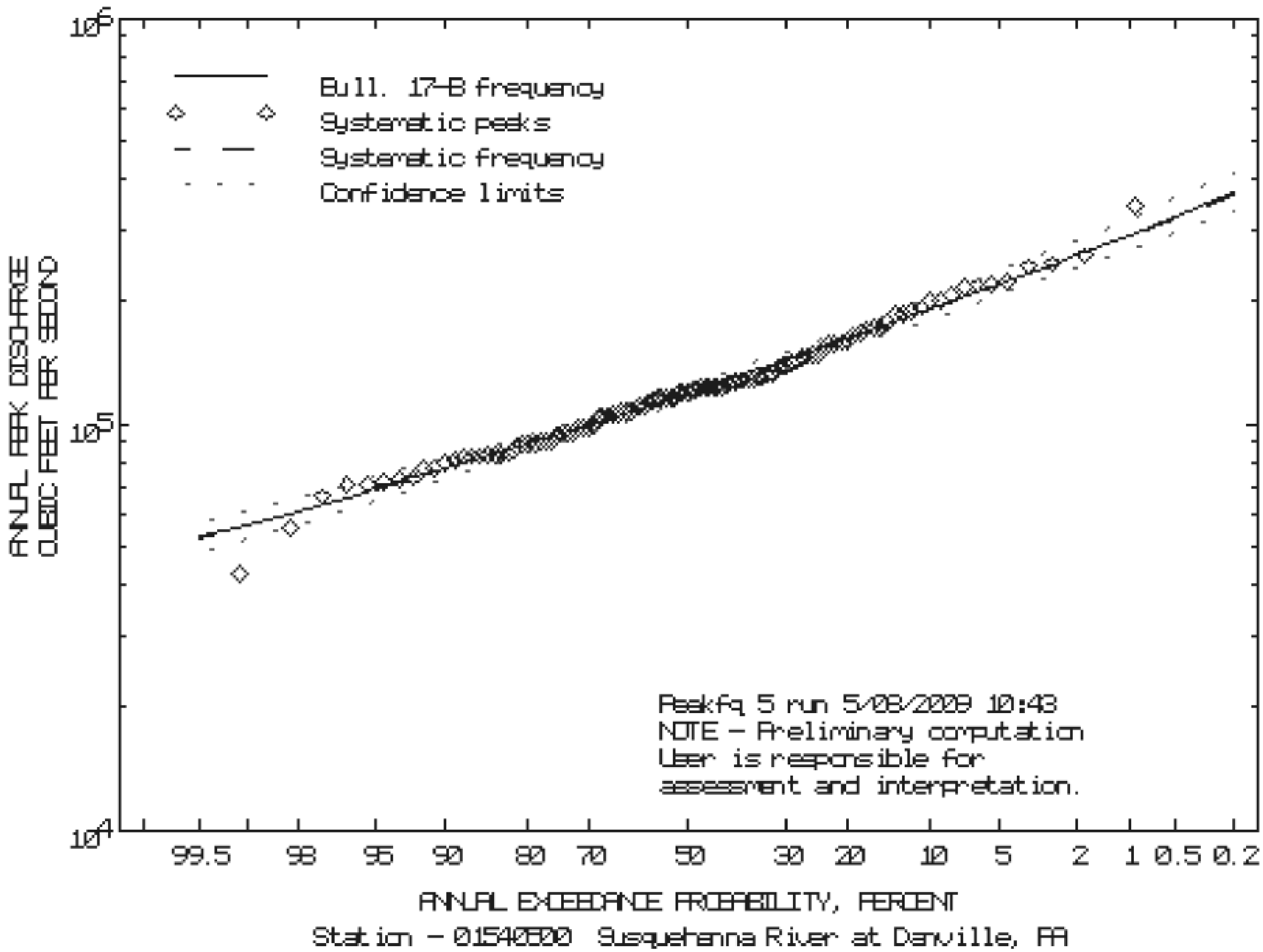
USGS PEAK-FQ


Bull. 17-B frequency
Systematic peaks
Systematic frequency
Confidence limits
Peakfq 5 run 5/08/2009 10:43
NOTE - Preliminary computation
User is responsible for
assessment and interpretation.
ANNUAL PEAK DISCHARGE
CUBIC FEET PER SECOND
10^6
10^5
10^4
99.5 98 95 90 80 70 50 30 20 10 5 2 1 0.5 0.2
ANNUAL EXCEEDANCE PROBABILITY, PERCENT
Station - 01540500 Susquehanna River at Danville, PA

Application to Rainfall