PDHonline Course L105G (12 PDH)

GPS Surveying

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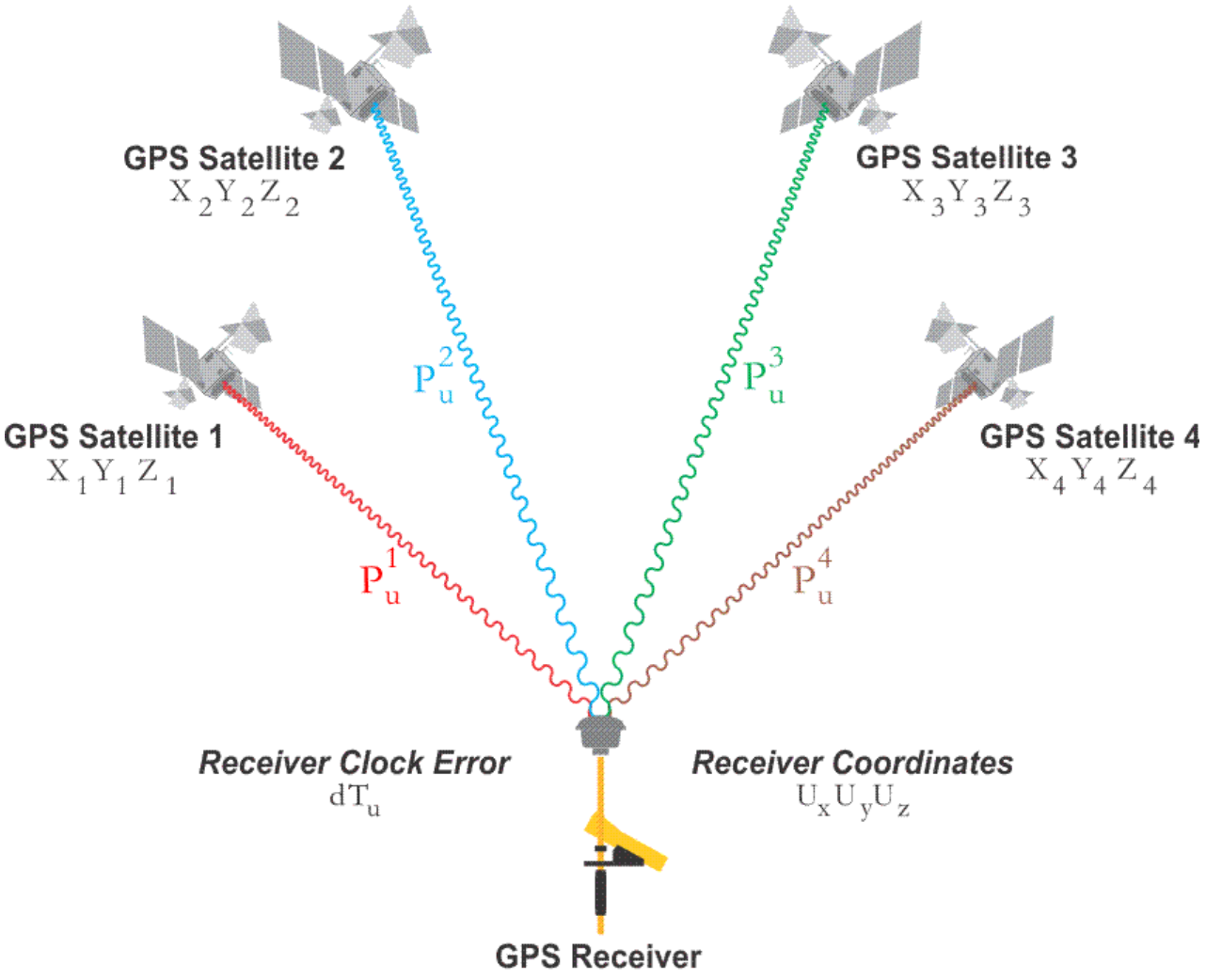
Module 3

Point Positioning

In point positioning, the positions of the satellites are available from the data in their broadcast ephemerides. The satellite clock offset and the ionospheric corrections are also available to a GPS receiver once it has locked on. That is once it has correlated its code with the codes it is receiving. Then it can immediately read the Navigation messages of all four or more satellites it is tracking. But the receiver must assume that all these corrections in the Navigation message are absolutely correct, which is a bit of an exaggeration. In fact, there are errors in these corrections. And along with the timing errors between the receiver and satellite clocks, receiver noise, and any multipath, they still contaminate the position that results. (See module 2 for a more complete discussion of these biases.)

Nevertheless this kind of positioning is the fulfillment of the original idea behind GPS. It relies on a coded pseudorange measurement and can be used for virtually instantaneous positioning. Pseudoranges from the receiver to three GPS satellites provide enough data to solve three Cartesian coordinates, u_x, u_y, and u_z of the receiver. The fourth pseudorange to the fourth satellite provides the information for the solution of the receiver's clock offset, dT_u, as illustrated in the equations below and Figure 3.1.



$$p_u^1 = \sqrt{(X_1 - U_x)^2 + (Y_1 - U_y)^2 + (Z_1 - U_z)^2} + c(dT_u)$$

$$p_u^2 = \sqrt{(X_2 - U_x)^2 + (Y_2 - U_y)^2 + (Z_2 - U_z)^2} + c(dT_u)$$

$$p_u^3 = \sqrt{(X_3 - U_x)^2 + (Y_3 - U_y)^2 + (Z_3 - U_z)^2} + c(dT_u)$$

$$p_u^4 = \sqrt{(X_4 - U_x)^2 + (Y_4 - U_y)^2 + (Z_4 - U_z)^2} + c(dT_u)$$

Figure 3.1

The ability to achieve so much redundancy in the measurement of the receiver's clock error, is one reason the moderate stability of quartz crystal clock technology is entirely adequate as a receiver oscillator.

A unique solution is found here because the number of unknowns is not greater than the number of observations, obviously observing more satellites makes the solution even better. But when the receiver tracks at least four satellites simultaneously; these four equations can be solved simultaneously for every *epoch* of the observation. An epoch in GPS is a very short period of observation time, and is generally just a small part of a longer measurement. However, theoretically there is enough information in any single epoch to solve these equations.

Relative Positioning

When two or more receivers are available relative positioning is possible and the accuracy of GPS positioning improves. Relative positioning can attain higher accuracy than point positioning because of the extensive correlation between observations taken to the same satellites at the same time from separate stations. In other words, two receivers operating simultaneously, collecting signals from the same satellites will record errors, but very similar errors. This is especially true when you consider that the distances between such stations on the earth are very short compared with the 20,000-km altitude of the GPS satellites. Relative positioning generally relies on carrier phase ranging instead of code pseudoranges.

Carrier Phase

Carrier phase is the observable at the center of high accuracy surveying applications of GPS. It is also known as the carrier beat phase observable. It depends on the carrier waves themselves, the unmodulated L1, L2 and L5, rather than their P and C/A codes. There is an interesting approximation that illustrates the potential of carrier phase ranging, " The One-Percent Rule of Thumb."

Assume for the moment most of the errors in the receiver and satellite clocks, receiver noise, ionospheric delay, broadcast ephermeris errors or multipath are gone. Under such a circumstance you could expect pseudoranges to be accurate within about one percent of the chipping rate of the code used, whether it is the P code or the C/A code. In practice, positions derived from these codes are rather less reliable than that, of course, because they must rely on the Navigation message corrections for the most part.

But consider this a P code chip occurs every 0.0978 of a microsecond. So by using this rule, a P code based measurement could have a maximum precision of about 1- percent of about a tenth of a microsecond, or one nanosecond. One nanosecond multiplied by the speed of light is approximately 30 centimeters, 1 percent of the length of a single P-code chip.

The C/A code based pseudorange is 10 times less precise. Its chipping rate is 10 times slower. A C/A code pseudo-range would have a maximum resolution of about 3 meters, that is 1 percent of the length of a single C/A code chip.

Ok, now take a look at this same 1- percent rule of thumb applied to the carrier phase observable. Instead of a code chip as the basis of the measurement, it is the wavelength of the carrier. How long is one wavelength of the GPS carriers? The length of a single wavelength of each carrier can be calculated using the same formula as was used previously in Module 1.

$$\lambda = \frac{c_a}{f}$$

Where: λ = the length of each complete wavelength in meters;

c_a = the speed of light corrected for atmospheric effects;

f = the frequency in hertz.

The L1-1575.42 MHz carrier transmitted by GPS satellites has a wavelength of approximately 19 cm.

$$\lambda = \frac{c_a}{f}$$

$$\lambda = \frac{300*10^6\, mps}{1575.42*10^6\, Hz}$$

$$\lambda = 0.19m$$

The L2-1227.60 MHz frequency carrier transmitted by GPS satellites has a wavelength of approximately 24 cm.

$$\lambda = \frac{c_a}{f}$$

$$\lambda = \frac{300*10^6 mps}{1227.60*10^6 Hz}$$

$$\lambda = 0.24m$$

The L5-1176.45 MHz frequency carrier transmitted by GPS satellites has a wavelength of approximately 25 cm.

$$\lambda = \frac{c_a}{f}$$

$$\lambda = \frac{300*10^6 mps}{1176.45*10^6 Hz}$$

$$\lambda = 0.25m$$

Therefore, using the wavelength of any GPS carrier as the observable, the measurement resolved to 1 percent of the wavelength would be about 2 mm. It is no surprise then that the carrier phase observable is preferred for the higher precision work most surveyors have come to expect from GPS. However, first there is something called the *cycle ambiguity problem* that needs to be mentioned. We've seen before with the EDM, except it is a little more difficult to solve in GPS

An Illustration of the Cycle Ambiguity Problem

While carrier phase, or carrier beat phase as it is sometimes called, is different than the pseudorange, the basis of the measurements have some similarities. For example, the foundation of a pseudorange measurement is the correlation of the codes received from a GPS satellite with replicas of those codes generated within the receiver. The foundation of the carrier phase measurement is the combination of the unmodulated carrier itself received from a GPS satellite with a replica of that carrier generated within the receiver.

It's like a distance measurement by an EDM. As mentioned earlier an EDM sends a carrier wave to the reflector, and generates an identical internal reference. When the external beam returns from the reflector, it is compared with the reference wave. The difference in phase between the two reveals the fractional part of the measurement, even though the number of complete cycles between the EDM and the reflector may not be immediately apparent. That is until modulated carriers of longer wavelengths are used (See module 1 for a more complete discussion of this process).

Likewise, it is the phase difference between the incoming signal and the internal reference generated inside a GPS receiver that reveals the fractional part of the carrier phase measurement in GPS. The incoming signal is from a satellite rather than a reflector of course, but like an EDM measurement, the internal reference derived from the receiver's oscillator is used for comparison. And just like an EDM measurement it is the difference in phase between the two reveals the fractional part of the range measurement. And as the 1- percent rule of thumb illustrates this comparison is capable of a much more accurate measurement than pseudoranges.

But there is a problem. Again, just like the EDM measurement the number of complete cycles is not immediately known. In other words, when the receiver locks onto the carrier it cannot know how many full wavelengths stand between the satellite and itself at that instant and without that information it isn't possible to know the full distance, of course.

There is one helpful fact about the cycle ambiguity though. It has to be an integer. The full cycles between the receiver and the satellite at lock on must be a whole number. So it is also called the integer cycle ambiguity.

The situation is a lot like an unofficial technique used by some nineteenth century contract surveyors on the Great Plains. The procedure can be used as a rough illustration of the cycle ambiguity problem in GPS.

It was known as the buggy wheel method of chaining. Some of the lines of the public land system that crossed open prairies were originally surveyed by loading a wagon with stones or stakes and tying a cloth to a spoke of the wheel. One man drove the team, another kept the wagon on line with a compass and a third counted the revolutions of the flagged wheel to measure the distance. When there had been enough turns of the improvised odometer to measure half a mile they set a stone or stake to mark the corner and then rolled on, counting their way to the next corner.

A GPS receiver is like the man assigned to count the turns of the wheel. He is supposed to begin his count from the moment the crew leaves the newly set corner, but instead, suppose he jumps into the wagon, gets comfortable and takes an unscheduled nap. When he wakes up the wagon is on

the move. Trying to make up for his laxness, he immediately begins counting. But at that moment the wheel is at a half turn, a fractional part of a cycle. He counts the subsequent half turn and then, back on the job, he intently counts each and every full revolution as they come around. His tally grows as the cycles accumulate, but he is in trouble and he knows it. He cannot tell how far the wagon has traveled; he was asleep for the first part of the trip. He has no way of knowing how far they had come before he woke up and started counting. He is like a GPS receiver that cannot know how far it is from the satellite when it starts counting phase cycles. They can tell it nothing about how many cycles stood between itself and the satellite when the receiver was locked on and began tracking. Those unknown cycles are the cycle ambiguity. The 360° cycles in the carrier phase observable are wavelengths λ, not revolutions of a wheel. But there is a solution to the cycle ambiguity problem. It is called *differencing* .

Differencing

In GPS the word *differencing* has come to represent several types of simultaneous baseline solutions of combined measurements. Now, obviously, differencing is only possible using two or more receivers and relative positioning. And it usually involves carrier phase, or carrier beat phase, measurements. The lines between the pairs of receivers are called vectors or baselines. In these illustrations only two receivers are used, but there could certainly be more and the principles would be the same. OK, the most frequently used differencing methods are known as the *single difference*, *double difference*, and *triple difference*.

Single Difference

A single difference, also known as a *between-receivers difference*, can refer to the difference in the simultaneous carrier phase measurements from one GPS satellite as measured by two different receivers (Figure 3.2)

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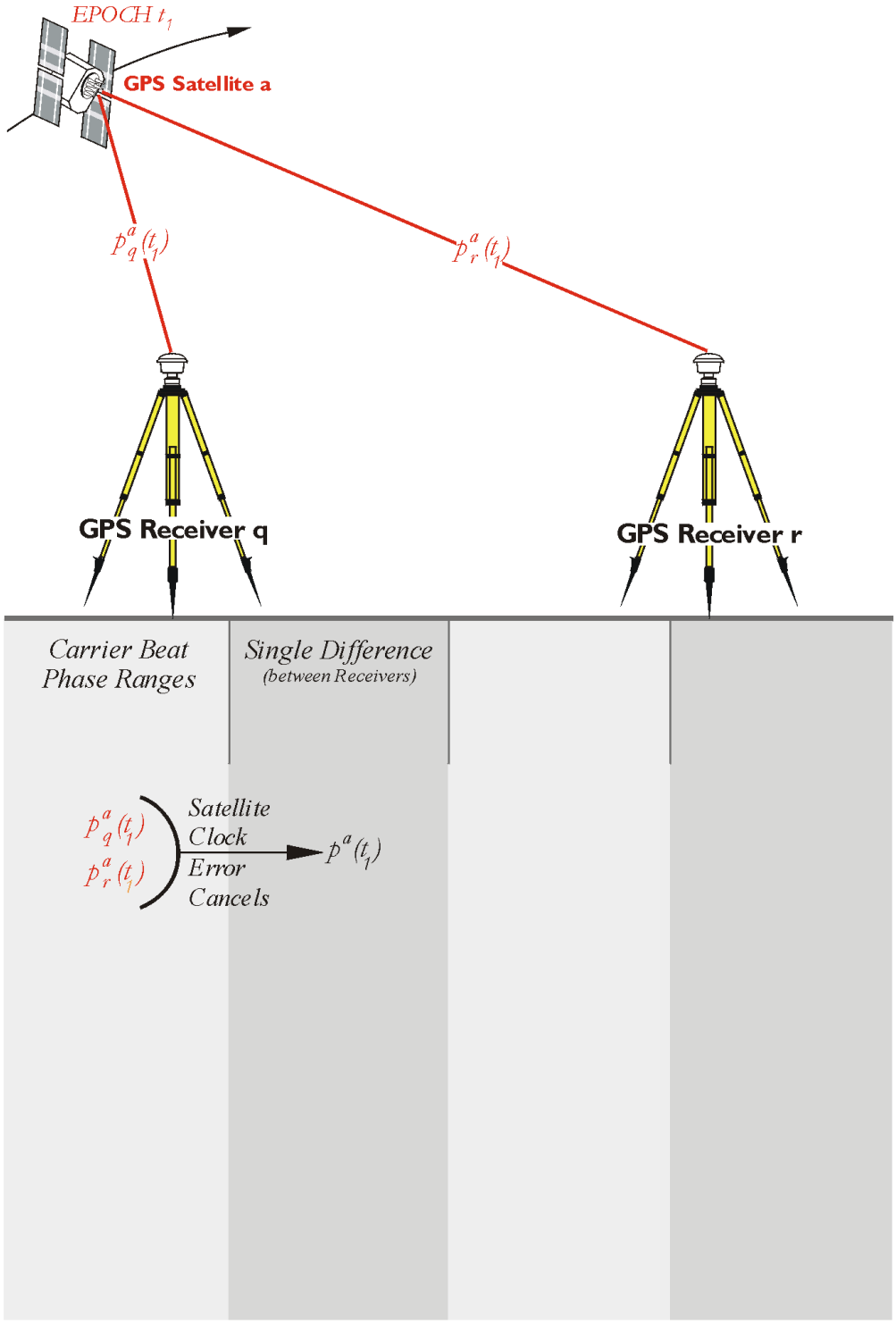


Carrier Beat Phase Measurements from One GPS as Measured by Two Different Receivers

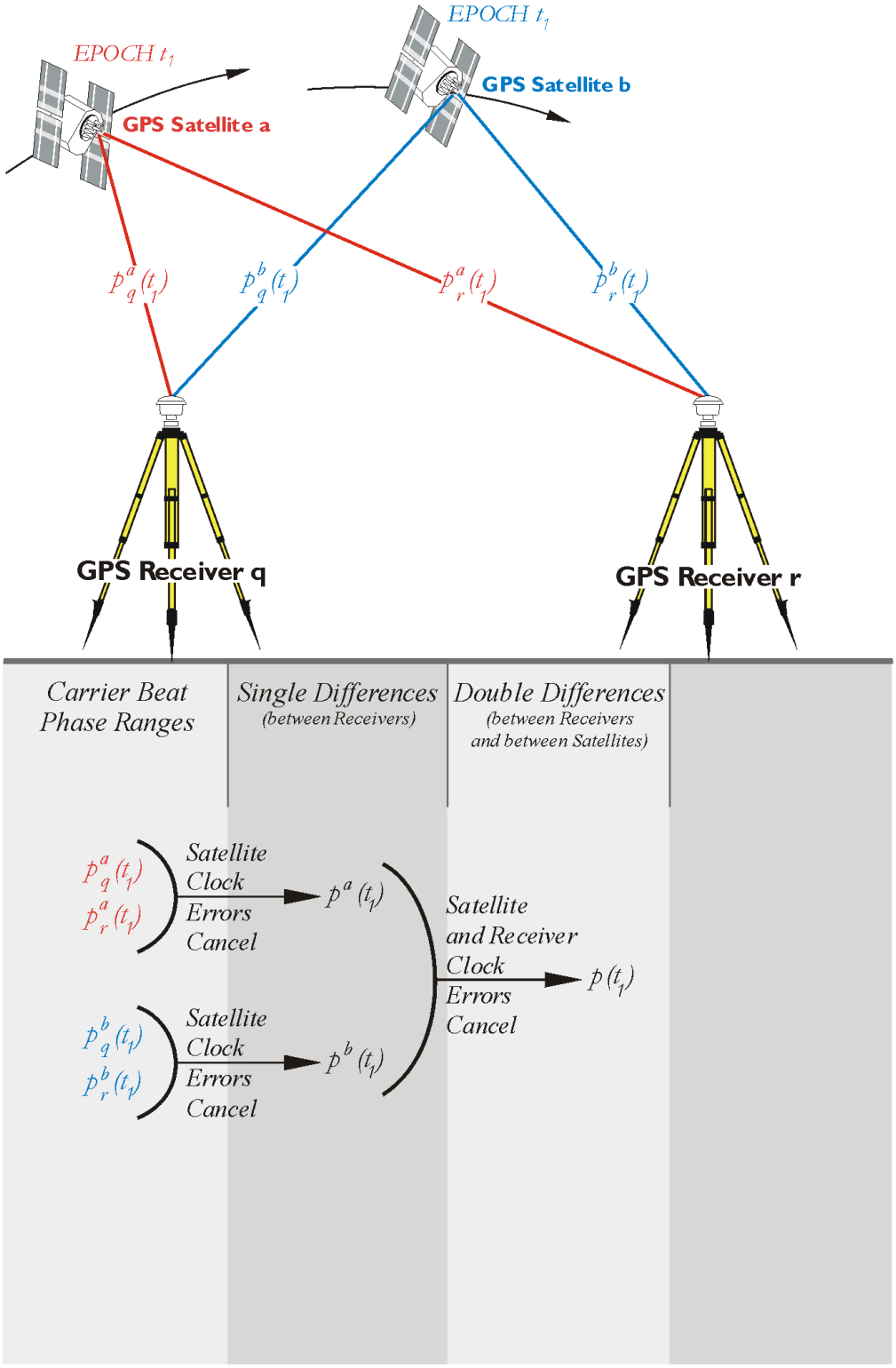
Figure 3.2

Since the two receivers are both observing the same satellite at the same time the satellite clock error is eliminated in a single-difference. It is a bonus that the atmospheric biases and the orbital errors recorded by the two receivers in this solution are also nearly identical, and can be virtually eliminated too.

Unfortunately, there are still two factors in the carrier beat phase observable that are not eliminated by single differencing. The difference between the integer cycle ambiguities at each receiver and the difference between the receiver clock errors remain.

Double Difference

There is a GPS solution that will eliminate the receiver clock errors. It involves the addition of what might be called another kind of single difference, also known as a *between-satellites difference*. This term refers to the difference in the measurement of signals from two GPS satellites, as measured simultaneously at a single receiver (Figure 3.3).



Carrier Beat Phase Measurement from Two GPS Satellites as Measured at Two Receivers.

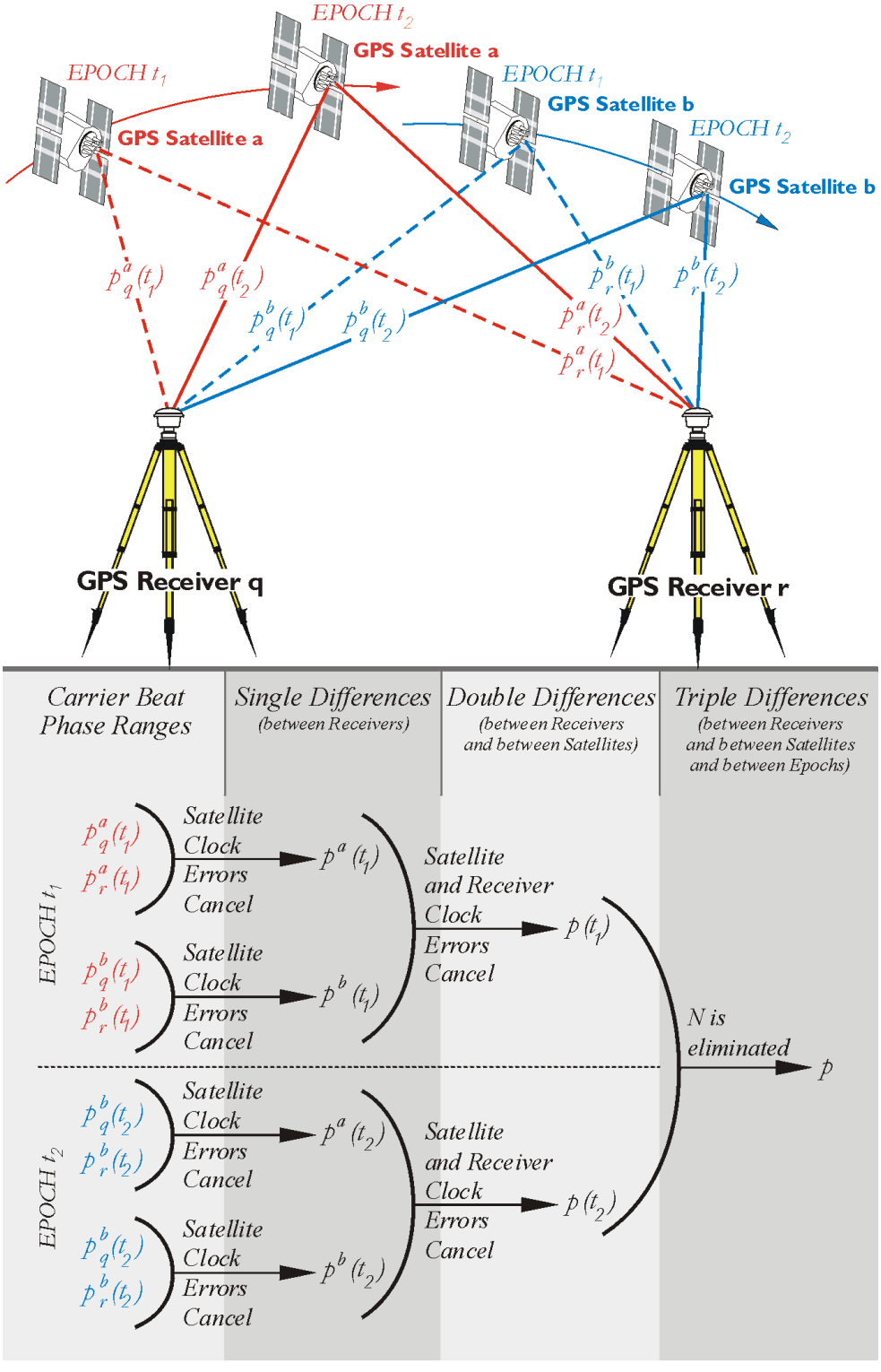
Figure 3.3

The data available from the between-satellites difference allow the elimination of the receiver clock error. In this situation, there can be no difference in the clock since only one receiver is involved in each of the component single differences. And the atmospheric effects on the two satellite signals are again nearly identical as they come into the lone receiver, so the effects of the ionospheric and tropospheric delays are virtually eliminated as well.

By using both the between-receivers difference and the between-satellites difference, a double difference is created. This combination is virtually free of receiver clock errors and satellite clock errors, but still there is one stubborn factor in the carrier beat phase observable that is not eliminated. The integer cycle ambiguity is still in there.

Triple Difference

Combining two double differences creates A third kind of differencing. Each of the double differences involves two satellites and two receivers. The difference next derived is between two epochs. The triple difference is also known as the *receiver-satellite-time triple difference* (Figure 3.4), the difference of two double differences of two different epochs.



The Receiver-Satellite-Time Triple Difference

Figure 3.4

In the triple difference two receivers observe the same two satellites during two consecutive epochs. This solution can be used to quantify the integer cycle ambiguity because if all is as it should be it is constant over the two observed epochs.

Actually, a triple difference is not sufficiently accurate for short baselines. It is used to find the integer cycle ambiguity. Once the cycle ambiguity is determined it can be used with the double difference solution to calculate the actual carrier phase measurement. Here's how it works, from the moment of a receiver's lock onto a particular satellite; there are actually three components to the total carrier phase observable.

$$\phi = \alpha + \beta + N$$

$$where:$$

$$\phi = total\ phase$$

$$\alpha = fractional initial\ phase$$

$$\beta = observed cycle\ count$$

$$N = cycle\ amiguity$$

First is the *fractional initial phase*, which occurs at the receiver at the first instant of the lock-on. The receiver starts tracking the incoming phase from the satellite. The receiver grabs onto the satellite's signal at some fractional part of a phase. It is interesting to note that this fractional part does not change for the duration of the observation and so is called the fractional initial phase. It is symbolized here by α.

Second is the integer number of full cycles of phase that occur from the moment of the lock to the end of the observation. It is symbolized by β, the observed cycle count. This element is the receiver's consecutive counting of the change in full phase cycles, 1, 2, 3, 4 . . ., between the receiver and the satellite as the satellite flies over. Of the three terms, β is only number that changes - that is, if the observation proceeds correctly.

Third is the integer cycle ambiguity *N*. It represents the number of full phase cycles between the receiver and the satellite at the first instant of the receiver's lock-on. N does not change from the moment of the lock onward, unless that lock is lost. (Figure 3.5)

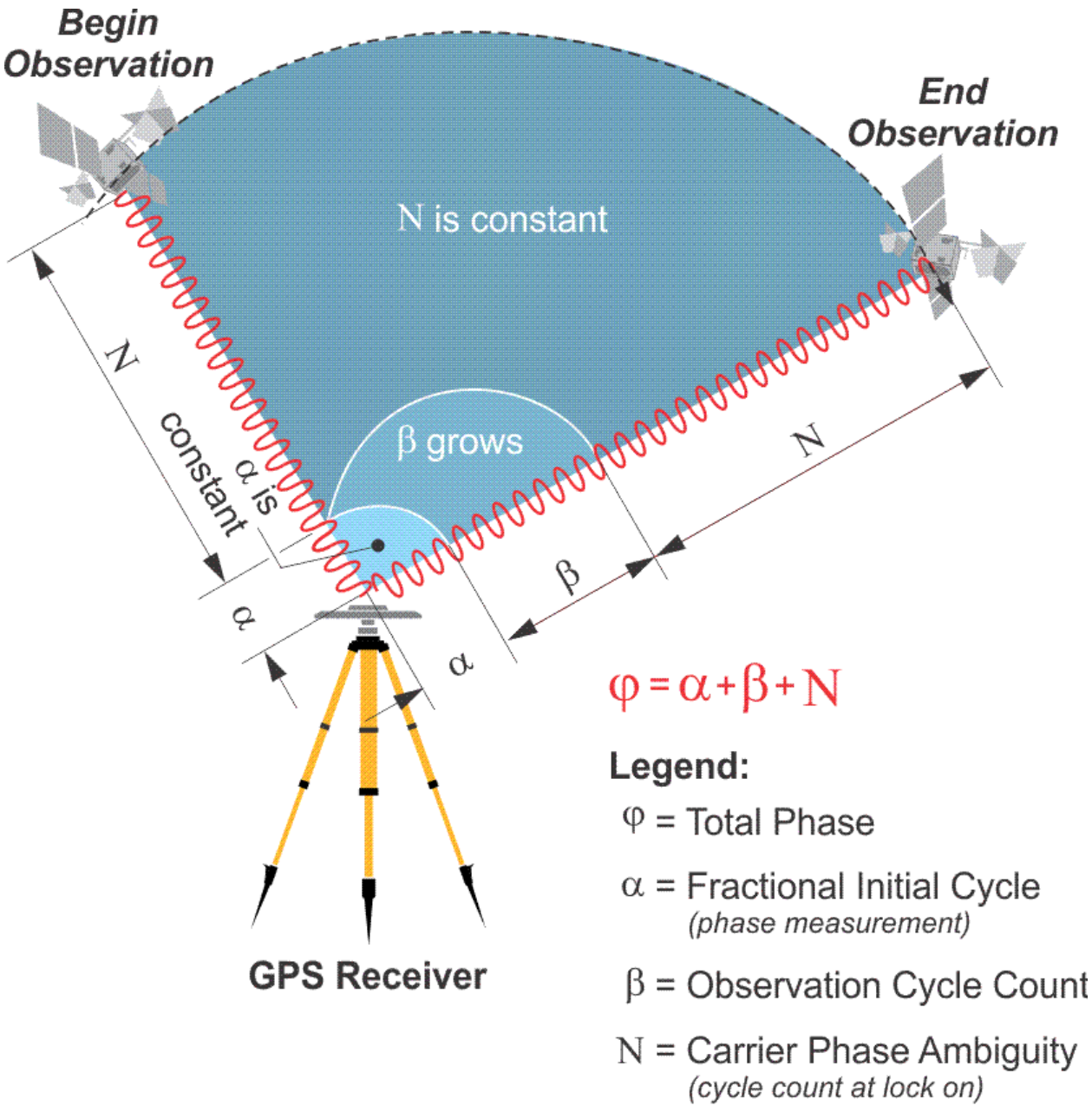

Begin Observation
End Observation
N is constant
N
constant
α is
β grows
N
β
α
α
GPS Receiver
φ = α + β + N
Legend:
φ = Total Phase
α = Fractional Initial Cycle
(phase measurement)
β = Observation Cycle Count
N = Carrier Phase Ambiguity
(cycle count at lock on)

Figure 3.5

In other words, the total carrier phase observable consists of two values that do not change during the observation, the fractional phase α, and the integer cycle ambiguity N. Only the observed cycle count β, changes, unless there is a cycle slip.

Cycle Slips

In fact, the triple difference makes the detection and elimination of cycle slips relatively easy. A *cycle slip* is a discontinuity in a receiver's continuous phase lock on a satellites signal. The coded pseudorange measurement is immune from this difficulty, but the carrier beat phase is not. (Figure 3.6)

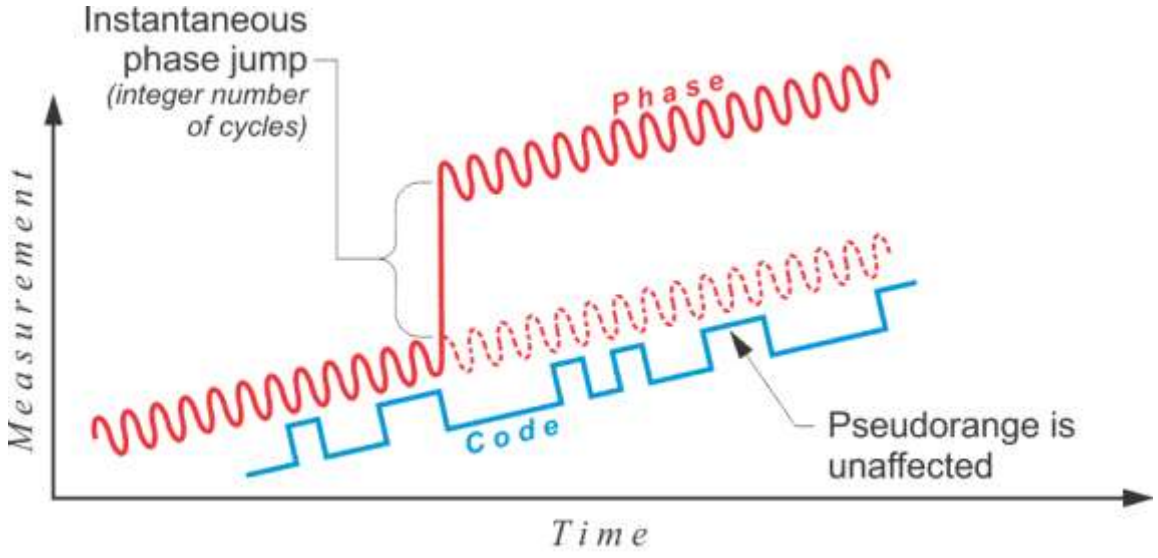


Figure 3.6

When lock is lost a cycle slip occurs. A power loss, an obstruction, a very low signal-to-noise ratio, or any other event that breaks the receiver's continuous reception of the satellite's signal causes a cycle slip. That is, the receiver loses its place in its count of the integer number of cycles β and, as a result, N is completely lost and the receiver has to start over from scratch.

There are several methods that may be used to regain a lost integer phase value, N. The triple difference is one of the better alternatives in this regard, as stated earlier the triple difference does not depend on the initial integer ambiguity, because it is a constant in time. Therefore, when a large residual does appear in its component double differences it is very likely caused by a cycle slip. Even better the obstructed signal can be singled-out by isolating all available satellite pairs until the problem is found. This utility in fixing cycle slips is the primary appeal of the triple difference. It can be used as a preprocessing step to weed out cycle slips and provide a first position for the receivers.

Summary

Relative positioning by carrier phase measurement is the primary vehicle for high-accuracy GPS surveying. Simultaneous observations, double differencing in postprocessing, and the subsequent construction of networks from GPS baselines are the hallmarks of geodetic and control work in the field. The strengths of these methods generally outweigh their weaknesses, particularly where there can be an unobstructed sky and relatively short baselines and where the length of observation sessions is not severely restricted.

However, conditions are not always so ideal. Where obstructions threaten to produce cycle slips, coded pseudorange measurements may offer an important advantage over carrier phase. Pseudorange measurements also may be preferred where accuracy requirements are low and production demands are high.

Differencing is an ingenious approach to minimizing the effect of errors in carrier phase ranging. It is a technique that largely overcomes the impossibility of perfect time synchronization. Double differencing is the most widely used formulation. Double differencing still contains the initial integer ambiguities, of course. And the estimates of the ambiguities generated by the initial processing are usually not integers, in other words, some orbital errors; atmospheric errors, etc. remain. But with the knowledge that the ambiguities ought to be integers, during subsequent processing it is possible to force estimates for the ambiguities that are in fact integers. When the integers are so fixed, the results are known as a *fixed solution*, rather than a *float solution*. It is the double differenced carrier phase based fixed solution that makes the very high accuracy possible with GPS.

However, in this discussion of errors it is important to remember that multipath, cycle slips, incorrect instrument heights, and a score of other errors whose effects can be minimized or eliminated by good practice are simply not within the purview of differencing at all. The unavoidable biases that can be managed by differencing - including clock, atmospheric, and orbital errors - can have their effects drastically reduced by the proper selection of baselines, the optimal length of the observation sessions, and several other considerations included in the design of a GPS survey. But such decisions require an understanding of the sources of these biases and the

conditions that govern their magnitudes. The adage of, "garbage in, garbage out," is as true of GPS as any other surveying procedure. The management of errors cannot be relegated to mathematics alone.