



PDHonline Course M298 (3 PDH)

Mechanical Considerations for Servo Motor and Gearhead Sizing

Instructor: Chad A. Thompson, P.E.

2020

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

An Approved Continuing Education Provider

Mechanical Considerations for Servo Motor and Gearhead Sizing

Chad A. Thompson, P.E.

Course Content

Introduction

This course is intended to familiarize the designer with the mechanical issues involved when specifying a rotary servo motor and gearhead combination. While there are other things that should be taken into account when selecting a servo system, such as available electrical power and, as always, component cost, this course will concentrate on the mechanical considerations for the servo motor and gearhead.

Servo motors are most often used in industrial applications where there is a need for quick, accurate, and variable motion. Constant positional feedback to a controller, which is the “brains” of a servo motor, allows constant dynamic adjustment of the motor torque and speed in order to maintain the desired motion. While there are other types of servos available, this course will focus on the traditional rotary servo motor.

In addition, the designer should understand that, although many types of gearheads are available, servo applications usually require gearheads which are specifically made for servos. These gearheads are designed to withstand the rigors of sudden changes in the speed and direction of the motor, as well as meet the accuracy and repeatability requirements needed in a typical servo application. This is the general type of gearhead being considered for this course.

In this course, we will first discuss the basic application parameters that are needed to select the components of a servo/gearhead system, then go through the procedure for making these selections. It often becomes an iterative process to determine which combination of motor and gearhead is appropriate for a given application. It comes down to making some educated guesses as to which combination of motor and gearhead might work, evaluating whether those components meet the application requirements, then making adjustments to the selections as needed to achieve the desired results.

Finally, we will run through a detailed example where we will select a servo and gearhead for a specific application.

Evaluating Application Parameters

The first step towards determining an appropriate servo/gearhead combination for a given application is to define the mechanical characteristics of the payload. The primary mechanical parameter for a rotating payload is the *polar mass moment of inertia*. This value is analogous to the mass of an object in the familiar formula $F=ma$, where F is an applied force, m is the mass of an object, and a is the resulting acceleration of the object. In the case of a rotating object, the formula becomes $T=J\alpha$, where T is an applied torque, J is the polar mass moment of inertia, and α is a resulting angular acceleration.

A discussion of how to calculate J for a particular payload is beyond the scope of this course. Many sources are available, including *Machinery's Handbook*, which explain this procedure. Most modern solid modeling software packages will evaluate J automatically, assuming of course the appropriate data has been entered into the program. Lacking this, it is usually in the designer's best interest to mentally simplify the payload as much as possible in order to avoid overly complex manual calculations. Good estimates of mass concentrations and where those mass concentrations are located in relation to the axis of rotation are the critical parameters here.

The other necessary determination to be made is to define the motion profile through which the payload must be rotated. Most servo programming software automates, at least to some extent, the determination of the exact motion profile. The programmer simply tells the software where the motion should start, where it should end, and how long it should take to get there. Other parameters, such as the maximum speed at which the motor should run, may also be specified. It is a good idea, however, for the mechanical designer to at least have a basic understanding of the motion profile being produced by the servo motor so that loads on the gearhead can be properly evaluated.

Evaluating Gearhead Output Requirements

Once the basic parameters of payload moment of inertia and desired motion profile have been determined, it is then necessary to define how these parameters translate into torque and speed requirements for the output of the gearhead. Gearhead specifications typically list, among many other things, ratings for *peak output torque*, *continuous output torque*, *peak output speed*, and *continuous output speed*. Manufacturers use different terminology for these values, such as *allowable intermittent speed*, or *allowable acceleration/deceleration torque*, but most servo gearhead specifications list values that correspond to these basic parameters. As a first step in gearhead selection, these ratings should be compared to the application requirements.

The *peak torque rating* should be compared to the maximum torque seen on the gearhead output during normal operation. This is usually when the payload is being accelerated or decelerated, and is assumed by the gearhead manufacturer to be of short duration, typically a second or less. This rating is not to be confused with the *momentary maximum torque rating* of the gearhead, discussed later.

The *continuous torque rating* of a gearhead should be compared to the r.m.s. (root mean square) torque, or T_{RMS} , required, calculated with the formula

$$T_{RMS} = [(T_1^2 t_1 + T_2^2 t_2 + \dots + T_n^2 t_n) / (t_1 + t_2 + \dots + t_n)]^{1/2},$$

where t is time. Use of this formula will be clarified later in the application example.

The *peak speed rating* is to be compared to the maximum rotational velocity of the payload, and the *continuous speed rating* should be evaluated against the r.m.s. speed of the application. Usually the average speed is an acceptable approximation for this value. Again, this will be clarified further in the application example to follow.

Other ratings for the gearhead should be evaluated as well for a particular application. The capacity ratings for the internal bearings in the gearhead should be compared to actual loads placed on the gearhead, and the *torsional rigidity* and *backlash* of the gearhead should be evaluated against the accuracy requirements of the application. Also, gearhead specifications often include a *momentary maximum allowable torque*, which usually comes into play during an emergency stop or mechanical jam, when the gearhead experiences an unexpected and abrupt stop that is not part of normal operation.

Selecting A Gear Ratio

As with any motor/gearhead combination, servo or not, the gear ratio determines the relationship between the output of the motor and the output of the gearhead. The following standard equations are applicable.

$$\omega_{INPUT} = \omega_{OUTPUT} * N$$

$$T_{INPUT} = (T_{OUTPUT} + T_{NOLOAD}) / (N * \eta)$$

where ω_{INPUT} = rotational speed of the gearhead input

ω_{OUTPUT} = rotational speed of the gearhead output

T_{INPUT} = torque on the input side of the gearhead

T_{OUTPUT} = torque on the output side of the gearhead

T_{NOLOAD} = additional output torque required due to
internal inertia of the gearhead

N = gear ratio

η = gearhead efficiency

For a servo application, another function of the gear ratio needs to be considered, the *reflected inertia*. This is the polar mass moment of inertia of the payload as it is translated back through the gearhead to the motor. It is calculated using the equation

$$J_{REFLECTED} = J_{PAYLOAD} / N^2.$$

As you can see from this equation, since the reflected inertia is a function of the square of the gear ratio, a change in gearing can, for better or worse, greatly impact the ability of a servo motor to carry out its desired function. This value becomes important when evaluating the *inertia ratio* of the system, discussed later.

A gear ratio must be selected which meets all the requirements for torque, speed and inertia. The parameters for the servo motor itself must also be evaluated before a final determination for the gear ratio can be made.

Evaluating Servo Motor Requirements

In addition to meeting the torque and speed requirements of the application, the servo motor must also meet the requirements for the *inertia ratio* of the system. This is the ratio between the moment of inertia of the rotor of the servo motor and the moment of inertia that the output shaft of the motor “sees” from the masses it must rotate. This ratio is important because it is a measure of how well the system can perform during the sudden starts, stops, reversals, etc. that are common with servo applications. The inertia ratio R is determined with the equation

$$R = (J_{\text{REFLECTED}} + J_{\text{INPUT}}) / J_{\text{ROTOR}}.$$

J_{INPUT} represents the moment(s) of inertia of anything being rotated by the motor on the input side of the gearhead, that is, any mass that is directly rotated by the motor shaft and not being translated through the gearhead. One common component which fits this description is a shaft coupling. Couplings are often employed to accurately translate motion from the motor shaft to the gearhead input shaft while accommodating slight misalignments between the two. This prevents undue side loads from being placed on either shaft, as well as provides an easily modified component to accommodate various motor shaft sizes.

The moment of inertia for the rotor of the servo motor, J_{ROTOR} , is generally listed in the specifications for a servo motor. If it is not listed, consult the manufacturer. They should have this value readily available, as it is a commonly used parameter for servos.

As a rule of thumb, an inertia ratio of 10:1 or less is desirable for a servo application. For a particularly demanding application, this ratio might need to be significantly lower, even down to a 1:1 ratio. Conversely, a less demanding application might accept a higher ratio. Consultation with the servo manufacturer is recommended to aid the designer in selecting an appropriate inertia ratio for a given application.

A wide range of servo motors are available for use with both AC and DC current, various voltages, and with different types of controllers. A discussion of all the various types of servo systems is beyond the scope of this course, but the wide variety of electrical options available for servos generally means that the application dictates the servo power requirements, and not vice versa.

Also, integral brakes are often available for servo motors which can be used when there is a need to hold a load in place for an extended period of time. This prevents unnecessary wear and tear on the servo which would result from applying a continuous torque to hold the load.

Evaluating The Complete System

Once the designer runs through an evaluation of the initial servo/gearhead combination, it may be necessary to try different motors and/or gearheads if all of the application requirements are not met. For instance, if the torque and speed requirements are met, but the inertia ratio is too high, it may be necessary to specify a larger motor or increase the gear ratio.

There may be other limitations that prevent a particular motor from working with a particular gearhead. Sometimes physical size precludes a particular combination, even though all other specifications meet the needs of the application. For instance, it may be impractical for a large motor to be adapted to a small gearhead.

Application Example

Let us now go through a simple representative application, to see how the concepts previously discussed might be applied in the real world. Figure 1 illustrates a typical servo gearhead mounted to a fixed surface. A payload is attached to the output flange of the gearhead. A servo motor is attached to input side of the gearhead by means of an adapter flange, and the motion of the motor shaft is transmitted to the input shaft of the gearhead by means of a coupling.

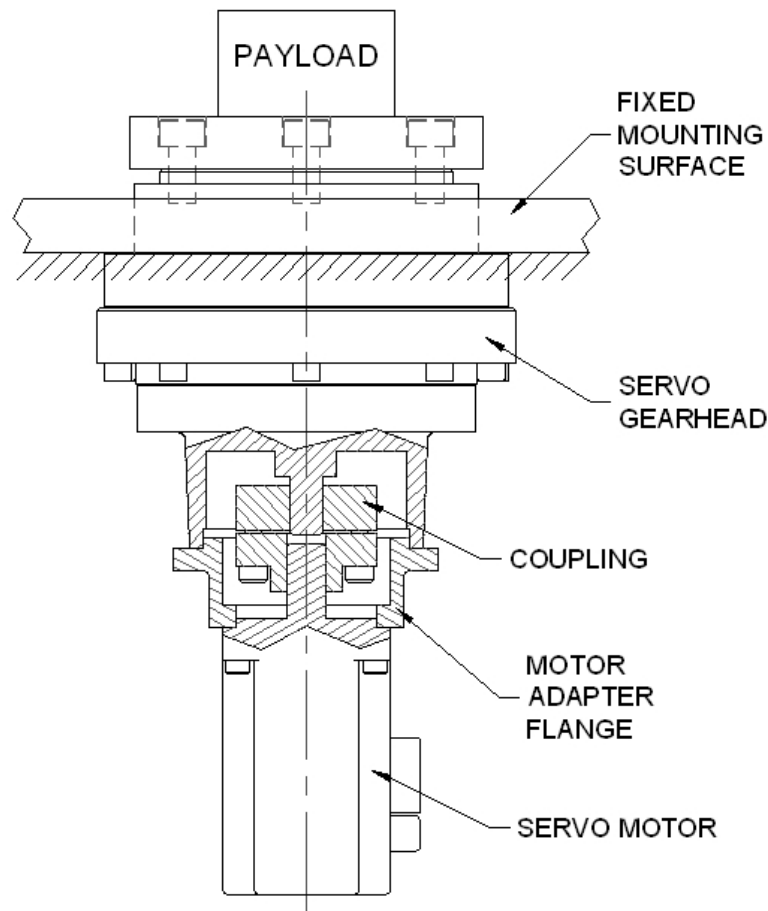


Figure 1

We will say that the purchasing department has mandated that we use servo motors from the fictitious Heaton Servo Company, and likewise our gearheads must come from the make-believe Gitten Gearhead Company. Appendix A shows the available models from each company with their relevant specifications. We will be referring to these as we go through the application. Let us also say for the sake of simplicity that every Heaton servo motor listed can be adapted to every Gitten gearhead listed.

For our example, we will say that the polar mass moment of inertia of the payload attached to the gearhead output flange has been evaluated and determined to be

$$J_{\text{PAYLOAD}} = 25 \text{ kg-m}^2.$$

Let us also say that our desired motion profile for the payload is as illustrated in Figure 2.

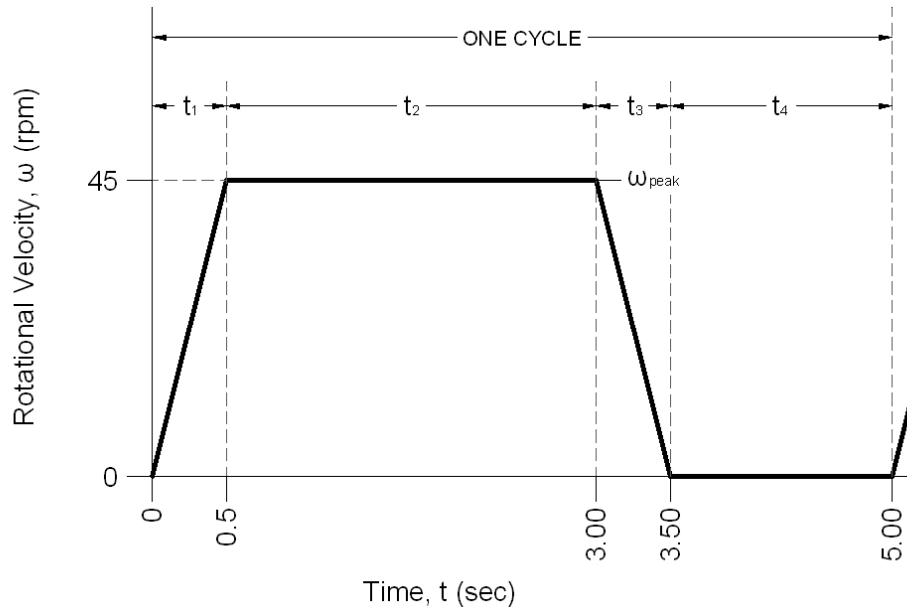


Figure 2

In addition, the servo motor is required to rotate the payload through the desired motion profile repeatedly for an indefinite amount of time. That is to say, the motion profile represents one cycle, and this cycle repeats indefinitely.

Finally, let us say that an inertia ratio of 10:1 or less is appropriate for this application.

To find the required *peak output torque*, that is the torque required on the output flange of the gearhead to accelerate the payload from 0 to 45 rpm in 0.5 seconds, simply use the standard equations of motion $T = J\alpha$ and $\alpha = \omega/t$, or $T = J\omega/t$. (Note: $(45 \text{ rpm})(2\pi \text{ rad/rev}) / (60 \text{ sec/min}) = 4.71 \text{ rad/sec}$.)

$$\begin{aligned} T_{\text{PEAK,OUT}} &= J_{\text{PAYLOAD}} * (\omega_{\text{PEAK}} - 0) / t_1 \\ &= (25 \text{ kg-m}^2) * (4.71 \text{ rad/sec}) / (0.5 \text{ sec}) \end{aligned}$$

$$T_{\text{PEAK,OUT}} = 236 \text{ N-m}$$

Note that since deceleration also occurs over the same amount of time as acceleration, the required torque to decelerate is equal to and opposite the acceleration torque.

Other factors besides the payload inertia usually come into play when evaluating output torque requirements for many applications. However, as is the case for many real applications, we will consider these other factors to be negligible for this example. This is not to be confused with factors within the motor/gearhead system, such as inefficiencies in the gearhead and the use of a coupling. These will be considered later when calculating the necessary motor output torque. The required output torque profile would look like Figure 3.

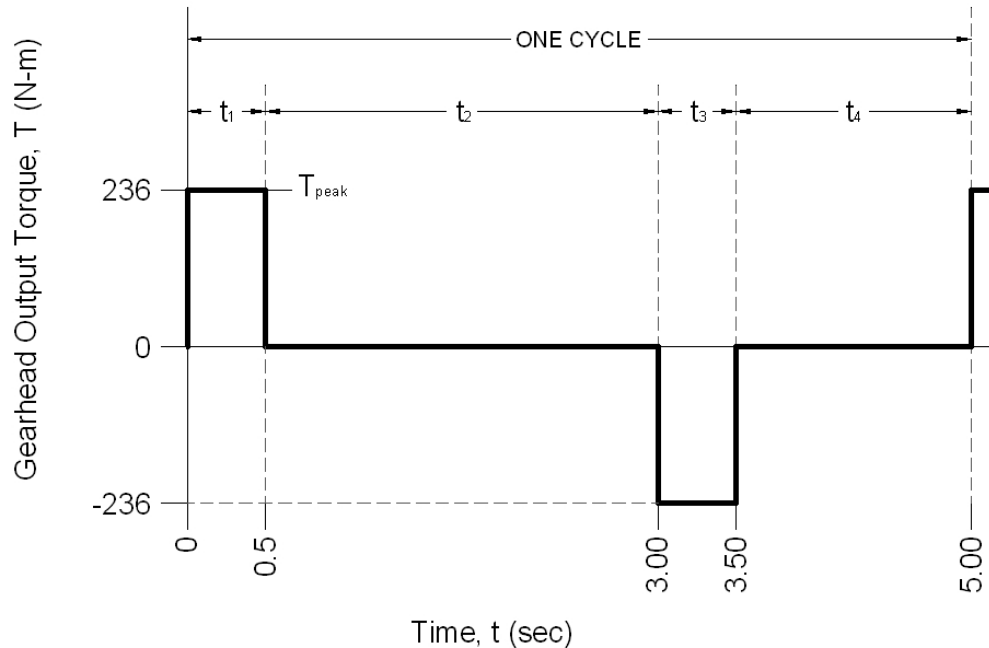


Figure 3

The required *continuous output torque* is equivalent to the r.m.s. output torque. Using the equation mentioned in the *Evaluating Gearhead Output Requirements* section, the continuous output torque is

$$T_{\text{CONT,OUT}} = \left[\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4} \right]^{1/2}$$

$$T_{\text{CONT,OUT}} = \left[\frac{(236 \text{ N-m})^2(0.5 \text{ sec}) + (0)^2(2.5 \text{ sec}) + (236 \text{ N-m})^2(0.5 \text{ sec}) + (0)^2(1.5 \text{ sec})}{0.5 \text{ sec} + 2.5 \text{ sec} + 0.5 \text{ sec} + 1.5 \text{ sec}} \right]^{1/2}$$

$$T_{\text{CONT,OUT}} = 106 \text{ N-m}$$

Note that we have used the absolute value for the torque during each time period, because the power requirements to rotate the load are the same, regardless of which direction it is turning.

The *peak output speed* is already defined in the motion profile as

$$\omega_{\text{PEAK,OUT}} = 45 \text{ rpm}$$

The *continuous output speed* is approximated by finding the average speed using

$$\omega_{\text{CONT,OUT}} = \frac{0.5\omega_1t_1 + \omega_2t_2 + 0.5\omega_3t_3 + \omega_4t_4}{t_1 + t_2 + t_3 + t_4}$$

$$\omega_{\text{CONT,OUT}} = \frac{(0.5)(45 \text{ rpm})(0.5 \text{ sec}) + (45 \text{ rpm})(2.5 \text{ sec}) + (0.5)(45 \text{ rpm})(0.5 \text{ sec}) + (0)(1.5 \text{ sec})}{0.5 \text{ sec} + 2.5 \text{ sec} + 0.5 \text{ sec} + 1.5 \text{ sec}}$$

$$\omega_{\text{CONT,OUT}} = 27 \text{ rpm}$$

So, to summarize our relevant required output torques and speeds:

peak output torque	$T_{\text{PEAK,OUT}} = 236 \text{ N-m}$
continuous output torque	$T_{\text{CONT,OUT}} = 106 \text{ N-m}$
peak output speed	$\omega_{\text{PEAK,OUT}} = 45 \text{ rpm}$
continuous output speed	$\omega_{\text{CONT,OUT}} = 27 \text{ rpm}$

Upon examination of our available gearheads in Appendix A, it appears that Gitten Model #G200, with a peak torque rating of 412 N-m, a continuous torque rating of 167 N-m, a peak speed rating of 75 rpm, and a continuous speed rating of 45 rpm, is the smallest gearhead which meets our requirements. We have yet to determine what gear ratio we need, however.

It would also be appropriate at this point to verify that the load capacity and accuracy of the gearhead meet the application requirements. Possible jam and emergency stop scenarios should be evaluated and compared to the momentary maximum torque rating of the gearhead as well. We will assume for this example that these criteria have been met.

To start getting an idea of what gear ratio we need, let's take a look at what the reflected inertia from the payload would be if we used the gear ratio right in the middle of our range of choices, the 81:1 ratio. With this ratio, our reflected inertia would be

$$J_{\text{REFLECTED}} = 25 \text{ kg-m}^2 / (81)^2 = 38.1 \times 10^{-4} \text{ kg-m}^2.$$

Disregarding the coupling for now, if we are shooting for a 10:1 inertia ratio or less, this would mean that we would need a moment of inertia for the rotor of the motor of at least $3.81 \times 10^{-4} \text{ kg-m}^2$.

Looking at our list of available Heaton Servo motors in Appendix A, this value exceeds the rotor inertia for even the largest motor. It is clear we need to look at a higher gear ratio. Let's try the next one, 105:1.

$$J_{\text{REFLECTED}} = 25 \text{ kg-m}^2 / (105)^2 = 22.6 \times 10^{-4} \text{ kg-m}^2.$$

A 10:1 inertia ratio in this case would mean a rotor inertia of at least $2.26 \times 10^{-4} \text{ kg-m}^2$. This is still obviously too high. Let's try our highest available gear ratio, 161:1.

$$J_{\text{REFLECTED}} = 25 \text{ kg-m}^2 / (161)^2 = 9.65 \times 10^{-4} \text{ kg-m}^2.$$

Here, a 10:1 inertia ratio would mean a minimum rotor inertia of $0.965 \times 10^{-4} \text{ kg-m}^2$. That looks like a number we can live with, so let's set our gear ratio at

$$N = 161.$$

Let's now see what this means for the torque and speed requirements that the servo motor must supply to the input of the gearhead. Referring to the graph in Appendix A for no-load running torque of the gearhead, the torque increases as the output speed goes up. So, to give ourselves a worst-case scenario, let's use our maximum output speed of 45 rpm, which gives us a no-load torque of about

$$T_{\text{NOLOAD}} = 22 \text{ N-m}.$$

Looking at the graph for the efficiency of the Model #G200, using the continuous output torque and speed of our application should give us a reasonable value for efficiency. So for 106 N-m of torque at 25 rpm, the efficiency appears to be about

$$\eta = 86\%.$$

So, calculating the required *peak input torque* gives us

$$T_{\text{PEAK,IN}} = (T_{\text{PEAK,OUT}} + T_{\text{NOLOAD}}) / (N * \eta)$$

$$T_{\text{PEAK,IN}} = (236 \text{ N-m} + 22 \text{ N-m}) / (161 * 0.86)$$

$$T_{\text{PEAK,IN}} = 1.86 \text{ N-m}$$

Our required *continuous input torque* is

$$T_{\text{CONT,IN}} = (T_{\text{CONT,OUT}} + T_{\text{NOLOAD}}) / (N * \eta)$$

$$T_{\text{CONT,IN}} = (106 \text{ N-m} + 22 \text{ N-m}) / (161 * 0.86)$$

$$T_{\text{CONT,IN}} = 0.92 \text{ N-m}.$$

Required *peak input speed* is

$$\omega_{\text{PEAK,IN}} = N * \omega_{\text{PEAK,OUT}} = 161 * 45 \text{ rpm} = 7245 \text{ rpm.}$$

Required *continuous input speed* is

$$\omega_{\text{CONT,IN}} = N * \omega_{\text{CONT,OUT}} = 161 * 27 \text{ rpm} = 4347 \text{ rpm.}$$

So to summarize our gearbox input requirements, and therefore, our motor requirements:

peak input torque	$T_{\text{PEAK,IN}} = 1.86 \text{ N-m}$
continuous input torque	$T_{\text{CONT,IN}} = 0.92 \text{ N-m}$
peak input speed	$\omega_{\text{PEAK,IN}} = 7245 \text{ rpm}$
continuous input speed	$\omega_{\text{CONT,IN}} = 4347 \text{ rpm}$

Examining our list of available servo motors in Appendix A, it appears that the Heaton Model #S2100 is the smallest motor that meets our input requirements. It has a peak torque rating of 2.78 N-m, a continuous torque rating of 1.12 N-m, a peak speed rating of 8000 rpm, and a continuous speed rating of 6000 rpm.

Let's check our inertia ratio. Looking in Appendix A again, for the Heaton Model #S2100, the rotor inertia is $0.87 \times 10^{-4} \text{ kg-m}^2$. For the Gitten Model #G200 gearhead, the coupling inertia is $1.70 \times 10^{-4} \text{ kg-m}^2$. So our inertia ratio is

$$R = (J_{\text{REFLECTED}} + J_{\text{COUPLING}}) / J_{\text{ROTOR}}$$

$$R = (9.65 \times 10^{-4} \text{ kg-m}^2 + 1.70 \times 10^{-4} \text{ kg-m}^2) / 0.87 \times 10^{-4} \text{ kg-m}^2$$

$$R = 13.0$$

This ratio is a little higher than what we're looking for. Let's see if we can find another choice for the motor. Model #S3100 is the next smallest motor that meets our torque and speed requirements, with a peak torque rating of 5.31 N-m, a continuous torque rating of 1.62 N-m, a peak speed rating of 8000 rpm, and a continuous speed rating of 5500 rpm. Let's check the inertia ratio for this motor, which has a rotor inertia of $1.36 \times 10^{-4} \text{ kg-m}^2$.

$$R = (9.65 \times 10^{-4} \text{ kg-m}^2 + 1.70 \times 10^{-4} \text{ kg-m}^2) / 1.36 \times 10^{-4} \text{ kg-m}^2$$

$$R = 8.3$$

That looks like a good number.

Let's look at one more issue before we wrap up. The Heaton Model #S3100 servo we have decided on is capable of a peak torque of 5.31 N-m. Rewriting our previous torque equation to see what this gives us on the output torque yields

$$T_{\text{PEAK,OUT}} = (T_{\text{PEAK,IN}})(N)(\eta) - T_{\text{NOLOAD}}$$

$$T_{\text{PEAK,OUT}} = (5.31 \text{ N-m})(161)(0.86) - (22 \text{ N-m}) = 713 \text{ N-m.}$$

Looking back at the ratings for the Gitten Model #G200 gearhead we selected, the rating for peak output torque is 412 N-m. This tells us that the motor is capable of putting more torque into the gearhead than the gearhead is rated to handle.

The gearhead peak torque rating gives us plenty of headroom over the 236 N-m peak torque requirement we calculated. Even if we were off on our estimated payload moment of inertia, or if we decided we wanted to accelerate a little faster, we still have plenty of leeway. It is unlikely that we would need a peak output torque that exceeds the gearhead rating. Still, it is a point of caution. Servo software generally allows the programmer to put a hard limit on motor torque. That would be appropriate in this case.

So, our final selection is the

Gitten Model #G200, 161:1 ratio gearhead

and the

Heaton Model #S3100 servo motor.

Appendix A

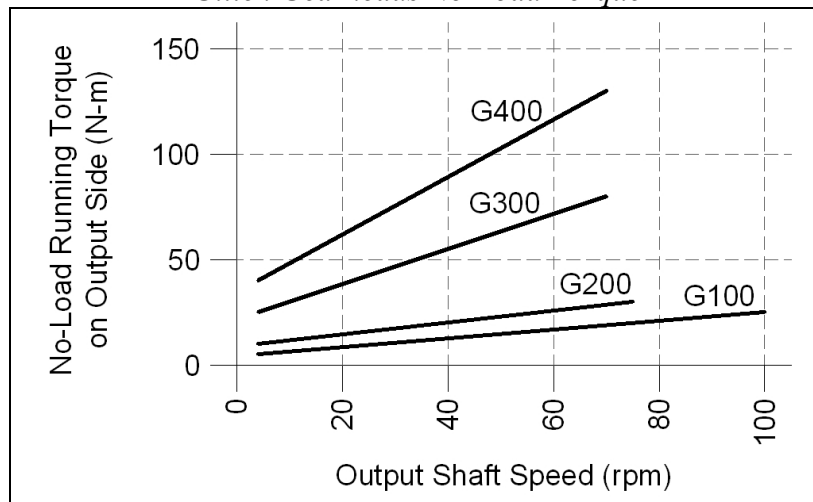
Heaton Servo Motors Product Specifications

Model #	Rated Torque (N-m)	Peak Torque (N-m)	Rated Speed (rpm)	Max Speed (rpm)	Rotor Moment of Inertia (kg-m ²)
S1000	0.36	1.04	4500	9000	0.31 x 10 ⁻⁴
S1100	0.32	0.96	6000	9000	0.31 x 10 ⁻⁴
S2000	1.19	3.02	4000	8000	0.87 x 10 ⁻⁴
S2100	1.12	2.78	6000	8000	0.87 x 10 ⁻⁴
S3000	1.94	5.76	4000	7500	1.36 x 10 ⁻⁴
S3100	1.62	5.31	5500	8000	1.36 x 10 ⁻⁴
S4000	2.88	8.50	3500	7000	1.88 x 10 ⁻⁴
S4100	2.74	8.02	4500	8000	1.88 x 10 ⁻⁴

Gitten Gearheads Product Specifications

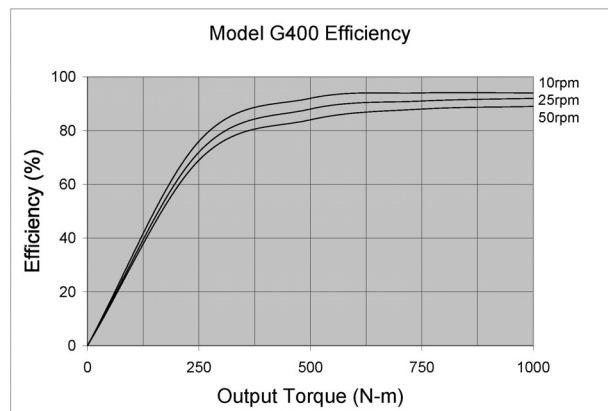
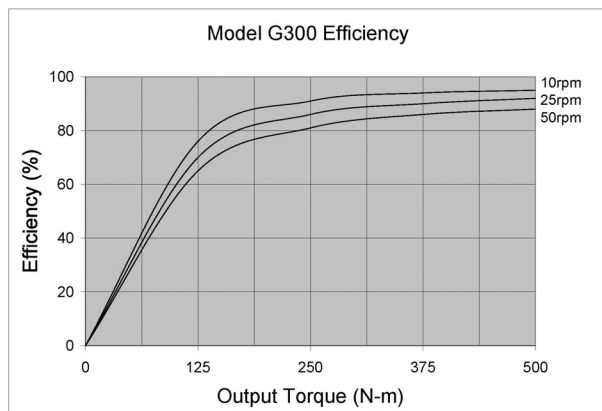
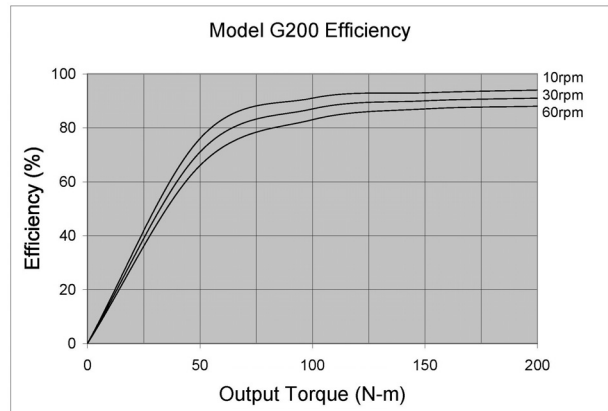
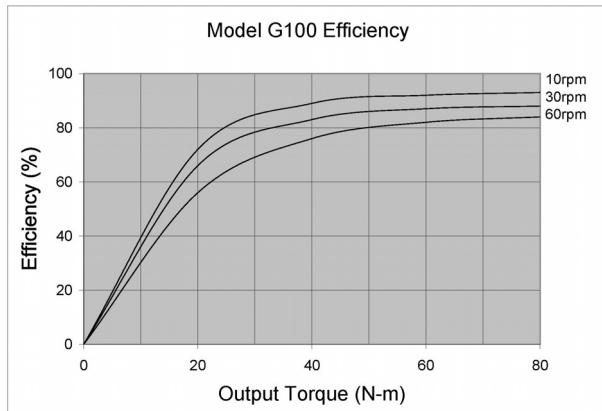
Model #	Available Gear Ratios	Rated Output Torque (N-m)	Peak Output Torque (N-m)	Rated Output Speed (rpm)	Peak Output Speed (rpm)
G100	31 43 54 79 103	58	117	60	100
G200	41 57 81 105 161	167	412	45	75
G300	41 57 81 105 153	412	1029	42	70
G400	41 57 81 101 153	784	1960	42	70

Gitten Gearheads No-Load Torque



Appendix A (cont'd)

Gitten Gearheads Efficiency Charts



Gitten Gearhead Coupling Moments of Inertia

Model #	Coupling Moment of Inertia (kg-m ²)
G100	1.30 x 10 ⁻⁴
G200	1.70 x 10 ⁻⁴
G300	2.40 x 10 ⁻⁴
G400	3.90 x 10 ⁻⁴