PDHonline Course P204 (3 PDH)

Interest Rates

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Interest Rates

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Table of Contents

Introduction

Interest is, in effect, the rent paid for the use of money. It can be considered as a fee to borrow capital and the cost of credit. An interest rate, or more appropriately, the cost of capital, is a critical item in the economic analysis of engineering projects.

Interest is the price of using the property of another for a period of time. For instance, if you borrow money from a bank, you are in effect paying them rent for the use of their money. Just like renting an apartment, it is still their money and it must be returned to the owner at some point. This can be done at the end of the 'rental period' or the money can be returned over a period of time through an amortization of the loan amount.

At one time in history, charging interest was considered inappropriate and in the middle-ages the Roman Catholic Church condemned the charging of interest as the 'sin of usury'. This view gradually fell into disfavor (and was generally ignored anyway), and the Church defined certain conditions where interest could be charged. Basically, the Church said money is an item just like any other commodity that can be rented and there is a risk in renting that should be compensated.

Today there are many types of interest rates. Just look in the financial pages of any major newspaper and you will see many different interest rates published. The problem then is "which interest rate do I use for my analysis?" This course reviews the different types of interest rates, the make-up of an interest rate, and important interest rate terms.

The concept of interest is based on opportunity costs for the owner of the capital. As interest rates rise investors will want a higher rate for their investment, which means that fewer projects will qualify for capital. This means that fewer capital plant additions will occur and fewer workers will be hired and ultimately the economy will slow down. Likewise, lower interest rates make more projects attractive and make it easier for companies to acquire capital for new projects.

The term 'opportunity cost' comes up when discussing interest rates. An investor has several options for the use of his money. He may invest it in a short-term project, a long-term project, a bank note, or use the money to buy a tangible product (e.g. a new car). If he decides to invest in a long-term project, then the money is not available for other - perhaps better - opportunities that may occur during the interim period. This 'lost opportunity' to use money for a better investment must be considered when loaning money. Opportunity costs are reflected in an interest rate as a risk premium added to the interest rate to account for the potential lost opportunity.

Of course, interest rates follow the economics of supply and demand. When money is plentiful and opportunities are few, investors are willing to take a lower interest rate for the use of their money. In contrast, in tight money markets, investors may be more selective in which projects they will consider, thereby raising the cost of borrowing money.

In essence, interest rates control the flow of money in the economy. Interest rates alone do not tell the story of how the economy is doing. The direction of interest rates, in conjunction with the consumer price index (CPI) and the gross domestic product (GDP) can yield clues as to what is happening with the economy. For instance, look at the following table for a good overview of how these indicators may describe the economic conditions.

Impact of interest rates on the economy		
Interest Rates	**GDP / CPI**	**Affect on Economy**
Rising	CPI decreasing	Stable economy
Decreasing	CPI increasing	De-stabilizing economy (inflation)
Decreasing	GDP increasing	Improving economy
Rising	GDP decreasing	Slowing economy

As you can see, rising interest rates may be okay if the consumer price index is falling. Decreasing interest rates may not be a good indicator if the CPI is increasing since this may be a sign of inflation. If the consumer price index is stable, then the decreasing interest rates coupled with an increasing GDP is a sign of an improving economy. And rising interest rates with a decreasing GDP is a sure sign of a slowing economy. In general, high interest rates slow inflation, but also slow the economy, while low interest rates spur the economy and increase the chances of inflation.

Interest rates are generally thought of as either short-term rates or long-term rates. The definition of these two terms is not completely clear, but anything less than one-year is definitely short-term and anything longer than 10-years is considered long-term. Between one and ten years the definition would need to be based on what interest rate index is being considered. The Federal government essentially sets the short-term rates through its open market policies. However, as we will see shortly, the Federal government does not actually 'set' the interest rate, they merely set the targeted interest rate and their actions will tend to drive the actual rate toward their target. Long term rates are set by the market based on expectations of economic growth, inflation, and other market forces.

In this course we will look at the various terms that are used when describing interest rates. Terms such as nominal interest rates, real interest rates, basis points, annual effective rate, yield curves, etc, will be addressed in the next section.

Section II of the course is an overview of the various components that make up an interest rate including risk-free interest rate, inflation premium, default risk premium, maturity risk premium, and liquidity premium.

Section III includes a discussion of what several different interest rates such as the Federal Funds Rate, Discount Rate, Treasury Rates, Prime Rate, LIBOR and EURIBOR.

Chapter 1
Interest Rate Terms

Nominal interest rate, real interest rate, annual effective rate, annual percentage rate and annual percentage yield are all forms of interest rates. The variety of interest terms can be confusing. In this Chapter, we will look at what each of these interest rates mean and we will look at several other terms that describe interest rates.

Nominal versus Real Interest Rates

A *nominal interest rate* is often called the "quoted" or "stated" interest rate. It is the basic interest rate used in a financial transaction. If someone agrees to pay you 5% interest for the use of $1,000 for a year, they have just quoted the nominal interest rate. At the end of the year you will have $1,050, which is your original $1,000 plus the $50 in interest. Because of inflation though, the $50 you receive at the end of the year will not be worth the same as $50 today. The *real interest rate* takes into account the effect of inflation on interest. In simple terms, if the inflation during the year of our example is 3.5%, then at the end of the year the buying power of the $50 in interest income would only be $15 since inflation would have consumed $35 of the interest income. In this case, the real interest rate is only 1.5%.

The formula for converting from a nominal interest rate to the real interest rate is,

$$\mathbf{r} = \left(\frac{(1+\mathbf{R})}{(1+\boldsymbol{\pi})} - 1\right) * 100$$

Where,
r = Real interest rate, percent.
R = Nominal interest rate, decimal value (i.e. 0.035).
π = expected inflation rate, decimal value.

For instance in our previous example, if the nominal interest rate is 5% and the expected rate of inflation is 3.5% then the real interest rate is,

r = {[(1+0.05) / (1+0.035)] – 1} * 100

r = 1.45%

Most people just approximate the real interest rate by subtracting the expected inflation rate from the nominal rate (e.g. 5% - 3.5% = 1.5%).

Inflation is a risk factor in a nominal interest rate since it is unknown before the transaction takes place. The expected inflation rate is sometimes referred to as the 'inflation rate *ex-ante'*, which means 'before the fact'. Once the inflation rate is actually known, which is after the transaction occurs the term 'inflation rate *ex-post'* is used. The person loaning the money is making an assumption about what the inflation rate will be ex-post. The person loaning the money is hoping the inflation rate will be lower than expected so that his real interest rate earned will be higher than expected. In contrast, the person borrowing the money will benefit from a higher inflation rate because he will be paying the loan back with cheaper dollars. In Section II we will look at how other risk factors affect an interest rate.

Effective Interest Rate

The *effective interest rate* is the nominal interest rate adjusted for the effects of any compounding of interest. The effective interest rate is sometimes referred to as the annual effective rate (AER), effective annual interest rate, or just the effective rate.

In our previous example a nominal interest rate of 5% was applied to $1,000 loan for one year with no compounding (e.g. the interest rate was calculated at the end of the loan period.) Quite often interest charges are calculated on more frequent time intervals increasing the effective interest rate. This compounding of interest is commonly used in all banking transactions from loans to simple demand deposits. Compounding may occur on any time period including, yearly, monthly, daily, or even continuously. Let's look at the impact of various compounding periods on the interest rate. The formula for compounding is,

$$\mathbf{AER = [(1 + \frac{i}{n})^n - 1] * 100}$$

Where,
AER = Annual effective interest rate, percent.
i= Annual Nominal interest rate, decimal value.
n = number of compounding periods.

For example, if the 5% interest rate is compounded monthly the annual effective rate is,

$AER = [(1+(.05/12))^{12} - 1] * 100$

AER = 5.1162%

For daily compounding the annual effective interest rate is,

$AER = [(1+(.05/365))^{365} - 1] * 100$

AER = 5.1267%

So, a nominal interest rate of 5% on a $1,000 one year loan could actually result in a interest payment of $50, $51.16, or $51.26 depending on whether the interest rate is compounded annually, monthly, or daily.

The worse case interest rate is where the interest rate is compounded continuously. The formula for this special case of interest compounding is,

$$\textbf{AER} = (e^{i} - 1) * 100$$

Where,
AER = Annual effective interest rate, percent.
i = Annual Nominal interest rate, decimal value.
e = exponential term.

For example, a 5% nominal interest rate compounded continuously will result in an annual effective rate of,

$AER = (e^{0.05} – 1) * 100$

AER = 5.1271%

The annual effective rate is sometimes referred to as the annual percentage rate (APR), but as we will see shortly, there are differences between the annual effective rate and the annual percentage rate.

Annual Percentage Rate (APR)

An annual percentage rate (APR) is a method of representing an effective interest after taking into account all of the one-time fees associated with the loan. It provides a standardized methodology to compare rates with different fees such as "points" and origination fees. The intent is to let the borrower know the real cost of a loan. For non-commercial loans, including mortgage loans, the *Truth in Lending Act* requires the lender to disclose the APR for the loan.

An APR is only effective for comparing rates with the same term limits and should not be used to compare loans of different maturities since the impact of the one-time fees will be distorted. While the intent of an APR is noble, in reality there can be differences in loan rates even when using an APR. Lenders do not necessarily include the same fees in the calculation of the APR. For instance, the following are generally included in the calculation of an APR: points, pre-paid interest, origination fees (loan processing, document preparation, etc), private mortgage insurance, and closing agent's document fees. Some lenders will include application fees and mandatory life insurance fees in the calculation. Fees such as title fees, appraisal fees, and credit reports are generally not included in the calculation. As you can see, it is important to know what each lender included in their calculation of the APR.

Calculating an annual percentage rate is quite complex and requires multiple iterations of the formula to arrive at the true APR. For a simple, and relatively accurate, method for calculating APR follow these steps,

1. Calculate the loan payment using the quoted interest rate.
2. Add loan origination fees, discount points, and other front-end fees to the principal amount and re-calculate the loan amount.

3. Using the original principal and trial and error find a new interest rate that will yield the payment amount found in step 2. The final interest rate found in this step is the APR.

This method only works for front-end charges. Back-loaded charges, monthly amounts, etc, complicate the APR further. Fortunately most loan charges are front-end loaded

As an example consider a $100,000 loan that is offered at 7% for 30 years with a 1% origination fee, 2 discount points and 1,500 in other fees. What is the APR?

A loan amortization can be determined using the following formula,

$$\mathbf{PMT = PRIN * i * [\frac{(1+i)^n}{(1+i)^n - 1}]}$$

Where,
PMT = Payment.
PRIN = Principal loan amount.
i = Interest rate per period, decimal amount.
n = Number of interest rate periods.

For our example the interest rate per period is 0.07/12 = 0.005833 and the number of periods is 30 * 12 = 360 periods.

Step 1.
PMT = 100,000 * 0.005833 * [$(1+0.005833)^{360}$ / $(((1+0.005833)^{360}) – 1)$]

PMT = $665.28

Step 2.
Add the front-end fees ($4,500) to the principal amount and re-calculate the payment.

PMT = 104,500 * 0.005833 * [$(1+0.005833)^{360}$ / $(((1+0.005833)^{360}) – 1)$]

PMT = $695.21

Step 3.
Now the trial and error starts. As a guess, try an interest rate of 7.5%.

PMT = 100,000 * 0.00625 * [$(1+0.00625)^{360}$ / $(((1+0.00625)^{360}) – 1)$]

PMT = $699.21

7.5% is close, so try an interest rate of 7.4%,

PMT = 100,000 * 0.006167 * [$(1+0.006167)^{360}$ / $(((1+0.006167)^{360}) – 1)$]

PMT = $692.38

If we continue this process, we will eventually arrive at the actual APR rate of 7.442%.

A note of caution: Credit card issuers use a slightly unusual approach to calculating APR. First, credit cards are based on "APR compounded monthly", so it is, in effect, not a true APR and must be converted to an annual effective interest rate. Second, Federal regulators allow credit card issuers to quote the APR to two decimal places and to round any digits beyond two decimals. Therefore, it is a good bet that a credit card rate that is quoted as 12.99% is probably 12.9949% since the bank is allowed to quote the lower number, but collect interest on the higher value. The annual effective rate of a credit card offered at 12.99% is probably,

$$\text{Annual Effective Rate} = ((1+0.129949/12)^{12} -1) * 100 = 13.80\%$$

The intent of the APR method is to allow two different interest rate offerings to be evaluated on a comparable basis. As you can see, the APR method does not a guarantee that the evaluation is on an equivalent basis.

Basis Points

When someone says an interest rate, that was previously 5%, just went up one percent do they mean the rate went up from 5% to 6% or did the rate actually just go one percent to 5.05% (i.e. 5% *1.01 = 5.05%)?

To minimize the ambiguity than can occur when talking about percentages, financial markets refer to differences in percentage rates in terms of basis points. A *basis point* is a unit that is equal to 1/100th of a percentage point. Therefore, if an interest rate went from 5% to 6% then the rate went up 100 basis points (bps). If a 5% interest rate increased by 1%, then the rate went up 5 bps to 5.05%.

Financial instruments will frequently have a reference to basis points over an index such as "the loan rate is quoted at 200 basis points over prime" for example. Remember that 100 basis points equal 1.00%.

Day Count Convention

The determination of a compound interest rate will be affected by the number of days in the calculation period. That sounds like an obvious statement, but there are several different methods to determine the days within a period. This is known as the *day count*. The day count convention defines how the varying number of days in the months of the year, as well as leap year, is accounted for. The four most common methods are known as the Actual/Actual, 30/360, Actual/365, and Actual/360.

With the *Actual/Actual* method the actual number of days in a period is used as well as the actual number of days in the year (remember that a year will have either 365 days in a normal year or 366 days in a leap year.) As an example, consider a loan with a 7% nominal rate that is for a loan from October 1st through December 31st of a leap year. What is the monthly rate and annualized rate, if the day count convention is Actual/Actual? There are 31 days in October, 30 days in November, and 31 days in December, so the total number of days is 92. Since this is a leap year there are 366 days in the year. Therefore the rate for the quarter is 7% * 92/366 = 1.760% or 7.04% annualized. Unless otherwise noted, most loans use the Actual/Actual method of counting days.

The 30/360 method assumes that all months have 30 days and all years have 360 days. This makes the math simple and was commonly used for bond calculations before the advent of computers. This method is still used for some corporate bonds and mortgage backed securities. Continuing our previous example, the quarterly interest rate is 7% * 90/360 = 1.750% or 7.00% annualized.

With the Actual/365 method the actual number of days in the period is used but the year is assumed to contain 365 days, including leap year. The quarterly interest rate from our example is 7% * 92/365 = 1.764% or 7.06% annualized. An advantage of this method is that the daily interest rate is the same regardless of whether it is a normal year or a leap year.

Finally, the Actual/360 method uses the actual number of days in the period but assumes that there are 360 days in the year. The quarterly interest rate from our example is 7% * 92/360 = 1.789% or 7.16% annualized.

Add-on Interest

An interest rate calculation that is commonly used with vehicle financing is called add-on interest. Other terms include 'pre-computed' or 'flat-rate' interest. Add-on interest is a method where the entire interest for the life of the loan is added to the principal at the beginning of the loan and the total is divided by the number of months in the loan to arrive at the monthly payment. Add-on interest programs result in significantly higher interest rates than the quoted rate.

Consider an example where a $25,000 car loan is offered at 8% interest, pre-computed for 48 months. The interest is 0.08 * 25,000 * 4 = $8,000. Adding interest to the loan amount results in a total cost of $33,000 or $687.50 per month. The interest rate required on $25,000 for 48 months to generate a payment of $687.50 is 14.35%, which is 80% more than the quoted rate.

Note, the actual interest rate was found using an iterative process,

$$PMT = 25{,}000 * 0.01196 * [\ (1+0.01196)^{48} / (((1+0.01196)^{48}) - 1)] = \$687.58$$

Rule of 78's

A variation of the pre-computed, or add-on interest method is a concept known as the Rule of 78's, which is also called the sum-of-the-digits method. This method also adds-on the interest at the beginning of the loan and then recovers the interest over the life of the loan. The unusual feature of this method is that a large percentage of the interest is recovered early in the loan.

The *Rule of 78's* gets its name from sum of the number of months in a year (i.e. if you add up the numbers 1 through 12 the total is 78.) For a 12 month loan, 12/78s of the interest (15.38%) is recovered in the first month and 1/78 (1.28%) is recovered in the last month. For loans of other than 12 months, the sum-of-the-digits is calculated by,

$$\mathbf{SYD = n * \frac{(n+1)}{2}}$$

Where,
SYD = Sum-of-the-years digits.
n = number of months in the loan.

For a 48-month loan, the sum-of-the-digits is,

SYD = 48 * (48+1)/2 = 1,176

In a 48-month loan, the interest in the first month is 48/1176 or 4.08%.

For a simple example, look at the following table that has a one-year, $1,000 loan with an 8% interest rate. The interest for the period is $80.

Table 1
Rule of 78s Calculation

Month	Rule	Rate	Interest	Principal	Payment
1	12/78	15.38%	12.31	77.69	90.00
2	11/78	14.10%	11.28	78.72	90.00
3	10/78	12.82%	10.26	79.74	90.00
4	9/78	11.54%	9.23	80.77	90.00
5	8/78	10.26%	8.21	81.79	90.00
6	7/78	8.97%	7.18	82.82	90.00
7	6/78	7.69%	6.15	83.85	90.00
8	5/78	6.41%	5.13	84.87	90.00
9	4/78	5.13%	4.10	85.90	90.00
10	3/78	3.85%	3.08	86.92	90.00
11	2/78	2.56%	2.05	87.95	90.00
12	1/78	1.28%	1.03	88.97	90.00
Totals			80.00	1,000.00	1,080.00

As you can see in Table 1 the interest varies from $12.31 in the first month to only $1.03 in the final month. In fact, almost 75% of the interest is paid during the first six months of the loan.

The Rule of 78's benefits the lender if the loan is paid off early because so much of the interest cost is recovered early in the loan. If the loan is paid off early, the lender will need to credit the borrower with the unearned interest. The amount of the unearned interest can be found by,

$$\mathbf{I_U = Fin * k * \frac{(k+1)}{(n*(n+1))}}$$

Where,
I_U = Unearned interest.
Fin = Total finance charge.
k = Number of months that the loan is paid off early.
n = Total number of months.

From our previous example, what is the unearned interest to be returned if the loan is paid off four months early?

I_U = 80 * 4 * (4+1) / ((12 * (12+1))

I_U = $10.26.

This value can be verified by summing the interest charges in Table 1 above for the last for months (4.10+3.08+2.05+1.03 = 10.26.)

Because of the front-end loading of interest charges, the Rule of 78's is not allowed for mortgage refinancing loans and consumer loans of over 60 months.

Fixed vs. Variable Rates

Anyone who has ever taken out a mortgage to purchase a home probably has as opinion about whether a fixed or variable interest rate is a better choice. In actuality, neither is better or worse than the other, it is just a matter of assigning risk to the agreement. When using a fixed interest rate to make a capital investment, the borrower knows how the cost of borrowing will affect the economics of the capital addition. The disadvantage is that the borrowing is paying a risk premium to the lender in exchange for the guaranteed rate. This guarantee raises the cost of borrowing. Also with a fixed rate, the borrower cannot take advantage of lower interest rates if the economy slows during the term of the agreement.

With a variable rate, the borrower is not paying for the guarantee afforded by the fixed rate and therefore the economics of a capital addition can change as interest rates rise. With a variable rate, the borrower is not 'captive' to a different economic time. If the borrower takes out a loan during a high interest period and the economy changes such that interest rates decrease, the borrower benefits.

Yield Curve

Financial markets often talk about the shape of the yield curve. What they are referring to is the relationship between interest rates and the time to maturity for an investment. Normally long investment periods result it the highest interest rates. When interest rates are plotted against the time to maturity, a positively sloping yield curve would be expected and this is considered a *normal yield curve*. A normal yield curve reflects the conventional wisdom that higher risks are associated with long term investments and that there is less financial risk with shorter term investments. A normal yield curve implies that inflation is expected to higher in the future and future loans can be paid off with cheaper money. Investors tend to see a normal yield curve as a positive economic sign for the intermediate term. Figure 1 shows a normal yield curve. As you can see, the interest rate for a one-year investment is about 1.5%, at ten-years the rate is 4.5%, and at thirty-years the rate is approaching 6%.

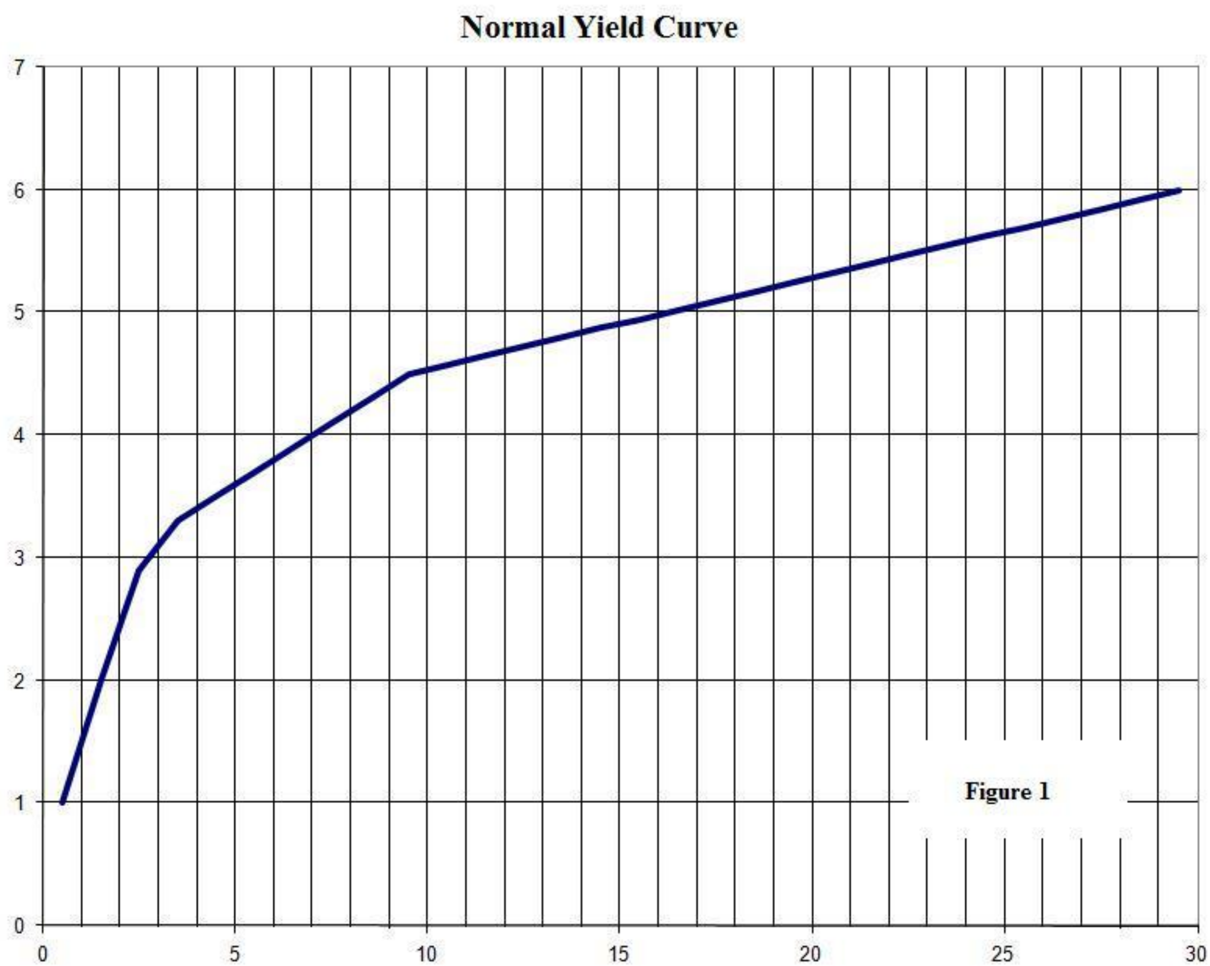


Figure 1

The opposite of a normal yield curve is an inverted yield curve. An *inverted yield curve* occurs when short-term rates are higher than long term rates. This condition usually occurs when the economy is experiencing higher than normal inflation and the Feds raise short-term interest rates to attempt to slow down the economy and inflation. An inverted yield curve is one sign of a potential slow-down in the economy. Figure 2 shows an inverted yield curve. In this example, the interest rate is 6.5% at one-year, at ten-years the rate is 4.5%, and at thirty-years the rate is below 4%.

The front end of a yield curve (i.e. the first few years) is sensitive to the Federal Reserve monetary policy and the long-end of the curve is affected by commodity prices. When commodities such as grain and oil are high, the long-end of the curve is usually higher because of the expectation of inflation.

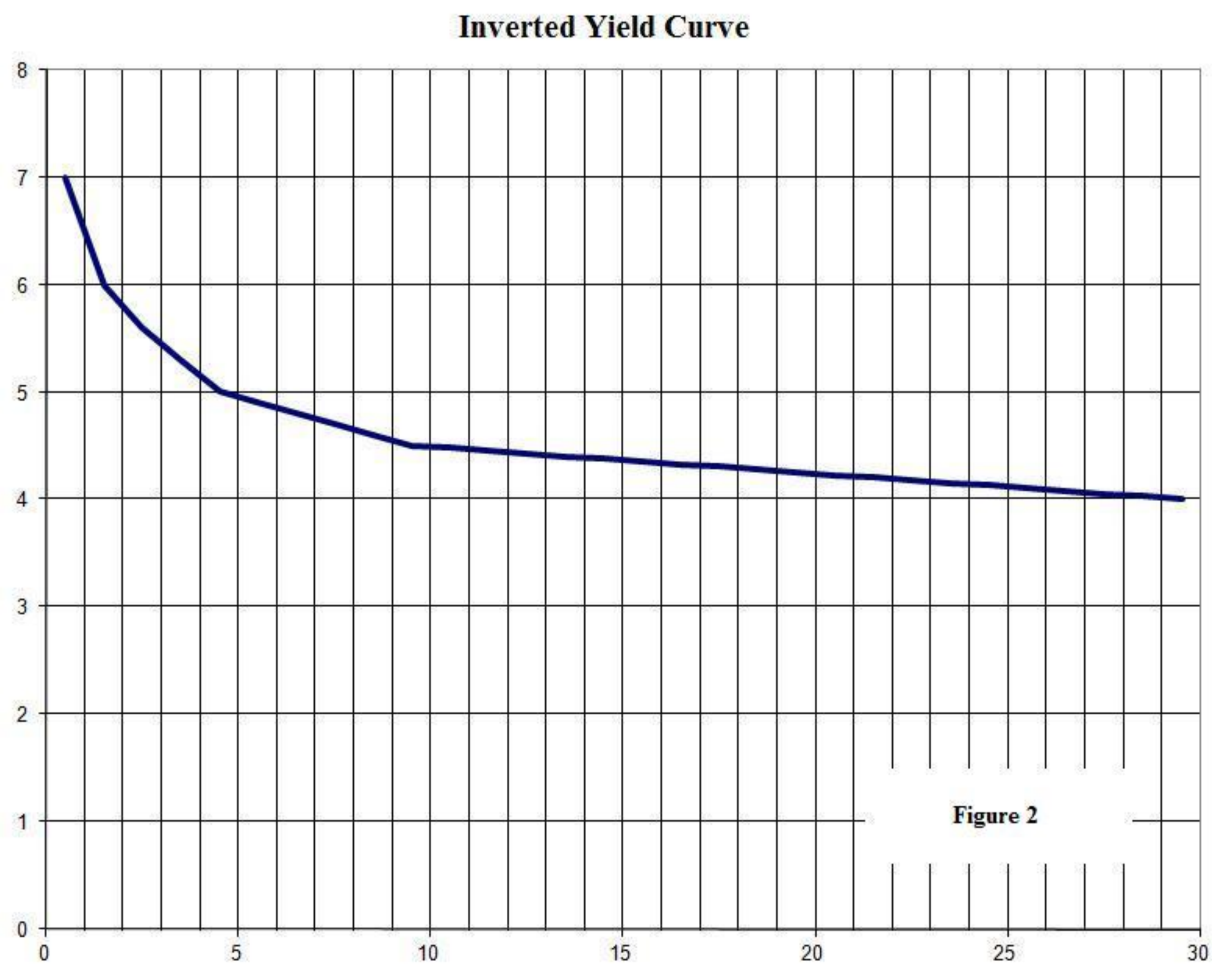


Figure 2

Interest Rate Swaps

An interest rate swap is a derivative contract where an entity exchanges a fixed interest rate stream of payments for a variable rate stream of payments. They are used by corporations to manage risk in financial transactions and are considered very liquid instruments.

> A **Derivative Contract** is a financial agreement where the value is derived from the value of underlying assets such as interest rates or currency rates.

There are many different types of interest rate swaps such as,

1. Fixed-for-Floating rate swap, same currency
2. Fixed-for-Floating rate swap, different currencies
3. Floating-for-Floating rate swap, same currency
4. Floating-for-Floating rate swap, different currencies

We will only consider the "Fixed-for-Floating rate swap, same currency" because it is the most popular and is commonly used to manage transactional risks. This type of interest rate swap is also known as a "vanilla swap".

With an interest rate swap, one party (known as the Payer) wants to pay a fixed interest rate to another party and in exchange will receive a floating interest rate payment from the other party. The party receiving the fixed interest payment is known as the Receiver. The Receiver will pay a floating interest rate payment to the Payer. The Receiver and the Payer are known as *counterparties* to the interest rate swap. Figure 3 shows the relationship between the Receiver and the Payer.

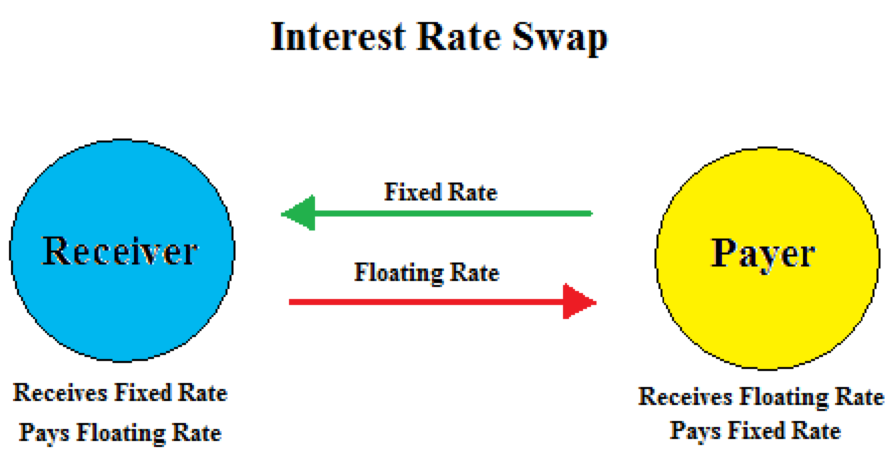


Figure 3

The *swap rate* is the fixed interest rate that the Receiver demands in exchange for taking the uncertainty risk of paying a floating rate to the Payer. The swap rate is typically based on a short-term LIBOR interest rate such as the one-month LIBOR or the three-month LIBOR.

The principal amounts are not normally exchanged in an interest rate swap – only the *interest payment stream* is exchanged. Since the interest payment stream must be based on some principal amount, an interest rate swap uses what is known as a *notational principal amount* to define the interest payment stream. For instance, a 5% swap rate on a notational principal amount of $1 million is $50,000. The $1 million principal amount is not exchanged in the swap, only the interest payment of $50,000 is exchanged. The notational principal amount is only used to define the size of the interest rate payment.

The concept of interest rate swaps can be confusing to grasp. The easiest way to visualize an interest rate swap is to consider two corporations, each with a $1 million loan. Corporation "A" has a 10-year floating rate loan with an interest rate of 4%. Corporation “B” has a 10-year fixed rate loan with an interest rate payment of 6%. Corporation “A” prefers the stability of a fixed rate and is willing to pay a higher interest rate for a fixed rate. Corporation “B” needs to reduce their monthly interest payments and is willing to take on the risk of a lower rate variable loan to free up the cash. In this case, a swap between the corporations will allow each to meet their objectives. Since they can’t just trade loans, they develop an agreement to exchange the interest rate payment streams.

A short-term LIBOR is typically used to define an interest rate swap with some adder for the interest rate risk in the transaction. An interest rate swap may look like this,

> Corporation “A” agrees to pay a 5% fixed interest rate is US dollars to Corporation “B” to receive an floating rate indexed to 3M Libor+25bps on a notational $1 million US dollars in quarterly payments.

In this example, the floating rate is based on the three-month LIBOR plus 25 basis points. If the three-month LIBOR is 2.79%, then the floating rate is 3.04%. On a $1 million notational amount the interest payment stream that corporation “A” will receive is $30,400 per year.

Chapter 2
Interest Rate Components

The quoted or nominal interest rate for a loan is composed of several factors. An interest rate is composed of the real, risk-free, rate of interest, along with several factors that are in essence premiums that reflect the lenders view of inflation, riskiness of the loan, the length of the loan, and the marketability (liquidity) of the loan.

These relationships can be expressed mathematically as,

$$\mathbf{k = k_{real} + IP + DRP + MRP + LP}$$

Where,
k = the rate of interest
k_{real} = A risk free, real interest rate
IP = the inflation premium
DRP = the default risk premium
MRP = the maturity risk premium
LP = the liquidity premium

We will look at each of these terms individually.

The real, risk-free, rate of interest, (k_{real})
The real, risk-free, rate of interest is the interest rate that would exist on a riskless security where no inflation is expected. A very short-term U.S. Treasury rate is a good example of a risk-free rate of interest. It is assumed that the U.S. government Treasury bill is a secure investment and therefore, the interest is virtually risk-free. And the time period is short enough to negate any inflation worries. An example is a one-month treasury rate. The implication is that anything other than a short-term government security carries some amount of risk, such as inflation risks and credit risks that should be rewarded with higher interest rates.

Inflation Premium (IP)
An *inflation premium* is the average inflation rate expected by the lender over the term of the loan. Remember, it is the expected inflation rate and not the current rate. It is based on the lenders opinion of what inflation will do. Since US Treasuries are a good risk free investment, we can basically assume that any difference in the terms of the Treasures is based on inflation. For instance, if the one-month Treasury rate is 2% and the one-year Treasury rate is 5%, then we can assume that the Feds are betting on a 3% annual inflation rate. Therefore, the inflation premium is 3%.

Default Risk Premium (DRP)
The *default risk premium* is a factor to compensate the lender for the risk that the borrower may not repay the loan. For a US Treasury note we assume the default risk is zero. For corporations, the default risk will rise with a decrease in the credit-worthiness of the corporation. A good

method to view the default risk premium is to compare the US Treasury rate with bond offerings of corporations with the same maturity. The difference between their bond offering and the US Treasury rate will define their default risk premium. The credit-worthiness of a corporation (as measured by its bond rating) correlates well with the default risk premium.

Look at Table 2. The difference between a 30-year US Treasury (4.70%) and an AAA rated 30-year corporate bond is 0.7%, so the default risk premium is 0.7%.

Table 2
Default Risk Premium (30-year) note

Instrument	Rate	Default Risk Premium
US Treasury	4.70%	-
AAA Bond	5.40%	0.7%
AA Bond	5.50%	0.8%
A Bond	5.60%	0.9%
BBB	5.80%	1.1%
BB+	6.40%	1.7%

In Table 2 we see that a BB+ grade bond has an interest rate of 6.4%, so the default risk premium is 1.7%. BB+ grade bonds are below what is normally called investment grade bonds.

Note that the figures in Table 2 are representative only. These relationships change over time.

Maturity Risk Premium (MRP)
Since loaning money ties up the money for some period of time, the investor loses the opportunity to invest the money in other projects. The *maturity risk premium* compensates the lender for the lost opportunity for long term loans. Longer term loans will carry a higher maturity risk premium than shorter term loans. The maturity risk premium is difficult to measure but it is in the range of 0.1% per year of the loan up to maximum of about 1.5-2.0%. So, a 10 year loan might have a MRP of 1.0% while a thirty year loan may only carry an MRP of 1.5%.

An MRP is affected by the volatility and uncertainty of interest rates. The MRP is higher when interest rates are volatile and the MRP seems to be less during times of stable interest rates.

Liquidity Premium (LP)
A *liquidity premium* is used to compensate the lender for the risk that the loan security cannot be quickly converted to cash. It is difficult to assess how the markets assess liquidity risk, but in general, there can be a 200-400 basis point difference in a liquid investment versus a non-liquid investment. A short-term car loan is considered a fairly liquid investment since a car can be quickly sold (converted to cash) in an established market. Artwork is probably on the higher end of the liquidity risk curve. Interestingly, the liquidity risk premium for real estate varies inversely with the length of the loan term. In the short term, real estate is considered somewhat non-liquid, but in the long term the risk is assumed to be less.

Country Risk
Although not expressly defined in the interest rate equation, there is risk associated with borrowing or lending money in other countries. One risk is the stability of the government of the other country. Investing in Mozambique is probably more risky than investing in Great Britain. Another risk is the currency exchange. The exchange rate can have a profound impact on the profitability of an investment.

For an example of determining an interest rate consider the following example. A commercial bank is asked to make a 30-year loan to an AAA-rated company for a new manufacturing plant. The one-month Treasure rate is current 1.5%, and the 30-year Treasury rate is currently 4.5%. The bank is assigning a maturity risk premium of 1.5% to this loan and a liquidity premium of 200 basis points. What interest rate is the bank likely to offer this company?

The risk-free real interest rate can be assumed to equal the one-month Treasury note, or 1.5%. The inflation premium is the difference between the one-month Treasury note and the 30-year Treasury note or 3.0% (4.5%-1.5% = 3.0%). The default risk premium can be approximated from the values in Table 2. Therefore the interest rate is found from the formula,

$k = k_{real} + IP + DRP + MRP + LP$

Therefore, the interest rate is,

$k = 1.5 + 3.0 + 0.7 + 1.5 + 2.0$

$k = 8.7\%$.

Since this is a 30-year loan it carries a fairly substantial premium for inflation and maturity risk. If the loan was only for five years the interest rate would likely be slightly over 6%, which is about 1% over the prime rate for this time period.

Note: All of the assumptions about risk premiums in this section are examples and may not represent the risk assignments that a commercial bank might use.

Chapter 3
Types of Interest Rates

In this chapter we will look at a few of the major interest rates used in the financial industry. We will review the Federal Funds rate, the discount rate, prime rate, Treasuries, LIBOR, and EURIBOR. First, let's look at Table 3 which shows the relative values of these different interest rate values at a specific point in time.

The values shown in Table 3 are available in publications such as the Wall Street Journal.

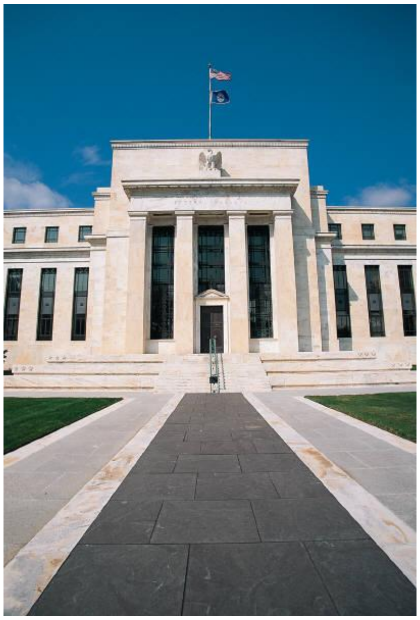
Table 3
Interest Rate Indices

Interest Rate	Rate
Federal Funds Rate (target)	2.00%
Discount Rate	2.25%
Prime (WSJ)	5.00%
Libor (3-month)	2.79%
Money Market	2.49%
Treasuries	
1-month	1.75%
1-year	2.25%
2-years	2.60%
5-years	3.25%
10-years	4.00%
30-years	4.70%

Federal Funds Rate

The Federal funds rate is one of the most powerful, and maybe least understood interest rates in use today. The Federal funds rate is the interest rate that banks use to lend balances in their Federal Reserve accounts to other banks for short time periods.

To explain the Federal funds rate we need to first look briefly at how the U.S. banking system operates. The Feds require banks to keep a certain amount of funds in reserve, either in the bank (cash) or on account at the Federal Reserve. When a bank makes a loan it must increase its reserve to compensate for the loan. It can do this by getting more people to deposit funds in the bank or by borrowing funds from the Federal Reserve accounts of other banks that have a surplus or directly from the Federal government. Since at any given time, some banks will have a surplus in the Federal accounts and others will have a deficit a market exists for short-term loans of Federal Reserve funds. The interest rates for these loans are negotiated between the banks and the average interest rate of all of the Federal funds loans is known as the *Federal funds effective rate*.



The Federal funds rate is a target rate the Federal Reserve sets in an effort to keep the economy healthy. When we see a newspaper headline such as,

"TODAY THE FEDS CHANGED THE FEDERAL FUNDS RATE TO 2.0%"

We need to remember that the Feds did not actually change the rate, what they have changed is the target rate. We call the Federal funds rate a *target* rate because the Federal Reserve sets the goal for the interest rate, but the market determines the actual rate through daily inter-bank loans. The goal, or target, rate is also called the *Federal funds nominal rate*.

Once the target rate is set, the Feds try to control the effective rate by buying and selling U.S. Treasuries. When the Feds want to lower the effective rate, they buy securities on the open market. They pay for the securities by creating money in the form of additional deposits at the Federal Reserve. The increase supply of money at the Federal Reserve tends to lower the overnight interest rates the banks charge each other since the supply of money has increased. To raise interest rates, the Feds sell U.S. Treasuries and the funds are removed from the Federal Reserve, decreasing the money available to banks, which tends to increase the effective interest rate. The Feds make daily decisions to buy and sell Treasuries to keep the target rate near the goal set by the Board of Governors.

The Federal funds rate is one of the main tools the Federal Government can use to control the economy. During low-growth, recessionary periods, the Feds will set a low Federal funds rate to help stimulate the economy. During high inflation periods, the Feds will raise rates. In the early eighties, the Feds set the Federal funds rate at almost 20% in an effort to slow the economy and control inflation. See the graph in Figure 4 below for a history of the Federal funds rate.

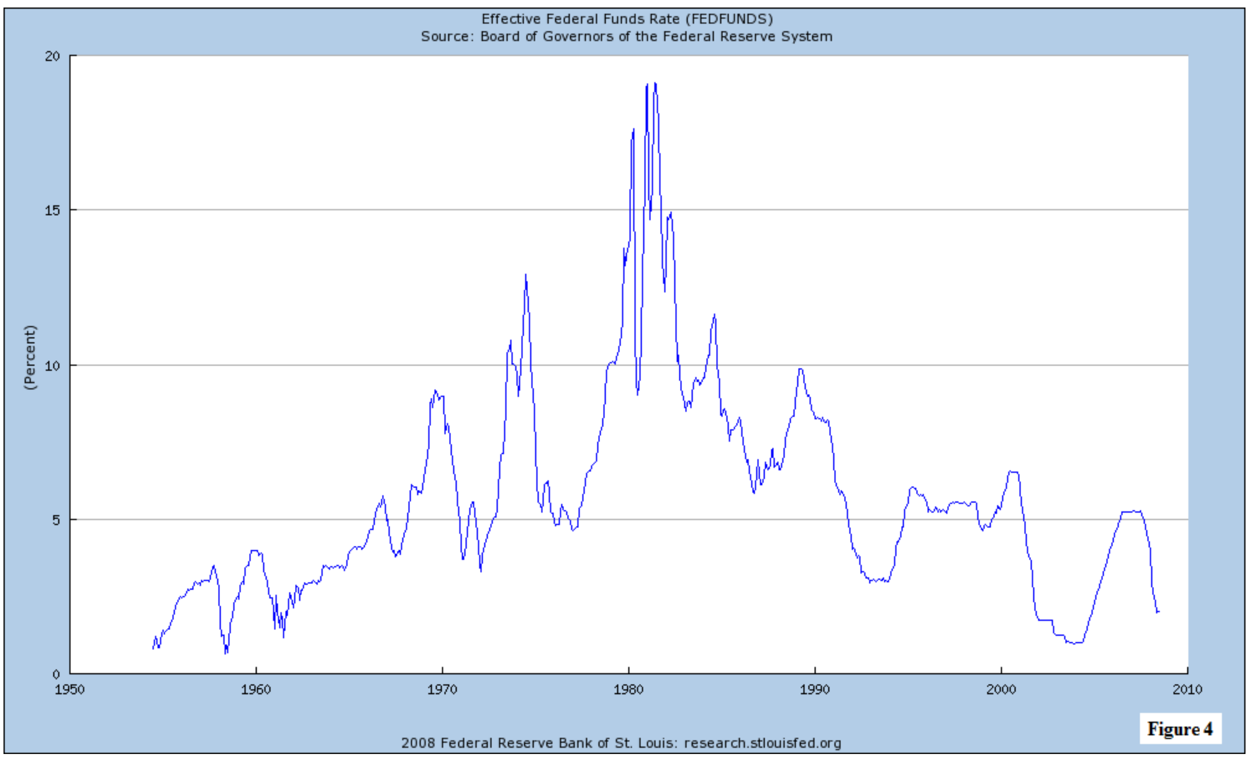


Figure 4

People sometimes question why mortgage rates don't fall when the Feds adjust the Federal funds target rate. The reason is that mortgage rates are tied to long-term rates such as bonds and 30-year Treasuries. Mortgage rates are more related to the perception of the economy and lenders optimism, or lack thereof, of future economic conditions. One retail rate that is impacted by the Federal funds rate is the Prime rate.

Discount Rate

Another tool at the disposal of the Feds is the discount rate. In addition to the inter-bank loans at the Federal Reserve, the banks may also borrow overnight funds directly from the Federal Reserve. The discount rate is the interest rate charged to banks on loans they receive from the Federal Reserve Bank's lending facility - the discount window. The Federal Reserve Banks offer three discount window programs to depository institutions: primary credit, secondary credit, and seasonal credit, each with its own interest rate. All discount window loans are fully secured.

Under the primary credit program, loans are extended for a very short term, usually overnight, to banks in generally sound financial condition. Banking institutions that are not eligible for primary credit may apply for secondary credit to meet short-term liquidity needs or to resolve severe financial difficulties. Seasonal credit is extended to relatively small banks that have recurring intra-year fluctuations in funding needs, such as banks in agricultural or seasonal resort communities.

The discount rate charged for primary credit is set above the usual level of short-term market interest rates. The discount rate on secondary credit is above the rate on primary credit. The discount rate for seasonal credit is an average of selected market rates. Discount rates are established by each Reserve Bank's board of directors, subject to the review and determination of the Board of Governors of the Federal Reserve System.

Treasuries

Treasury securities are government bonds issued by the United States Department of the Treasury. They are the debt financing instruments of the U.S. Federal government, and they are often referred to simply as Treasuries or Treasurys. There are four types of marketable treasury securities: Treasury bills, Treasury notes, Treasury bonds, and Treasury Inflation Protected Securities. Marketable Treasury securities are very liquid and are heavily traded on the secondary market. Non-marketable securities, such as savings bonds cannot be transferred through market sales. Marketable securities are issued directly by the Feds. Let’s briefly look at each of the different types.

<u>Treasury bill</u>
Treasury bills, or T-bills, mature in one year or less. They do not pay interest prior to maturity; instead they are sold at a discount of the par (face) value. Treasury bills are the least risky investment available to U.S. investors.

Regular T-Bills are commonly issued with maturity dates of 28 days, 91 days, and 182 days and are known as one-month, three-month, and six-month Treasurys respectively. Treasury Bills are sold by single price auctions held weekly. Banks and financial institutions, especially primary dealers, are the largest purchasers of T-Bills.

During periods when Treasury cash balances are particularly low, the Treasury may sell cash management bills (or CMBs). These are sold at a discount and by auction just like weekly Treasury bills. They differ in that they are irregular in amount, term, issuance, and maturity.

Treasury bills are quoted for purchase and sale in the secondary market on an annualized percentage yield to maturity, or basis.

The general calculation for the yield on T-bills is,

Yield = (Par - Price)/Par * (360/DTM) * 100

Where,
Yield = Percent yield on the T-bill.
Par = Par Value, dollars.
Price = Purchase price, dollars.
DTM = Days to Maturity.

For an example, consider a $1,000 T-bill that is sold for $950 at one-year (360 days) before maturity. The yield is,

Yield = (1,000 - 950)/1,000 * (360/360) * 100

Yield = 5.00%.

If this same T-bill is 90 days from maturity and is now selling at $990, then the yield is,

Yield = (1,000 - 990)/1,000 * (360/90) * 100

Yield = 4.00%.

In this example, the yield has dropped slightly as the T-bill approaches maturity.

<u>Treasury note</u>
Treasury notes, or T-Notes, mature in two to ten years. They have a coupon payment every six months, and are commonly issued with maturities dates of 2, 5 or 10 years, for denominations from $1,000 to $1,000,000.

T-Notes and T-Bonds are quoted on the secondary market at a percentage of par-value in thirty-seconds (1/32) of a point. For example, a quote of 98-05 on a note indicates that it is trading at a discount of 98 5/32% (98.156%) or $981.56 for a $1,000 bond.

The 10-year Treasury note has become the security most frequently quoted when discussing the performance of the U.S. government-bond market and is used to convey the market's take on longer-term macroeconomic expectations.

<u>Treasury bond</u>

Treasury bonds, T-Bonds, have the longest maturity, from ten years to thirty years. They have coupon payment every six months like T-Notes, and are commonly issued with maturity of thirty years. The secondary market is highly liquid, so the yield on the most recent T-Bond offering is commonly used as a proxy for long-term interest rates in general.

Treasury Inflation-Protected Securities

Treasury Inflation-Protected Securities, or TIPS, are inflation-indexed bonds issued by the U.S. Treasury. The principal is adjusted to the Consumer Price Index, the commonly used measure of inflation. The coupon rate is constant, but generates a different amount of interest when multiplied by the inflation-adjusted principal, thus protecting the holder against inflation. TIPS are currently offered in 5-year, 7-year, 10-year and 20-year maturities.

In addition to their value for a borrower who desires protection against inflation, TIPS can also be a useful information source for policy makers: the interest-rate differential between TIPS and conventional US Treasury bonds is what borrowers are willing to give up in order to avoid inflation risk. Therefore, changes in this differential to indicate that market expectations about inflation over the term of the bonds have changed.

Prime Rate

The prime rate is another interest rate that sometimes gets confused by media coverage of the financial markets. The prime rate is not set by the Feds. The prime rate is the interest rate charged by banks to their most credit-worthy customers. As the loan risk to a bank increases they will often quote a rate such as “prime plus 1%”, etc. to cover their risk.

There is not one, universal, prime rate. Every bank is free to set its own prime rate. The Wall Street Journal quotes a prime rate that is based on the prime rates charged by the nation’s 30 largest banks. Generally the prime rate will be about 300 basis points above the Federal funds rate. Figure 4 shows the historical rates for prime for a recent 10-year period.

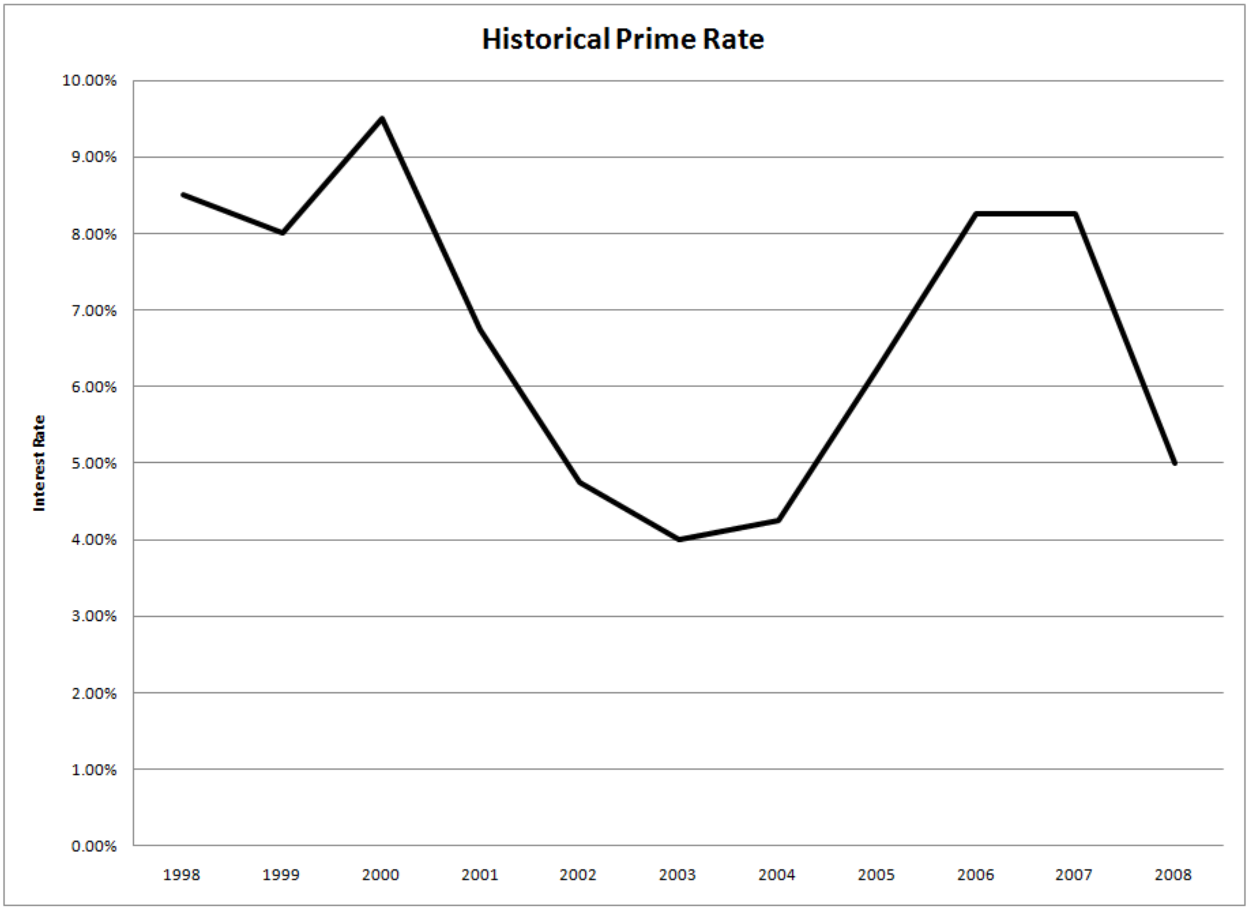


Figure 5

As you can see from Figure 5, the prime rate has varied from 10% to a low of about 4% during this 10-year period (these values were as of August of each year.)

LIBOR

The London Inter-bank Offered Rate (LIBOR) is another daily interest rate reference that is similar to the Federal funds rate. The LIBOR is used by British banking industry to make short-term interbank loans.

LIBOR is published each day by the British Banking Association (BBA) and it is based on the interbank interest loan rates of 16 British banks. The average of the 8 middle values of the 16 bank rates is used to compute the LIBOR. The BBA reports several LIBOR rates such as the overnight rate, 3-month rate, and a 6-month rate. Many adjustable rate mortgages are tied to the 6-month LIBOR. LIBOR rates are also popular for interest rate swaps involving either US dollars or the British Sterling.

EURIBOR

The Euro Interbank Offered Rate (EURIBOR) is becoming a widely used index that is similar to the LIBOR. The EURIBOR is based on the prime rate that is being quoted between banks in the European Union for short term loans of one week to one year. To determine the EURIBOR, the highest and lowest 15% of the quotes received are eliminated and the remaining values are averaged to determine the EURIBOR rate.

Around the world there are other similar types of rates such as the TIBOR (Tokyo) and HIBOR (Hong Kong) Inter-bank offered rates.

Summary

In this course we have reviewed the various terms that are used when discussing interest rates. Terms such as nominal interest rates, real interest rates, basis points, annual effective rate, yield curves, interest rate swaps, etc, were reviewed.

The various components that make up an interest rate including risk-free interest rate, inflation premium, default risk premium, maturity risk premium, and liquidity premium have been discussed and finally we reviewed several of the major interest indices in use today such as the Federal Funds Rate, Discount Rate, Treasury Rates, Prime Rate, LIBOR and EURIBOR.

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