PDHonline Course P209 (3 PDH)

Statistical Methods for Process Improvement - Part 3: Using Process Control Charts

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Statistical Methods for Process Improvement
Part 3: Using Process Control Charts

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"Industrial processes are much like small children, they behave better within well defined limits of behavior." – M. J. Moroney

Introduction to Process Control Charts

The control chart is primarily a tool used to analyze data which is generated over a period of time. Developed by Dr. Walter A. Shewhart in 1924, the control chart remains one of the most important tools in the SPC arsenal. Although simple to use, they are very powerful for process analysis.

In Part 3 of Statistical Methods for Process Improvement we will learn about using several types of control charts and how to interpret what a process is 'telling' us. However, before considering any specific chart, let's examine control charts in general.

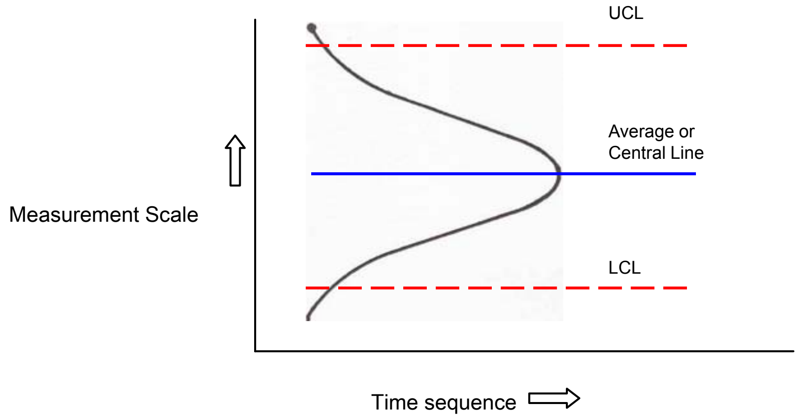
Process Control Charts

What are they? First of all, while the term "control chart" has been universally accepted and used, it must be understood that the chart does not control anything. It only provides information which serves as a basis for action. Control charts are effective only if those responsible for making decisions act on the information the chart reveals. In other words, control charts are tools which can be used, misused, or not used at all. Their effectiveness is directly related to the understanding of and proper application by the people using them, you and your colleagues.

To adequately control any repetitive operation requires continuous monitoring. The control chart is a graphical representation of this monitoring process.

How do they work? Control charts are all basically like the conceptual chart shown below. Along the vertical axis a scale is established consistent with the data being collected. The horizontal axis reflects the data sequence or time interval. As data is obtained, it can be plotted relative to these two axes.

The Conceptual Control Chart



Since we normally obtain data by sampling, let's assume we take a certain sample size from a source at regular intervals. Sample statistics (mean, median, range, fraction defective, etc.) are calculated. We know from the Central Limit Theorem that with proper sampling, these will vary in a specific way; namely, a normal distribution. By calculating the mean and standard deviation for each statistic and applying our knowledge of the normal distribution, we can estimate, or predict the group behavior of each sample statistic. This is done by calculating the grand mean and plotting it on the chart as a solid central line. We then measure up 3 standard deviations to plot the upper control limit and down 3 standard deviations to plot the lower control limit. These are shown on the chart as dashed lines.

Actually, what we have constructed is the normal distribution curve of the sample statistic and its $\pm 3\sigma$ limits. We know that for a normal distribution, the $\pm 3\sigma$ limits represent 99.73% of the distribution. We would expect that future samples would result in a sample statistic that would fall within the upper and lower control limits 99.73% of the time. This is a realistic expectation only if nothing has changed in the original distribution or source of the data.

What do they tell us? A control chart provides guides (control limits) which will indicate whether a population has shifted. If the limits are representative of the process we are monitoring, they also indicate the extent that common cause variation is influencing the distribution. Variation between the upper and lower limits can be expected and considered normal. Operator efforts to reduce this variation are ineffective; and in fact, such actions generally lead to more rather than less variation, a condition known as over control. A chart operating within the limits is saying, common cause variation at work--*do not adjust*. Since only common cause variation is present, special causes are absent; and we conclude that the process is in statistical control.

Data points which fall outside the control limits are very unlikely. If this situation occurs, it is an indication that the distribution has shifted. Such shifts are the result of special cause variation. Because special cause variation is present, we conclude that the process is not in control. Investigations should begin immediately to determine the source of the special cause variation so that corrective action can be taken to bring the process back under control.

Control charts simply tell us if the *process is in control or out of control*, that is, whether a search for special causes is warranted or the process should be left alone. More on the interpretation of control charts will be presented later in this course.

What do they *not* tell us? Too often people want to judge the data plotted on a control chart by the blueprint tolerance or specifications. Usually this is an attempt to verify that the process is producing acceptable quality results. People who do this suffer from a serious misunderstanding or misinterpretation of control charts.

A typical control chart, say an $\bar{X}$ & R chart, is based on subgroup averages. Blueprint specifications are for individual measurements. Simply because a subgroup average falls within the specification limits does not mean that the individual measurements are within those limits. Conversely, if an individual measurement is outside the control limits, it is erroneous to conclude that the process is out of control.

Remember, *the primary function of a control chart is to indicate whether the process is in control or out of control, not to judge the acceptability of an individual data point or part.* It is possible for a process to be in control yet produce out-of-spec material. A process also may be out of control and produce entirely acceptable material.

Types of Control Charts

The two major divisions in control charts result from the fact that there are two types of data - variable data and attribute data.

Whenever a record is made of an actual measured quality characteristic, such as a dimension expressed in thousandths of an inch, the quality is said to be expressed by variables. Variables are measurable characteristics such as a dimension, weight, purity, temperature, yield, tensile strength, flow rate, angularity, etc.

If a record shows only the number of articles conforming and failing to conform to specified requirements, the quality is said to be a record by attributes. Attributes are countable characteristics. For example, most visual examinations concern attributes. A shaft is either cracked or not; a bearing surface has an acceptable finish or it doesn't; a pump seal fails or it does not; scratches are present or not.

Types of Control Charts

Variables Charts

$\bar{X}$ - R Average and Range

$\tilde{X}$ - R Median and Range

$\bar{X}$ - S Average and Standard Deviation

X - R_m Individual and Moving Range

Attributes Charts

p Fraction Nonconforming

np Number of Nonconforming Units

c Number of Nonconformities

u Number of Nonconformities per unit

Requirements for Using Control Charts

Before applying control chart techniques to any process or operation certain essential requirements must be satisfied. First, the people in decision-making positions should understand and support efforts to continually improve processes and quality through the use of statistical methods such as control charts. When this has been achieved, the following criteria must be satisfied for a business to really experience process improvement using control charts.

1. Understand the process being studied. Understanding is necessary in order to make decisions regarding chart selection, sample size, sampling frequency, characteristics to be monitored and actions which can be taken to reduce variability.

2. Determine what characteristics are to be monitored. Usually these would be the ones most critical for satisfactory functioning of the process or product or perhaps the ones with the tightest specifications. They could be the characteristics most often found to be nonconforming or the ones most expensive to repair. A technique such as Pareto analysis is often used to select a single characteristic to be charted.

3. Define and understand the measurement system. The data on the chart is going to be no better than the measuring process or device. The required accuracy must be established so that proper measuring equipment can be selected. Measuring equipment should be reviewed for accuracy and repeatability, and calibrated regularly.

4. Select a chart type to best accomplish the purpose. Often this is dictated by the cost of measurement and the loss incurred if important changes go undetected when and if they should occur. Also, the type of information needed may dictate which chart to use.

   In general use an attribute chart if

   a. Measurements are not possible, as in visual inspection.

   b. Measurements are not practical due to cost of measuring equipment or excessive testing time.

   c. The part has many characteristics to evaluate.

   d. Plans are to use 100% inspection.

   Use a variables chart if

   a. A critical characteristic or a variable characteristic is involved.

   b. More precise control is desired than is possible with attribute charts.

5. Determine the sample size and frequency. For variables charts, the most common sample size is five. The sample subgroup should be selected to allow minimum opportunity for variation within the group. However, the larger the sample size, the more normal the distribution of subgroup averages even if the population is non-normal. Economic factors also affect the selection of sample sizes. If subgroups cannot be used, an individual and moving range chart may be used. In part 4 of this 5 part series of courses, the impact of sample size will be discussed further.

   Attribute charts usually have a much larger sample size - from 30 to several hundred or more. Since attribute data may be available from preexisting quality reports or studies, the time and expense have already been incurred.

   It is difficult to offer any general rule for frequency of sampling. Each case is different and must be decided on its own merits after considering the costs, risks and potential benefits. Usually, when beginning a chart, frequent samples are taken to allow conclusions to be reached quickly. Later, sampling frequency possibly may be reduced based on what is learned about the process.

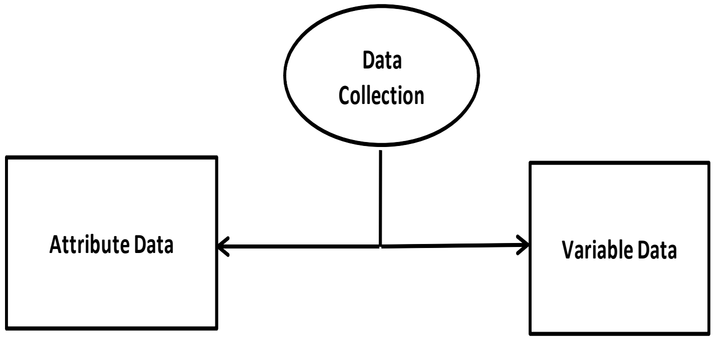
Control Chart Selection

In manufacturing plants some type of process trouble is, unfortunately, a rather common state of affairs. Control charts can be invaluable trouble-shooting aids for the engineer as well as for the operators. Control charts provide information in at least four areas of primary concern:

1. Basic variability of the characteristic
2. Consistency of performance
3. Average level of the characteristic
4. Type of variability present

The following chart depicts a general flow chart selection guide for the different types of basic control charts. Next is a table that presents a summary of the general control chart formulas for the more commonly used Shewhart control charts. These are discussed in the sections on variables and attribute charts.

Control Chart Selection Guide



Sample Size (n)	Chart Type
>20, not constant	p chart
>20, constant	np chart
>1; <20, not constant	u chart
1	c chart

Sample Size (n)	Chart Type
1	Individual and Moving Range
<10	Average & Range Median & Range
10 to 25	Average and Std. Deviation
>25	Average and Variance (σ^2)

p chart = fraction non conforming
np chart = number nonconforming

u chart = nonconforming/inspection unit
c chart = nonconforminginspection/unit

PROCESS CONTROL CHARTS

TYPE of DATA	CONTROL CHART	SAMPLE SIZE n	CONTROL CHARACTERISTIC	CONTROL FEATURE: CENTRAL LINE	CONTROL FEATURE: CONTROL LIMITS
VARIABLES	AVERAGE & RANGE $\bar{X}$-R	SMALL NORMALLY <10 USUALLY 3 to 5	$\bar{X}$-VARIATION of SAMPLE MEANS	$\bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$
			R-VARIATION of SAMPLE RANGES	$\bar{R}$	$UCL_R = D_4\bar{R}$ $LCL_R = D_3\bar{R}$
	AVERAGE & STANDARD DEVIATION $\bar{X}$-S	USUALLY 3 or GREATER	$\bar{X}$-VARIATION of SAMPLE MEANS	$\bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{S}$ $LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{S}$
			S-VARIATION of SAMPLE STANDARD DEVIATIONS	$\bar{S}$	$UCL_S = B_4\bar{S}$ $LCL_S = B_3\bar{S}$
	MEDIAN & RANGE $\tilde{X}$-R	SMALL NORMALLY <10 ODD NUMBER 3 or 5	$\tilde{X}$-VARIATION of SAMPLE MEDIANS	$\bar{\tilde{X}}$	$UCL_{\tilde{X}} = \bar{\tilde{X}} + \tilde{A}_2\bar{R}$ $LCL_{\tilde{X}} = \bar{\tilde{X}} - \tilde{A}_2\bar{R}$
			R-VARIATION of SAMPLE RANGES	$\bar{R}$	$UCL_R = D_4\bar{R}$ $LCL_R = D_3\bar{R}$
	CHART of INDIVIDUALS X-R_m	1	X-VARIATION of INDIVIDUALS	$\bar{X}$	$UCL_X = \bar{X} + E_2\bar{R}$ $LCL_X = \bar{X} - E_2\bar{R}$
			R-VARIATION between INDIVIDUALS	$\bar{R}$	$UCL_R = D_4\bar{R}$ $LCL_R = D_3\bar{R}$
ATTRIBUTE	p CONTROL CHART	VARIABLE >20	p: FRACTION NONCONFORMING $\bar{p} = \frac{\sum d}{\sum n}$	$\bar{p}$	$UCLp = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$ $LCLp = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$
	np CONTROL CHART	CONSTANT >20	np: No. of NONCONFORMING UNITS $n\bar{p} = \left(\frac{\sum d}{\sum n}\right)(n)$	$n\bar{p}$	$UCLnp = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ $LCLnp = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$
	c CONTROL CHART	CONSTANT >20	c: No. of NONCONFORMITIES $\bar{c} = \frac{\sum c}{k}$	$\bar{c}$	$UCLc = \bar{c} + 3\sqrt{\bar{c}}$ $LCLc = \bar{c} - 3\sqrt{\bar{c}}$
	u CONTROL CHART	VARIABLE >20	u: NONCONFORMITIES/INSP. UNIT $\bar{u} = \frac{\sum c}{\sum n}$	$\bar{u}$	$UCLu = \bar{u} + 3\sqrt{\bar{u}/n}$ $LCLu = \bar{u} - 3\sqrt{\bar{u}/n}$

Control Chart Formulas

Tables of Constants for Control Charts

Sample Size	All	Average & Range Charts			Median & Range Charts			Individual & Moving Range Charts			Average & Standard Deviation Charts			
(n)	d_2	A_2	D_3	D_4	$\tilde{A}_2$	D_3	D_4	E_2	D_3	D_4	A_3	B_3	B_4	C_4
2	1.128	1.880	0	3.267	—	0	3.3	2.659	0	3.267	2.659	0	3.267	0.7979
3	1.693	1.023	0	2.574	1.187	0	2.6	1.772	0	2.574	1.954	0	2.568	0.8862
4	2.059	0.729	0	2.282	—	0	2.3	1.457	0	2.282	1.628	0	2.266	0.9213
5	2.326	0.577	0	2.114	0.691	0	2.1	1.290	0	2.114	1.427	0	2.089	0.9400
6	2.534	0.483	0	2.004	—	0	2	1.184	0	2.004	1.287	0.030	1.97	0.9515
7	2.704	0.419	0.076	1.924	0.509	0.076	1.9	1.109	0.076	1.924	1.182	0.118	1.882	0.9594
8	2.847	0.373	0.136	1.864	—	0.136	1.9	1.054	0.136	1.864	1.099	0.185	1.815	0.9650
9	2.97	0.337	0.184	1.816	0.412	0.184	1.8	1.010	0.184	1.816	1.032	0.239	1.761	0.9693
10	3.078	0.308	0.223	1.777	—	0.223	1.8	0.975	0.223	1.777	0.975	0.284	1.716	0.9727

Summary

- Process control charts are tools which help us analyze data. Specifically, they distinguish between variability as a result of common causes or special causes.

- Processes are in control if only common causes are present.

- Processes are out of control if special causes are present.

- The two general categories of control charts are Variables Control Charts and Attribute Control Charts.

- Tables of constants have been developed for use with control charts to utilize the relationship between the samples and the population based on Shewhart's research. These are the only accepted methods of determining control limits.

- When the population is used to determine the control limits for an average and range chart, false indications of the state of control are evident because the control limits are to be based on the sampling distribution of averages and not the population distribution of individual values. The same holds true for the other variables charts.

Variables Control Charts

Introduction

By using variables control charts, the process performance can be recorded and monitored with respect to a selected variable. Variables are those quality characteristics which can be measured.

In this section, the types of variables charts to be discussed are as follows:

- Average and Range charts or "$\overline{X}$ and R" charts.
- Median and Range charts or "$\tilde{X}$ and R" charts.
- Average and Standard Deviation charts or "$\overline{X}$ and S" charts.
- Chart of Individuals or "X and R_m" charts.

The steps involved in developing all of these variables charts are basically the same. The differences are associated with the particular parameter to be plotted. These basic steps are presented below.

Steps for Developing and Plotting A Variables Control Chart

1. Enter the heading information on the chart. This may include such items as the name of the part, the part number, department, machine number, plant identification, operation being performed, etc.

2. Record the information about the sample. These are usually the date the sample was taken, the time, the initials of the checker, and shift on which the samples were taken.

3. Entered the measured data. In the appropriate places, enter the measurements that have been taken.

4. Calculate the statistic being monitored. This will be the average, the median, the range, the standard deviation, or other statistics.

5. Establish the scales for the graphs. Scales should be used which make it easy to do the plotting. As a general rule, the range of values for the central tendency measure should be either the product specification or twice the difference in the highest and lowest subgroup statistics. The chart for monitoring variability should have scales beginning at zero and extending to a value of about 1.5 to 2 times the maximum calculated measure of variability.

6. Plot the data.

7. Calculate and plot the central lines. There will be a central line for both portions of the chart - central tendency and variability. These should be drawn in as solid lines and labeled.

8. Calculate and plot the control limits. These are calculated using the specific formulas for the statistics being monitored. The formulas are shown on a previous page in this course. The necessary constants are shown as well. The control limits should be drawn in as dashed lines on the chart and labeled.

9. Interpret the chart. This will be discussed in detail later in this course.

Control Limit Calculations

As you remember from the section "Introduction to Control Charts", the upper and lower control limits are, in reality, simply the $\pm 3\sigma$ limits for the sample statistic. Let's look at the formulas which have been developed for $\overline{X}$ and R charts.

The upper control limit (UCL) and lower control limit (LCL) are calculated as follows:

$$UCL = \bar{X} + A_2 R$$
$$LCL = \bar{X} - A_2 R$$

$$UCL_R = D_4 R$$

$$LCL_R = D_3 R$$

These equations have the basic form of $\bar{X} \pm 3\sigma_{\bar{x}}$. Where $A_2\bar{R}$ is a means of determining 3 standard deviations of the of the sample statistic, in this case, $\bar{X}$. By calculating $\bar{R}$ and using the constant A_2 we can determine the 99.73% limits for the statistic we are monitoring. Although A_2 is a constant, it does depend on the sample size. For the Range chart, the distribution is not normal as it is bounded at Zero and Infinity, thus the calculations take that into consideration. For subgroup sizes ≤6 the lower control limit for the Range will be zero (see the table of constants).

A table of the constants used in variable control charting is shown in the table below.
Formulas have been derived for specific charts and constants established. But in every case, the control limit calculations allow you to calculate the ±3 standard deviation limits.

Sample Size	All	Average & Range Charts			Median & Range Charts			Individual & Moving Range Charts			Average & Standard Deviation Charts			
(n)	d_2	A_2	D_3	D_4	$\sim A_2$	D_3	D_4	E_2	D_3	D_4	A_3	B_3	B_4	C_4
2	1.128	1.880	0	3.267	—	0	3.3	2.659	0	3.267	2.659	0	3.267	0.7979
3	1.693	1.023	0	2.574	1.187	0	2.6	1.772	0	2.574	1.954	0	2.568	0.8862
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10	3.078	0.308	0.223	1.777	—	0.223	1.8	0.975	0.223	1.777	0.975	0.284	1.716	0.9727

Constants for Variables Control Charts

Average and Range ($\bar{X}$ - R) Charts

The keys to plotting an Average and Range chart are the central line calculations and the formulas for the control limits. These are shown below. Constants are from the table above.

<u>Central Lines</u>

$$\bar{\bar{X}} = (\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \ldots\ldots \bar{X}_n) \div n$$

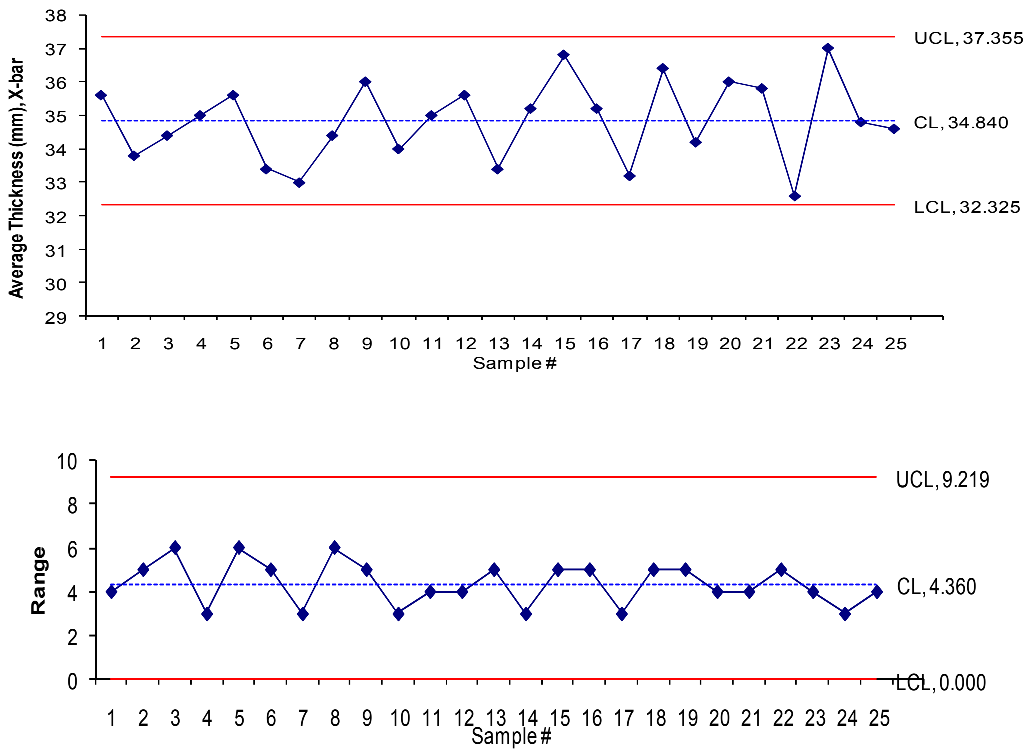
Or stated in words, it is the sum of the subgroup (sample) averages divided by the number of subgroups or samples

$$\bar{R} = \sum_{1}^{n} Ri \div n,$$

Or stated in words it is the sum of the Ranges divided by the number of subgroups or samples

<u>Control Limits</u>

$$UCL = \bar{\bar{X}} + A2\bar{R}$$
$$LCL = \bar{\bar{X}} - A2\bar{R}$$
$$UCL\,R = D4\bar{R}$$
$$LCL\,R = D3\bar{R}$$



Average and Range Control Chart

Note: By doing an internet search, one can find a variety of templates that allow you to simply enter the data and let excel® or another program do all the work for you. However, it is important to understand the statistics involved to enable proper interpretation of the results.

Just to be sure the concepts are clear, another example that shows the data by subgroups, calculations and control chart is shown below:

Data Collection for Average and Range Chart/Process Capability

Subgroup	X_1	X_2	X_3	X_4	X_5	Ave	Range
1	41	41	46	44	43	43.0	5
2	42	41	42	41	45	42.2	4
3	41	40	44	43	48	43.2	8
4	43	46	47	49	42	45.4	6
5	47	43	49	44	44	45.4	6
6	44	49	45	46	43	45.4	6
7	46	44	42	40	48	44.0	6
8	45	41	44	48	46	44.8	7
9	49	49	47	45	46	47.2	4
10	47	41	43	42	45	43.6	6
11	46	45	42	48	45	45.2	6
12	41	47	42	46	44	44.0	6
13	49	45	46	43	49	46.4	6
14	46	41	45	45	45	44.4	5
15	47	48	41	43	47	45.2	7
16	46	42	44	49	50	46.2	8
17	42	44	42	41	46	43.0	5
18	41	46	49	47	45	45.6	8
19	45	49	41	43	48	45.2	8
20	42	49	44	47	48	46.0	5
21	49	46	41	45	45	45.2	8
22	42	49	46	42	41	44.0	8
23	41	42	46	43	49	44.2	8
24	41	44	45	44	44	43.6	4
25	48	43	44	50	43	45.6	7
					Average	44.7	
					Ave. Range		6.3

Note: Range = hi-lo in each subgroup

Central Line = Average of Subgroup Averages = 44.7 and the Average Range = 6.3

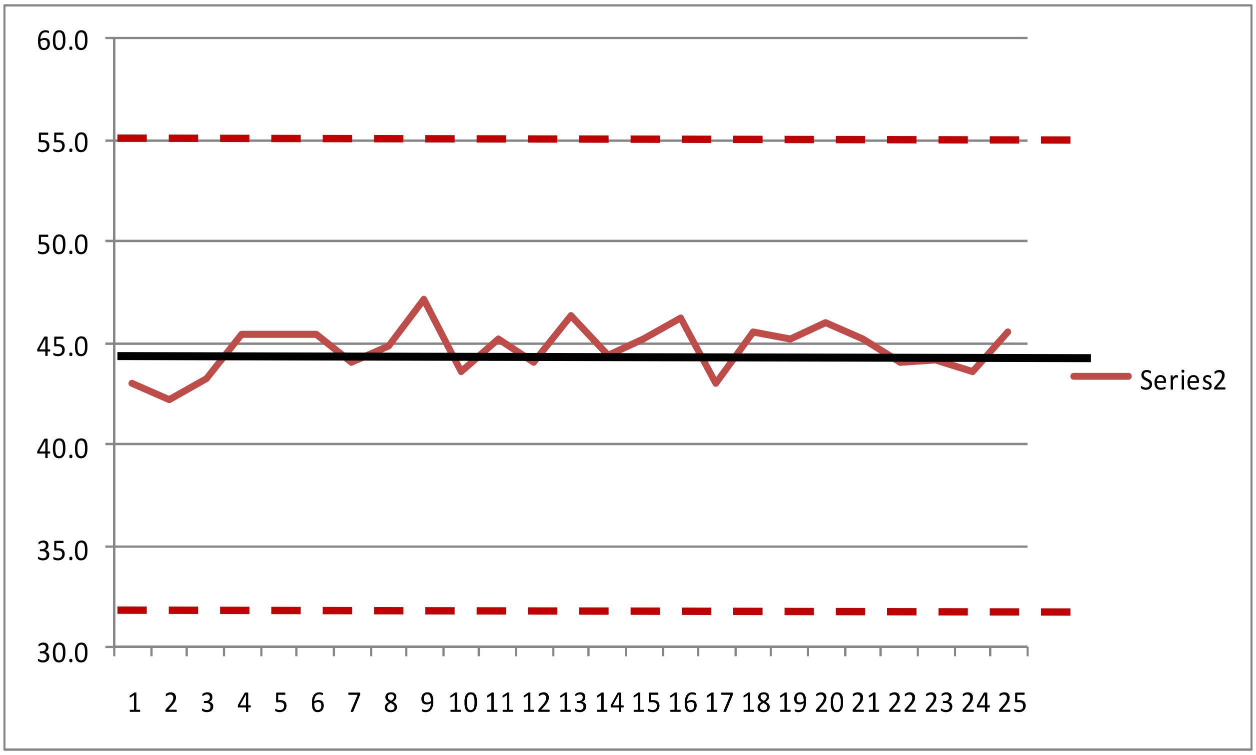
UCL = 44.7 + 3(0.577 x 6.3) = 44.7 + 10.9 = 55.6

LCL = 44.7 - 3(0.577 x 6.3) = 44.7 - 10.9 = 33.8

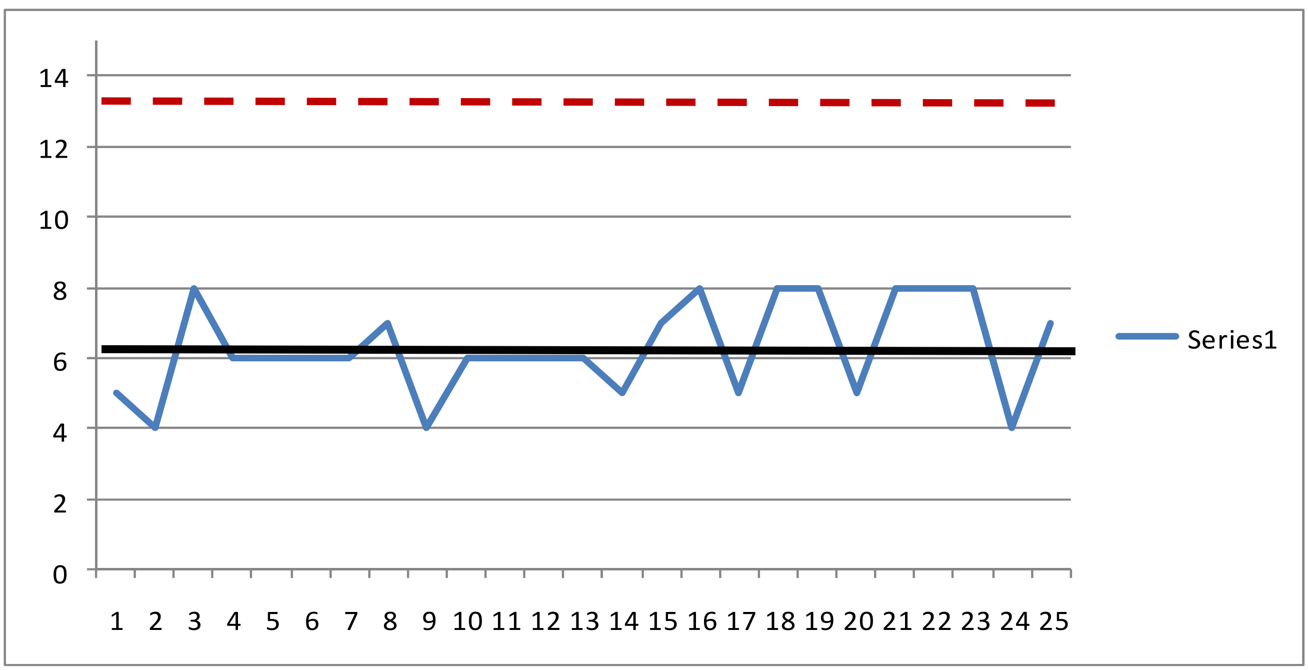
UCL_R = Ave R x D4 = 6.3 x 2.114 = 13.3

LCL_R = Ave R x D3 = 6.3 x 0 = 0

The Average Chart:



The Range Chart:



Median and Range ($\tilde{X}$ and R) charts

The calculations for establishing a median and range chart are like average and range charts, except the median is used for each subgroup and the overall median is used as the central line. The

R chart is constructed exactly like the R chart for an average and range chart. Median charts are seldom used, but are available if needed.

Average and Standard Deviation ($\bar{X}$ and S) Charts

The average and standard deviation chart has not been used much in industry because of the required calculation of the standard deviation. However, with excel® or other statistical packages, this chart could easily become a manufacturing fixture. The formulas for developing such a chart are: Again, it is mentioned herein for completeness and to expose the reader to all of the tools available. To see the formulas used, go to the table of control chart formulas.

Charts of Individuals (X and R_m)

With some processes, it may not be practical to utilize subgroups. Instead, individual measurements or readings are collected. Since it is not possible to determine a range with only one observation, a moving range is calculated instead. The variables plotted on the control chart are then the individual readings and the moving range.

The steps in developing this chart are outlined below:

1. Record the data.
2. Calculate the moving range, R_m. By taking the difference between the first and second observations, record it; then, the difference in the second and third, third and fourth, etc. Just as the range is the difference between largest and smallest values within a subgroup, the moving range is the difference between each subsequent individual value.
3. Plot the individual values, X, and the moving range on their respective charts.
4. Calculations of chart characteristics:

<u>Central Lines</u>

The central line $\bar{X} = \sum_{1}^{x} xi \,/n$

Or, in words simply add the individual values and divide by how many values you have.

The central line for $\bar{R}m = \sum_{1}^{n} Rm \,/n\text{-}1$

Or, in words simply add the moving ranges and divide by the number of moving ranges you have minus 1.

Control Limits

$$UCL = \bar{X} + E2\ \bar{R}m$$

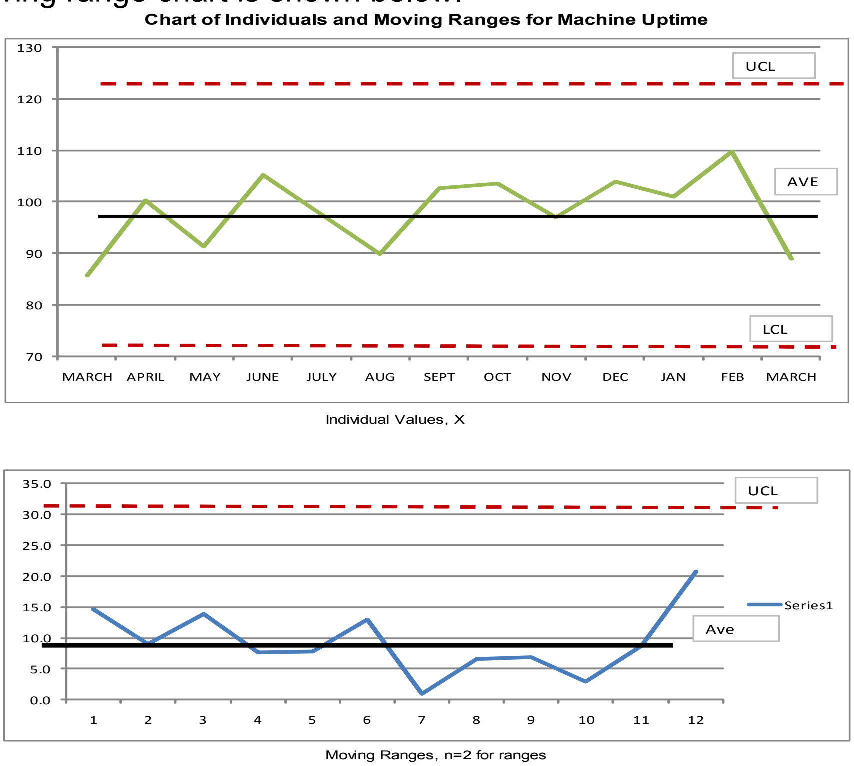
$$LCL = \bar{X} - E2\ \bar{R}m$$

$$UCL_{Rm} = D4\ \overline{Rm}$$

$$LCL_{Rm} = D3\ \overline{Rm}$$

5. Interpretation of the chart is similar to the other variables charts except the tests for non-randomness are less significant on the moving range.

An example of a moving range chart is shown below.



Individual and Moving Range Chart

Control Chart Interpretation

Once data have been recorded on a process control chart, the task of interpreting the chart begins. The control chart is used to indicate when special cause variation is occurring. The range of common cause variation or random, inherent dispersion is represented by the upper and lower control limits on the chart. Whenever nonrandom or special cause variation occurs it can manifest itself in several ways.

The first rule of chart interpretation is to **look for points that fall outside the control limits.** Remember, ***evaluate the RANGE chart first*** on a variables chart. If the range chart is out of control, it is essentially useless to attempt evaluation of the average (individual, median, etc.) chart. Whether the results are outside the upper or lower limits, the indication is that some unusual circumstances have developed which have shifted the distribution of the variable. Points falling outside the control limits are very strong indicators and the likelihood of a "false" alarm is extremely remote.

Next, look for trends in the data. Trends are depicted by **seven points in a row** in an increasing or decreasing pattern. Trends are examples of nonrandom patterns in the data. Another way the nonrandom behavior of the data exhibits itself is by a non-normal distribution of the control chart variable. The Central Limit Theorem states that the distribution of the sample means will form a normal distribution. If we find that this has not occurred, then something has influenced the data. A quick check for a normal distribution is to count the number of points in the middle third of the chart. Since the middle third represents the $\pm 1\sigma$ limits, approximately 2/3 of the points should be within this portion of the chart. (If you recall, from Part 2 of Statistical Methods for Process Improvement, Using the Normal Distribution, $\pm 1\sigma$ encompasses 68.26% of the data of a normal distribution.) With most of the data falling near the central line of the chart, we would expect to find only a few points near the control limits. Several simple rules for checking for non-randomness are shown below. These rules are based on probabilities that are essentially equivalent to finding one point outside the control limits.

Simple Tests for Non-randomness

2 of 3 consecutive points outside 2σ limits

4 of 5 consecutive points outside 1σ limits

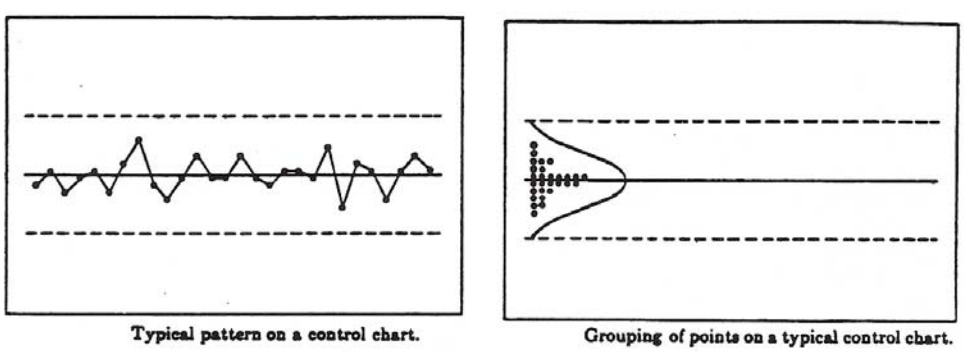
8 consecutive points on one side of the mean

15 consecutive points within the ±1σ limits

(Check the chart above and below the central line independently)

This last test is to determine if stratification is occurring. Stratification is associated with a stable mixture which exhibits an artificial or false stability. Instead of varying randomly within the control limits, stratified data will tend to hug the central line. Sometimes the distribution is referred to as "unnaturally quiet".

Do not make the mistake of concluding that a stratified pattern is showing the process to be in good statistical control. Just the opposite is true. Stratification results whenever samples are taken in a consistent manner from very different distributions. Sometimes this occurs by taking one part from each operator in a group, or one part from each machine, or one part from each cavity in a multi-cavity molding machine, etc. Usually people sample in this manner in an effort to make the samples "representative". Instead, the deliberate pattern leads to a non-randomness and subsequently to a lack of natural fluctuation.



Typical pattern on a control chart. Grouping of points on a typical control chart.

Control Chart Typical Random Pattern

There are several other simple tests, similar to these. However, these few tests will allow adequate interpretation of the chart. These tests are all based upon the probability of occurrences, and are indications of special causes of variability. One must remember that chart interpretation rules should be applied along with a good deal of common sense and judgment, based on your knowledge of the process and measurement system.

Other Tests

While there are “tests” for the number of runs above and below the central line or the length of runs, the simple tests listed above will provide a solid basis for chart interpretation. If you desire more information on control chart interpretation, see *Statistical Quality Control Handbook*, Western Electric.

Summary

1. Variables control charts are used for measurable characteristics.
2. There are four general types of variables charts that may be used to monitor a process:
    - Average and Range Charts
    - Median and Range Charts
    - Average and Standard Deviation Charts
    - Individuals and Moving Range Charts
3. Control charts must be interpreted for nonrandom variation which is symptomatic of special causes.
4. Interpretation of control charts will indicate whether a process is in control, out of control, or developing a trend.
5. The real work begins when the chart is interpreted and corrective or preventive actions are indicated.
6. Always look for the “test of reasonableness.” Ask yourself, “Does this make sense given what I know about this process?”

Attribute Control Charts

Introduction

An important distinction in process control is that between variables and attributes. When data is gathered on an actual measured quality characteristic, such as a dimension expressed in thousandths of an inch, the quality is said to be expressed in variables. When we look at the number of articles failing to conform to any specified requirement, it is expressed as an attribute. The section titled "Introduction to Control Charts" discusses attributes and variables in more detail.

Control Charts for Attributes

There are several different types of control charts for attributes. These are summarized below.

1. The p chart monitors the proportion of items nonconforming to specifications.
2. The np chart monitors the actual number of nonconforming items.
3. The c chart monitors the actual number of nonconformities.
4. The u chart monitors the number of nonconformities per unit.

At this point, it may be good to review the difference between nonconformities and nonconforming units. An article which does not conform in some way to a given specification can be described as nonconforming. Every nonconforming article contains one or more nonconformities.

Control Chart for Nonconforming Units

The most versatile and widely used attributes control chart is the p chart. This is the chart for the fraction rejected as nonconforming to specifications. It is applied to quality characteristics which can be observed as attributes only or to those that are considered as attributes. As long as the result of the inspection is the classification of an individual article as accepted or rejected, a single chart may be applied to one or any number of quality characteristics contained in the article.

To become better acquainted with the use of p charts, consider an experiment that fairly well describes the use of p charts in an industrial environment. Suppose 10,000 beads of the same size and density are placed in a container. Of these beads, 9,500 are white and 500 are red. The white beads represent articles which conform to specifications, and the red beads represent nonconforming articles. Samples of 50 beads are drawn from the 10,000 beads. If the beads are replaced after each sample has been drawn and all the beads are thoroughly mixed before the next drawing, the theory of probability enables us to calculate the relative frequency, in the long run, of getting exactly 0, 1, 2, 3, 4, 5, etc. red beads.

Any one sample of beads drawn from the container is a sample from a very large quantity of beads with 5% red. As a matter of chance, variations from sample to sample in the number of red beads are inevitable. In a similar way, we may think of a day's production (or other lot) of any manufactured article or part as a sample from a larger quantity with some unknown fraction which do not conform to specifications.

p Charts

Shown below are the basic calculations necessary to develop a p chart. Just as with variables charts, the calculations are for the central line and the control limits. On the following page, an example of a p chart is shown.

<u>Central Lines</u>

The central line is simply the average proportion nonconforming, or as a formula:

$\bar{p}$ = Total Number Nonconforming/Total Number Inspected, expressed as a decimal

<u>Control Limits</u>

$$UCL_p = \bar{p} + \frac{3\sqrt{\bar{p}(1-\bar{p})}}{n}$$

$$LCL_p = \bar{p} - \frac{3\sqrt{\bar{p}(1-\bar{p})}}{n}$$

Attribute Control Chart: Proportion Non-Conforming (p chart)

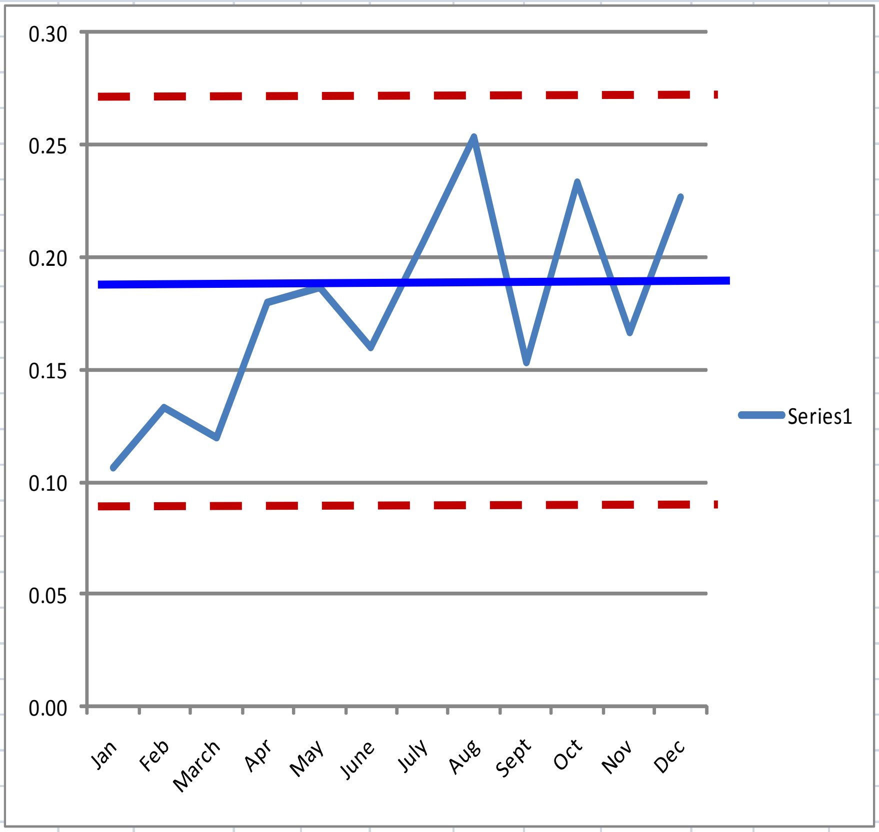
Nonconformities in Purchase Orders

N=150/month examined

Type of n/c	**Jan**	**Feb**	**March**	**Apr**	**May**	**June**	**July**	**Aug**	**Sept**	**Oct**	**Nov**	**Dec**	**Total**
Missing signature	1	2	1	7	4	6	5	9	7	6	1	9	58
Incorrect dates	3	1	0	3	9	4	1	9	1	5	7	5	48
Inadequate info	1	2	4	9	5	2	4	7	3	2	2	6	47
Supplier not approved	1	1	3	4	6	0	8	5	2	8	6	3	47
Exceeded limits	6	5	8	4	3	8	6	6	5	5	4	9	69
Other	4	9	2	0	1	4	7	2	5	9	5	2	50
Total non-conforming	**16**	**20**	**18**	**27**	**28**	**24**	**31**	**38**	**23**	**35**	**25**	**34**	**319**
Proportion non-conforming (p)	**0.11**	**0.13**	**0.12**	**0.18**	**0.19**	**0.16**	**0.21**	**0.25**	**0.15**	**0.23**	**0.17**	**0.23**	**0.18**
													(319/1800)

Ave. Proportion N/C = 0.18
(Central Line)

UCL = 0.27
LCL = 0.09



Example of p Chart for Purchase Order Non-conformities

np Charts

The choice among the control chart techniques for attribute data is based primarily upon convenience in the interpretation of the chart. p charts are used when the number of items inspected are constant or varying, and the statistic of interest is the fraction rejected.

Whenever the subgroup size is variable, the chart must show the fraction rejected rather than the actual number of rejects. *If the **subgroup size is constant**, the chart for actual number of rejects (non-conformances) may be used.* Such a chart is called an np chart.

The formula for np charts central line and control limits are shown below:

<u>Central Lines</u>

$n\overline{P}$ = Number Nonconforming

<u>Control Limits</u>

$$UCL_{np} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$$

$$LCL_{np} = \overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}$$

c Charts

We have already discussed that a nonconforming article is an article that in some way fails to conform to one or more given specifications. Each instance of the article's lack of conformance is a *nonconformity*. Where it is appropriate to make a total count of the number of nonconformities in each article, or group of articles, either a c chart or a u chart is used. Examples of c chart applications could be:

1. The number of n/c rivets in an aircraft wing.
2. The number of imperfections observed in a galvanized sheet.
3. The number of imperfections found in a bolt of cloth.

The basic formulas for charts are:

<u>Central Lines</u>

Simply the average number of nonconformities per unit and the sample size is one. ($\bar{C}$)

Control Limits

The control limits are simply the 3 standard deviation limits of the distribution, which in this distribution is actually the square root of the average, or $\sqrt{\bar{C}}$, thus the control limits are:

$$UCL_C = \bar{C} + 3\sqrt{\bar{C}}$$

$$LCL_C = \bar{C} - 3\sqrt{\bar{C}}$$

u Charts

When working with c charts, the quantity c was the number of nonconformities observed in some specified inspection of a constant size, thus the distribution is bounded at 0 and 1. This inspection unit could be a single unit of product, 10 products or any number of products. When plotting on control charts, the total number of nonconformities per inspection unit were plotted just as if the subgroup were a unit of product.

Whenever there is an evident change in the *area of opportunity* for occurrence of a nonconformity from subgroup to subgroup, a chart showing only the total number of nonconformities is not applicable. For example, if a number of units constitute a subgroup of size n, where n varies from subgroup to subgroup, defects per unit (c/n) may be an appropriate control statistic. In such cases, if total defects observed in each subgroup were plotted, the central line on the chart as well as the control limits would change from one subgroup to another.

The symbol 'u' is used to represent nonconformities per unit or c/n where c is the nonconformities found and n is the number of items. The central line on the chart would then be u.

Formulas for finding the central line and the control limits for u charts are shown below:

Central Lines

$$\overline{U} = \frac{Total\ nonconformities}{Total\ number\ inspected}$$

Control Limits

$$UCL_U = \overline{U} + \frac{3\sqrt{\overline{U}}}{n}$$

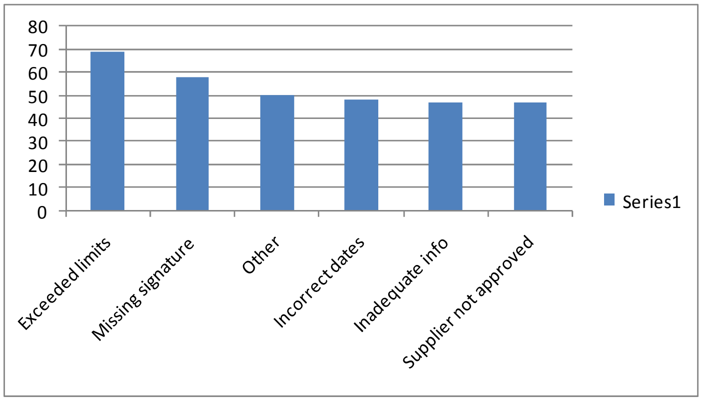
$$LCL_U = \overline{U} - \frac{3\sqrt{\overline{U}}}{n}$$

Chart Interpretation

For attribute charts, remember that if data points show up on a chart, something is not right with our process, nonconformities have occurred. So, there is no range chart as in variables charts. Also, these are not based on a normal distribution, so some of the typical rules do not apply. Instead, look for:

- A point outside the control limits
- Trends in the data
- Significant runs (8 or more points above or below the mean)

With attribute charts, it is as important to investigate a process performing “better than expected” as it is a process that is trending towards an upper control limit. Sometimes, with attribute charts we discover a process is actually improving and when that is the case we need to apply a problem solving approach to determine why it is better so we can work to sustain the improvements and/or transfer the knowledge to other processes. For example, let’s use the p chart for purchase orders we looked at earlier. When the data are summarized for a 12 month period, a Pareto Chart of the non conformities quickly shows us the top issue, is purchase orders that exceeded limits. Now, we can begin taking corrective action based on the process performance and not on an isolated incident.



Summary Control Charts

Why Use Summary Charts?

It is easy to become inundated with control charts that have been completed by operators, electronically or other personnel.

For control charts to be utilized as a management tool, it is important to establish an easy to use method of summarizing control chart information. Summary Control Charts are an easy way for one to determine the overall impact of SPC in an operation. Both Attribute Charts and Variables Charts can become summary charts.

p Charts as Summary Charts

The following procedure outlines the methods used:

1. Tabulate the total number of data points plotted on all control charts. This may be done daily, weekly or monthly. The control charts may be from one department, one process, one product or the entire plant. Record the total number of points on the p Chart. This will become the summary chart.

2. Tabulate the number of out of control points for the same data used in #1, above.

3. Calculate p for each set of data. This is the fraction non-conforming to control limits. Plot on the summary chart.

4. When sufficient data are available, calculate the control limits for the summary chart. You may use average sample size or the formula for moving control limits, depending upon the actual data.

5. Evaluate the chart as any other attribute chart.

The p Chart as a summary control chart allows you to evaluate the overall performance of SPC efforts in a given area by:

1. Observing the total number of data points plotted on charts.
2. Comparing how processes are performing in relation to control limits.
3. Improvements in performance can be observed.
4. More data are available for decision making.

Average and Standard Deviation Charts as Summary Charts

The following procedure outlines the methods used:

1. When a particular variables control chart is filled with data, it is time to summarize the data for future reference and use.

2. Calculate $\bar{X}$ (or $\bar{\bar{X}}$) for the data on the chart.

3. Calculate S or σ using $\bar{R}/d_2$ for the chart. You may calculate S directly from all the data points if you prefer.

4. Transfer the values from #2 and #3 to the Summary Chart as data for one sample for this particular variable.

5. After completing at least ten charts for a variable, and ten data points for the summary chart, calculate control limits for the summary chart.

6. Evaluate the summary chart as any other variables chart.

Summary variables charts enable one to view the data for a particular variable over a longer period of time, thus allowing evaluation by:

1. Results being monitored over time.
2. Long term trends or shifts are detected in a process or product.
3. Data are available for decision making concerning process changes, capital expenditures, etc.
4. Process data are made more manageable and useful.

Getting the Most Benefits From Control Charts in Your Business

To gain the most benefit it is important to define and implement a uniform method of reviewing control charts for continual improvement.

1. Define a routine review plan for control charts.
2. Summarize out of control conditions and special causes on an established frequency.
3. Maintain the summary control charts.
4. Review major special causes and do Pareto and/or other studies as required.
5. Re-calculate control limits as needed.
6. Assign specific responsibilities to specific personnel.
7. Take appropriate corrective actions using your corrective action process

These three general portions of a process control/corrective action plan can be expanded and tailored to any of your facilities to ensure consistent response to what our processes are "telling us".

Glossary of Statistical Terms

Accuracy - The closeness of agreement between and observed value and an accepted reference value.

Alternative Hypothesis - The hypothesis that is accepted if the null hypothesis is disproved.

Attribute Data - Qualitative data that typically shows only the number of articles conforming and the number of articles failing to conform to a specified criteria. Sometimes referred to as Countable Data.

Average - The sum of the numerical values in a sample divided by the number of observations.

Bar Chart - A chart that uses bars to represent data. This type of chart is usually used to show comparisons of data from different sources.

Batch - A definite quantity of some product or material produced under conditions that are considered uniform.

Bias - A systematic error which contributes to the difference between a population mean of measurements or test results and an accepted reference value.

Bimodal Distribution - A distribution with two modes that may indicate mixed data.

Binomial Distribution - A distribution resulting from measured data from independent evaluation, where each measurement results in either success or failure and where the true probability of success remains constant from sample to sample.

Cells - The bars on a histogram each representing a subgroup of data.

Common Cause - A factor or event that produces normal variation that is expected in a given process.

Confidence - The probability that an interval about a sample statistic actually includes the population parameter.

Control Chart - A chart that shows plotted values, a central line and one or two control limits and is used to monitor a process over time.

Control Limits - A line or lines on a control chart used as a basis for judging the significance of variation from subgroup to subgroup. Variation beyond a control limit shows that special causes may be affecting the process. Control limits are usually based on the 3 standard deviation limits around an average or central line.

Countable Data - The type of data obtained by counting -attribute data.

Data - Facts, usually expressed in numbers, used in making decisions.

Data Collection - The process of gathering information upon which decisions to improve the process can be based.

Detection - A form of product control, not process control, that is based on inspection that attempts to sort good and bad output. This is an ineffective and costly method.

Distribution - A group of data that is describable by a certain mathematical formula.

Frequency Distribution - A visual means of showing the variation that occurs in a given group of data. When enough data have been collected, a pattern can usually be observed.

Histogram - A bar chart that represents data in cells of equal width. The height of each cell is determined by the number of observations that occur in each cell.

k - The symbol that represents the number of subgroups of data. For example, the number of cells in a given histogram.

Lower Control Limit (LCL) - The line below the central line on a control chart.

Mean - The average value of a set of measurements, see Average.

Measurable Data - The type of data obtained by measurement. This is also referred to as variables data. An example would be diameter measured in millimeters.

Median - The middle value (or average of the two middle values) of a set of observations when the values have been ranked according to size.

Mode - The most frequent value in a distribution. The mode is the peak of a distribution.

n - The symbol that represents the number of items in a group or sample.

np - The symbol that represents the central line on an np chart.

Nonconformities - Something that doesn't conform to a drawing or specification; an error or reason for rejection.

Normal Distribution - A symmetrical, bell-shaped frequency distribution for data. This is a distribution that is often seen in industry.

Null Hypothesis - The hypothesis tested in test of significance that there is no difference (null) between the population of the sample and the specified population (or between the populations associated with each sample).

Out of Control - The condition describing a process from which all special causes of variation have not been eliminated. This condition is evident on a control chart by the presence of points outside the control limits or by nonrandom patterns within the control limits.

p - The symbol on a p chart that represents the proportion of nonconforming units in a sample.

Parameter - A constant or coefficient that describes some characteristics of a population (e.g. standard deviation, average, regression coefficient).

Pareto Charts - A bar chart that arranges data in order of importance. The bar representing the item that occurs or costs the most is placed on the left-hand side of the horizontal axis. The remaining items are placed on the axis in descending (most to least) order.

Population - All members, or elements, of a group of items. For example, the population of parts produced by a machine includes all of the parts the machine has made. Typically in SPC we use samples that are representative of the population.

Prevention - A process control strategy that improves quality by directing analysis and action towards process management, consistent with the philosophy of continuous quality improvement.

Process - Any set of conditions or set of causes working together to produce an outcome. For example, how a product is made.

Product - What is produced; the outcome of a process.

Quality - Conformance to requirements.

Random Sampling - A data collection method used to ensure that each member of a population has an equal chance of being part of the sample. This method leads to a sample that is representative of the entire population.

Range - The difference between the highest and lowest values in a subgroup.

Repeatability - The variation in measurements obtained when one operator uses the same test for measuring the identical characteristics of the same samples.

Reproducibility - The variation in the average of measurements made by different operators using the same test when measuring identical characteristics of the same samples. (In some situations it is the combination of operators, instruments and locations.)

Run Chart - A line chart that plots data from a process to indicate how it is operating.

Sample - A small portion of a population.

Sampling - A data collection method in which only a portion of everything made is checked on the basis of the sample being representative of the entire population.

Significance Level () - The risk we are willing to take of rejecting a null hypothesis that is actually true.

Skewed Distribution - A distribution that tapers off in one direction. It indicates that something other than normal, random factors are effecting the process. For example, TIR is usually a skewed distribution.

Special Cause - Intermittent source of variation that is unpredictable, or unstable; sometimes called an assignable cause. It is signaled by a point beyond the control limits or a run or other nonrandom pattern of points within the control limits. The goal of SPC is to control the special cause variation in a process.

Specification - The extent by which values in a distribution differ from one another; the amount of variation in the data.

Standard Deviation (σ) - The measure of dispersion that indicates how data spreads out from the mean. It gives information about the variation in a process.

Statistic - A quantity calculated from a sample of observations, most often to form an estimate of some population parameter.

Statistical Control - The condition describing a process from which all special causes of variation have been eliminated and only common causes remain, evidenced by the absence of points beyond the control limits and by the absence of nonrandom patterns or trends within the control limits.

Statistical Methods - The means of collecting, analyzing, interpreting and presenting data to improve the work process.

Statistical Process Control (SPC) - The use of statistical techniques to analyze data, to determine information, and to achieve predictability from a process.

Statistics - A branch of mathematics that involves collecting, analyzing, interpreting and presenting masses of numerical data.

Subgroup - A group of consecutively produced units or parts from a given process.

Tally or Frequency Tally - A display of the number of items of a certain measured value. A frequency tally is the beginning of data display and is similar to a histogram.

Tolerance - The allowable deviation from standard, i.e., the permitted range of variation about a nominal value. Tolerance is derived from the specification and is NOT to be confused with a control limit.

Trend - A pattern that changes consistently over time.

Type I Error (α) - The incorrect decision that a process is unacceptable when, in fact, perfect information would reveal that it is located within the "zone of acceptable processes".

Type II Error(β) - The incorrect decision that a process is acceptable when, in fact, perfect information would reveal that it is located within the "zone of rejectable processes".

u - The symbol used to represent the number of nonconformities per unit in a sample which may contain more than one unit.

Upper Control Limit (UCL) - The line above the central line on a control chart.

Variables - A part of a process that can be counted or measured, for example, speed, length, diameter, time, temperature and pressure.

Variable Data - Data that can be obtained by measuring. See Measurable Data.

Variation - The difference in product or process measurements. A change in the value of a measured characteristic. The two types of variation are within subgroup and between subgroup. The sources of variation can be grouped into two major classes: Common causes and Special Causes.

X - The symbol that represents an individual value upon which other subgroup statistics are based.

$\bar{X}$ (x bar) - The average of the values in a subgroup.

$\bar{\bar{X}}$(x double bar) - The average of the averages of subgroups.

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