PDHonline Course P211 (1 PDH)

Statistical Methods for Process Improvement - Part 5: Understanding Process Capability

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Statistical Methods for Process Improvement

Part 5: Understanding Process Capability

How to determine and use Cpk correctly

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Course Content

"Numbers are symbols for things; the number and the thing are not the same."
More funny Ashley-Perry Statistical Axioms quotes

Understanding Process Capability

Introduction

Process capability is perhaps the most misunderstood and abused value in the statistical arsenal. Process capability is actually the common cause variation in a process under certain conditions as compared to the allowable tolerances or specifications. It is the measure of how well a process in a state of statistical control will meet the specifications for a given dimension.

In manufacturing terminology process capability is "the inherent ability of the process to turn out a similar product." This is the best distribution that can be maintained in statistical control for a sustained period of time under a given set of conditions. This is described numerically with a process capability index, Cpk or Cpi. It is important to remember that the process capability is not the same as process performance, because performance over time includes all sorts of unnecessary variables and process "upsets" in the system of causes. The performance index is usually designated Ppk or Ppi.

Many times short term studies are conducted to define "process potential", which is just another term for "short term capability." In this class we will discuss the calculations and methods for determining process capability. Of course, one can certainly call this process potential and/or translate the same methods into process performance studies.

In general, a process potential study considers 30 - 300 measurements and process capability is for > 300 measurements over a period of time that represents the true process performance. This is not the "best ever" time period either!

Before we get into process capability, let's take this opportunity to review variability and how it affects process control. (These concepts are discussed in more detail in P207, P208 and P209 in this five part series of Statistical Methods for Process Improvement.)

Variability and Process Control

Process control can be summarized:

In control = only common causes present

Out of control = special cause(s) present

Using the tools of Statistical Process Control (SPC) allows us to separate the special causes from the common ones in a process. (See P209 for the proper application of control charts and SPC). Once this is done, we must identify, correct and eliminate the special causes. In this method, action is taken before the production of a large amount of nonconforming product. Using **prevention** rather than detection, scrap and rework can be reduced as well as other quality costs; and the overall quality level of processes can be improved.

The bottom line: Statistical methods can help an organization achieve an economic advantage in its operations.

Summarizing Variability

1. All processes have variability.

2. The control of quality is largely the control of variability.

3. Causes of variability are either common or special.

4. Special causes may be found and eliminated.

5. To improve a process in statistical control requires changing the system to reduce or eliminate common causes.

Process Capability/Process Potential Studies

In order to conduct a true capability or potential study of a given process, it is important that we observe several key rules:

1. The process must be in a state of statistical control during the study. This requires the use of control charts to collect and analyze the data.

2. The process standard deviation must be calculated using $\overline{R}/d_2$.

3. During the study, unnecessary upsets must be avoided. Remember, it is the **natural variation** in the process that is being analyzed.

4. By using averages, we are assuring a normal distribution of the subgroup averages, thus it is possible to calculate capability.

To calculate the process capability, we can use the Cpk or the Cp methods. A brief discussion of these calculations follows.

Process Capability Calculations

The Cpk value is the process average minus the lower tolerance divided by 3 standard deviations or the upper tolerance minus the process average divided by 3 standard deviations. The lower Cpk of these two is the reported value by definition. The Cpk value for a process considers the natural variation in the process (6 σ), the tolerances or specifications and the location of the distribution with respect to the specification limits. The formula for Cpk is:

$$Cpk = (\bar{X} - LSL) \div 3\,\hat{\sigma}$$

Or

$$Cpk = (USL - \bar{X}) \div 3\,\hat{\sigma}$$

Where:

USL= the upper specification
LSL = the lower specification

$\bar{X}$ = the process average

$\hat{\sigma}$ = the process standard deviation calculated using $\bar{R}/d_2$; where $\bar{R}$ is the average range from a process in control using a control chart and d_2 is the constant for the sample size.

These formulas can be used for one sided tolerances or two sided tolerances (specifications). If two sided tolerances are used, then select the one which is closest to the process average, assuming the process is not centered.

While one could argue that it is acceptable to use the calculated standard deviation, or σ, <u>directly from process data, this approach is fundamentally not correct because it includes special and common cause variation.</u> The **process capability index is based on a process in control** using the average range to determine the standard deviation.

When a process is in control, these two values approach the same result. So, if you want to take a risk go ahead and use the standard deviation without knowing the state of process control. The risk is that the Cpk is artificially high if there are a number of special causes present; or that it is artificially low if you have selected the “best case” for the process performance. That is why many times folks calculate a high Cpk which would indicate no bad parts or products and the next month ship bad stuff to the customer. Beware of shortcuts, they will usually come back to haunt you at a time when you can least afford it.

Another calculation is the Capability Index (Cp or Cpi). This calculation assumes the process is centered within the specifications or tolerances and compares the natural variation to these tolerances. The formula for Cp is:

$$Cp = (USL - LSL) \div 6\,\sigma$$

The terms in this formula are defined the same as in the Cpk calculations. So, for processes centered within the specifications, Cpk = Cp.

In general, it is desired to have a Cpk > l.33 or a Cp > l.33. This means that the average ± 4σ will fit within the specifications. The term 6 σ actually means that the average ± 6 standard deviations will fit within the specifications. When this level of capability is reached, then the customer should never see bad product! The goal for process capability is usually established by management for a unit or perhaps as a corporate target. Sometimes the customer establishes a minimum capability level.

Interpreting the Results

The results of a capability study can fall into 3 general classes:

I. The process variation (6σ) is much less than the specifications.

2. The process variation is approximately equal to the specifications.
3. The process variation is greater than the specifications.

Numerically, this is shown in the table below.

Cpk	Will fit within tolerances	% of Spec tolerance use by the natural process variation	Expected ppm outside specifications
1.0	$\bar{X} \pm 3\hat{\sigma}$	100%	2700 ppm
1.33	$\bar{X} \pm 4\hat{\sigma}$	75%	~64 ppm
1.67	$\bar{X} \pm 5\hat{\sigma}$	60%	~0.6 ppm
2.0	$\bar{X} \pm 6\hat{\sigma}$	50%	0.002 ppm

From the table one can readily see that the "6 sigma" concept is to have a Cpk of ≥ 2.0 to insure that essentially zero nonconforming product will be produced. Of course, in most cases this is a difficult ideal to achieve.

For a process in control and operating in the center of the allowable tolerances or specification, improving the process capability means the variation must be reduced or the specifications relaxed. The goal of process improvement is to reduce variation and thus improve process capability. It would be most unusual to be able to "improve" process capability by relaxing the specifications, although in my consulting practice over the years, I have seen this happen on a few isolated occasions for a non critical dimension or tolerance.

Again, this is all about planning for the customer!

Examples

1. An $\overline{X}$ and R chart is being maintained on melt flow rate for HXMC7 polymer. The

process is in a state of statistical control with $\overline{\overline{X}}$ = 7.2 and $\overline{R}$ = 0.25. The subgroup size is 2. Calculate the Cpk and Cp values for this process, using a specification limit of 6.5 to 7.7. The unit of measure is Melt Flow Rate, grams/10 minutes.

$$Cp = (USL - LSL) \div 6\,\hat{\sigma}$$

$\hat{\sigma}$ = $\overline{R}/d_2$ = 0.25/1.128 = 0.22

$6\hat{\sigma}$ = 6 × 0.22 = 1.32

USL-LSL = (7.7 - 6.5)

Cp = 1.2/1.32 = 0.91

$$Cpk = (\overline{X} - LSL) \div 3\,\hat{\sigma} \quad \textbf{or} \quad Cpk = (USL - \overline{X}) \div 3\,\hat{\sigma}$$

Since this process is centered closer to the upper specification, use the Cpk_u

USL - $\overline{X}$ = 7.7 - 7.2 = 0.5
$3\hat{\sigma}$ = 3 × 0.22

Cpk ≑ 0.76

From these calculations, even though the process is not centered, one can readily see that the process spread is greater than the tolerance and in fact, the process capability is not adequate. Using only the Cp value could mislead one into thinking the capability is "better" than it really is, thus the Cpk value is usually more meaningful.

The variability is too high and/or the specifications are too narrow. If the process is centered, the Cpk = Cp and is 0.91 which is the best case scenario given the data on this process.

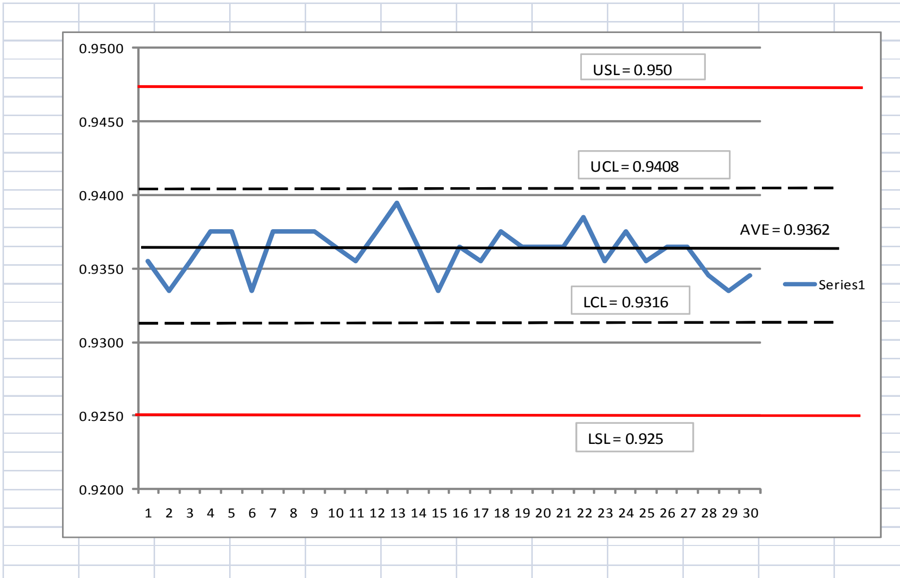
2. Let's look at another example with two different dimensions on a machined part.

Data	Dim "A"	Dim "B"
AVE	**0.9362**	**0.9360**
STD DEV	**0.0015**	**0.0032**
Target	**0.9350**	**0.9350**
USL	**0.945**	**0.945**
LSL	**0.925**	**0.925**
UCL	0.9408	0.9455
LCL	0.9316	0.9265

Cpk, USL	**1.92**	**0.94**
Cpk, LSL	**2.42**	1.16

Notice how the location (average) of the process affects the Cpk values. In this particular case the values to be reported would be 1.92 for "A" and 0.94 for "B." The lower value is the one that reflects the process capability.

The control chart for "B" indicates the process is in control and we have a valid Cpk indication. (Note, only the X chart is shown for brevity, the R chart must also be in control and in this case it was.)



Summary

Process capability is simply a measure of how well a process is "behaving" relative to the allowable tolerances or specifications. It ties control charts back to specifications. Remember, control limits are based on the natural variation in a process and are independent of the specifications (see P209 for more information on control charts and their applications). The Cpk value now brings the natural variation of a process in a state of control into the comparison to specifications. The average and standard deviations are both used in the calculations, thus we have fully described the process, know it is in control and then compare the data to the specifications.

Process capability is perhaps the most misunderstood and abused value in the statistical arsenal. Use it wisely and correctly.

One of the most blatant misinterpretations I have experienced was with a Plant Manager who asked me to determine a one number capability value for the entire plant. Cpk is based upon individual processes and product parameters as compared to the specifications or allowable tolerances for that particular parameter. Don't try to make more out of it than is really there! By the way, I explained that the number he wanted really did not exist in this realm. Most likely it would exist in the realm of profits or losses and not capability indices, if even then.

Glossary of Statistical Terms

Accuracy - The closeness of agreement between and observed value and an accepted reference value.

Alternative Hypothesis - The hypothesis that is accepted if the null hypothesis is disproved.

Attribute Data - Qualitative data that typically shows only the number of articles conforming and the number of articles failing to conform to a specified criteria. Sometimes referred to as Countable Data.

Average - The sum of the numerical values in a sample divided by the number of observations.

Bar Chart - A chart that uses bars to represent data. This type of chart is usually used to show comparisons of data from different sources.

Batch - A definite quantity of some product or material produced under conditions that are considered uniform.

Bias - A systematic error which contributes to the difference between a population mean of measurements or test results and an accepted reference value.

Bimodal Distribution - A distribution with two modes that may indicate mixed data.

Binomial Distribution - A distribution resulting from measured data from independent evaluation, where each measurement results in either success or failure and where the true probability of success remains constant from sample to sample.

Cells - The bars on a histogram each representing a subgroup of data.

Common Cause - A factor or event that produces normal variation that is expected in a given process.

Confidence - The probability that an interval about a sample statistic actually includes the population parameter.

Control Chart - A chart that shows plotted values, a central line and one or two control limits and is used to monitor a process over time.

Control Limits - A line or lines on a control chart used as a basis for judging the significance of variation from subgroup to subgroup. Variation beyond a control limit shows that special causes may be affecting the process. Control limits are usually based on the 3 standard deviation limits around an average or central line.

Countable Data - The type of data obtained by counting -attribute data.

Data - Facts, usually expressed in numbers, used in making decisions.

Data Collection - The process of gathering information upon which decisions to improve the process can be based.

Detection - A form of product control, not process control, that is based on inspection that attempts to sort good and bad output. This is an ineffective and costly method.

Distribution - A group of data that is describable by a certain mathematical formula.

Frequency Distribution - A visual means of showing the variation that occurs in a given group of data. When enough data have been collected, a pattern can usually be observed.

Histogram - A bar chart that represents data in cells of equal width. The height of each cell is determined by the number of observations that occur in each cell.

k - The symbol that represents the number of subgroups of data. For example, the number of cells in a given histogram.

Lower Control Limit (LCL) - The line below the central line on a control chart.

Mean - The average value of a set of measurements, see Average.

Measurable Data - The type of data obtained by measurement. This is also referred to as variables data. An example would be diameter measured in millimeters.

Median - The middle value (or average of the two middle values) of a set of observations when the values have been ranked according to size.

Mode - The most frequent value in a distribution. The mode is the peak of a distribution.

n - The symbol that represents the number of items in a group or sample.

np - The symbol that represents the central line on an np chart.

Nonconformities - Something that doesn't conform to a drawing or specification; an error or reason for rejection.

Normal Distribution - A symmetrical, bell-shaped frequency distribution for data. This is a distribution that is often seen in industry.

Null Hypothesis - The hypothesis tested in test of significance that there is no difference (null) between the population of the sample and the specified population (or between the populations associated with each sample).

Out of Control - The condition describing a process from which all special causes of variation have not been eliminated. This condition is evident on a control chart by the presence of points outside the control limits or by nonrandom patterns within the control limits.

p - The symbol on a p chart that represents the proportion of nonconforming units in a sample.

Parameter - A constant or coefficient that describes some characteristics of a population (e.g. standard deviation, average, regression coefficient).

Pareto Charts - A bar chart that arranges data in order of importance. The bar representing the item that occurs or costs the most is placed on the left-hand side of the horizontal axis. The remaining items are placed on the axis in descending (most to least) order.

Population - All members, or elements, of a group of items. For example, the population of parts produced by a machine includes all of the parts the machine has made. Typically in SPC we use samples that are representative of the population.

Prevention - A process control strategy that improves quality by directing analysis and action towards process management, consistent with the philosophy of continuous quality improvement.

Process - Any set of conditions or set of causes working together to produce an outcome. For example, how a product is made.

Product - What is produced; the outcome of a process.

Quality - Conformance to requirements.

Random Sampling - A data collection method used to ensure that each member of a population has an equal chance of being part of the sample. This method leads to a sample that is representative of the entire population.

Range - The difference between the highest and lowest values in a subgroup.

Repeatability - The variation in measurements obtained when one operator uses the same test for measuring the identical characteristics of the same samples.

Reproducibility - The variation in the average of measurements made by different operators using the same test when measuring identical characteristics of the same samples. (In some situations it is the combination of operators, instruments and locations.)

Run Chart - A line chart that plots data from a process to indicate how it is operating.

Sample - A small portion of a population.

Sampling - A data collection method in which only a portion of everything made is checked on the basis of the sample being representative of the entire population.

Significance Level () - The risk we are willing to take of rejecting a null hypothesis that is actually true.

Skewed Distribution - A distribution that tapers off in one direction. It indicates that something other than normal, random factors are effecting the process. For example, TIR is usually a skewed distribution.

Special Cause - Intermittent source of variation that is unpredictable, or unstable; sometimes called an assignable cause. It is signaled by a point beyond the control limits or a run or other nonrandom pattern of points within the control limits. The goal of SPC is to control the special cause variation in a process.

Specification - The extent by which values in a distribution differ from one another; the amount of variation in the data.

Standard Deviation (σ) - The measure of dispersion that indicates how data spreads out from the mean. It gives information about the variation in a process.

Statistic - A quantity calculated from a sample of observations, most often to form an estimate of some population parameter.

Statistical Control - The condition describing a process from which all special causes of variation have been eliminated and only common causes remain, evidenced by the absence of points beyond the control limits and by the absence of nonrandom patterns or trends within the control limits.

Statistical Methods - The means of collecting, analyzing, interpreting and presenting data to improve the work process.

Statistical Process Control (SPC) - The use of statistical techniques to analyze data, to determine information, and to achieve predictability from a process.

Statistics - A branch of mathematics that involves collecting, analyzing, interpreting and presenting masses of numerical data.

Subgroup - A group of consecutively produced units or parts from a given process.

Tally or Frequency Tally - A display of the number of items of a certain measured value. A frequency tally is the beginning of data display and is similar to a histogram.

Tolerance - The allowable deviation from standard, i.e., the permitted range of variation about a nominal value. Tolerance is derived from the specification and is NOT to be confused with a control limit.

Trend - A pattern that changes consistently over time.

Type I Error (α) - The incorrect decision that a process is unacceptable when, in fact, perfect information would reveal that it is located within the "zone of acceptable processes".

Type II Error(β) - The incorrect decision that a process is acceptable when, in fact, perfect information would reveal that it is located within the "zone of rejectable processes".

u - The symbol used to represent the number of nonconformities per unit in a sample which may contain more than one unit.

Upper Control Limit (UCL) - The line above the central line on a control chart.

Variables - A part of a process that can be counted or measured, for example, speed, length, diameter, time, temperature and pressure.

Variable Data - Data that can be obtained by measuring. See Measurable Data.

Variation - The difference in product or process measurements. A change in the value of a measured characteristic. The two types of variation are within subgroup and between subgroup. The sources of variation can be grouped into two major classes: Common causes and Special Causes.

X - The symbol that represents an individual value upon which other subgroup statistics are based.

$\bar{X}$ (x bar) - The average of the values in a subgroup.

$\bar{\bar{X}}$ (x double bar) - The average of the averages of subgroups.

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