PDHonline Course S121 (1 PDH)

Leaner Columns

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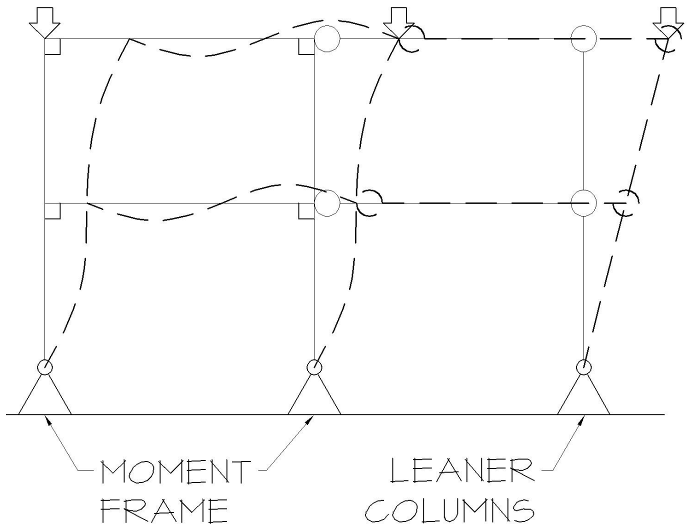
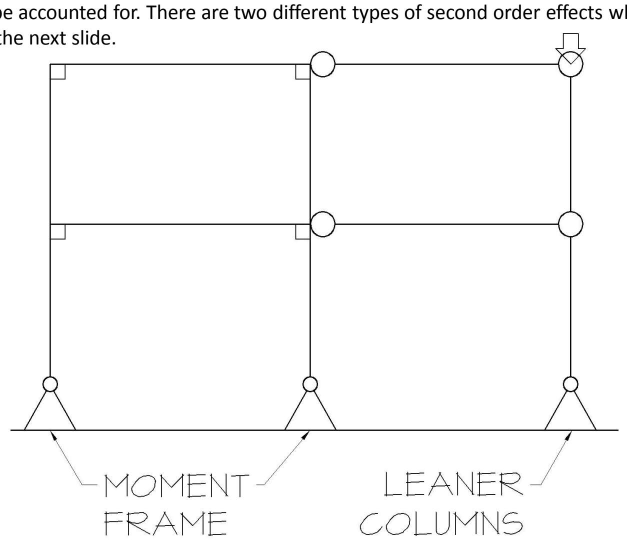
DEFINITION:

Leaner columns are columns that are pinned at each connection, and provide no bending restraint in a frame. Theoretically a leaner column has an infinite *kl,* therefore the strength should be zero.



DISCUSSION:

When analyzing leaner columns, the members are considered as simple pinned end columns, laterally supported at their ends. Therefore the members can be designed with an effective length equal to their actual length. Lateral stability provided by rigid or braced frames must be properly sized to provide restraint for all loading within the structure. This requires that second order effects be accounted for. There are two different types of second order effects which are illustrated on the next slide.



1. The first type of second order effect occurs when the ends of the column are prevented from displacing and deflection of the column is allowed only along the height or length of the member. This increase in moment due to member deflection is referred to as the member effect. (See Figure 1a)

2. The second type occurs when the structure is allowed to sway laterally. The moment required at the end of the column to maintain equilibrium in the displaced or swayed configuration is referred to as the P-Delta effect. This moment is referred to as the structure effect. (See Figure 1b)

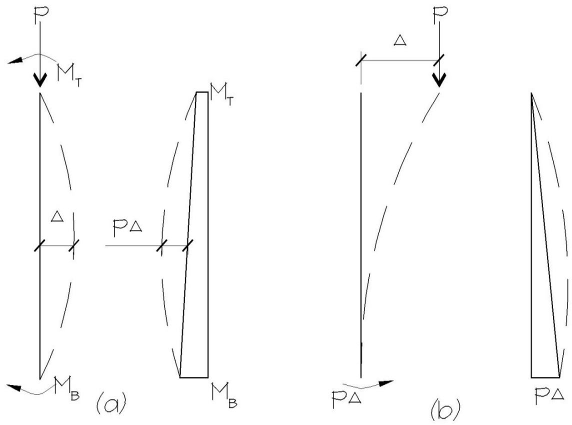


Figure 1 - Secondary Effects

Performing a second order analysis for individual columns is straightforward. However, when columns are combined to form frames, the interaction of all of the frame members increases the complexity of the solution. The addition of columns, which do not participate in lateral resistance but do carry gravity loads (leaner columns), further complicates the analysis. An example of this condition is represented below.

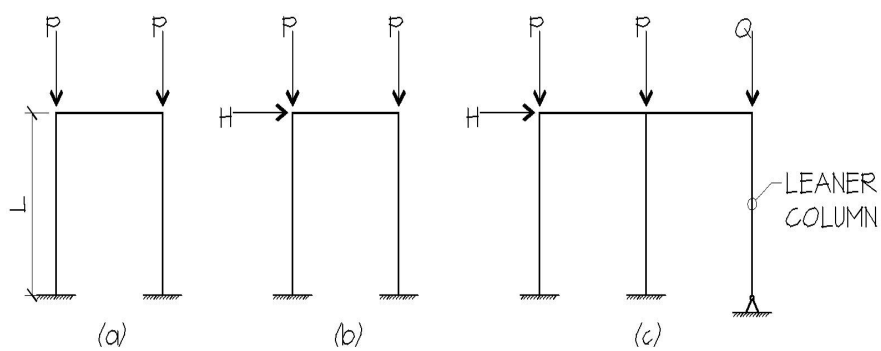


Figure 2 - Unbraced Frame

The frame in Figure 2 is composed of two identical columns carrying the load P. If the columns behave elastically their capacity would be determined using Euler's equation with K=2 for $P = (\pi)^2EI/4L^2$. The addition of lateral load H results in a reduction in the ability of the columns to carry axial load since now each column must resist an applied moment of approximately HL/2. This reduced capacity is accounted for through the use of the interaction equations provided by the AISC design specifications.

With the addition of a leaner column the original rigid frame columns must still resist the entire lateral load. If the leaner column has no impact on the frame columns as would appear from a first order analysis, the frame columns would continue to carry only the gravity load. However, since the addition of the leaner column does have an impact on the ability of the other columns to carry the load, then their load must be appropriately reduced. This is because the leaner column, as indicated previously can be designed with an effective length factor of k=1, therefore it must rely on the frame columns to keep its' upper end from moving laterally.

The normal approach to determining the axial load capacity of a column involves the determination of the effective length factor, K. Through the use of K-factors the actual critical buckling load of a column is related to the Euler buckling equation such that $P = (\pi)^2EI/KL^2$. The most commonly used method for finding the K-factor is through the use of the nomograph found in the AISC design specifications. Assumptions in the development of the design nomographs include:

1. All members behave elastically.

2. All columns in a story buckle simultaneously (i.e., any column in an unbraced frame will not contribute of the lateral sway resistance of another column).

Since the behavior of columns and structures indicates that leaner columns do in fact have axial capacity it is important to find a model that will predict the capacity of a frame including these leaner columns. Solutions for the proper modeling of leaner columns include both simple increases in the apparent axial load carrying capacity of the braced frame columns and modifications to the effective K-factor.

The axial load of a leaner column must be balanced by a horizontal force $Q\Delta/L$ to maintain equilibrium of the leaning column. This force must be applied at the restraining column. (See Figure 3)

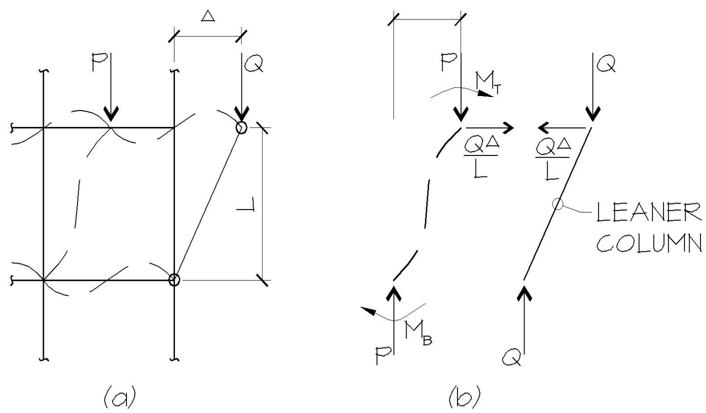


Figure 3 - Frame Model - Buckling with Leaner Column

There are different methods of accounting for the effects of leaner columns in a frame and include:

A. **Yura** (See Figure 4)

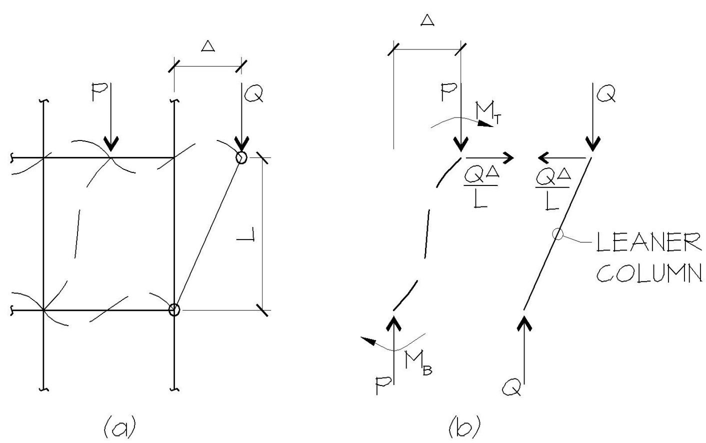


Figure 4 - Equilibrium Diagram - Yura Method

1. The Yura Method is the easiest but most conservative method.

2. When the frame column buckles, it buckles with the moment of ((P*D) + (Q*D)) at its' base. This is the same moment that would result if the individual frame column buckled under the axial load of (P+Q). Note: D = Δ (as indicated in Figure 4) such that (P*D) = (P x Δ) = (Axial Load multiplied by Δ). When the frame column buckles, it buckles with the moment of (P Δ + Q Δ) at its' base. This is the same moment that would result if the individual frame column buckled under the axial load of (P+Q).

3. The assumption that the buckling load (P+Q) is only slightly conservative because the deflected shape due to axial load and a lateral load differ only slightly for this frame column.

4. In order to assure sufficient lateral resistance to buckling for the leaner column the frame column must be designed to carry a fictitious load of (P+Q).

5. In order to compare this approach to the following two methods, it is helpful to convert it to an effective length approach. Since the frame column is to be designed to carry a load of (P+Q), a modified effective length factor will be required.

$$K_n = K_o\sqrt{\frac{P+Q}{P}}$$

K_n = the effective length factor that accounts for the leaning column.

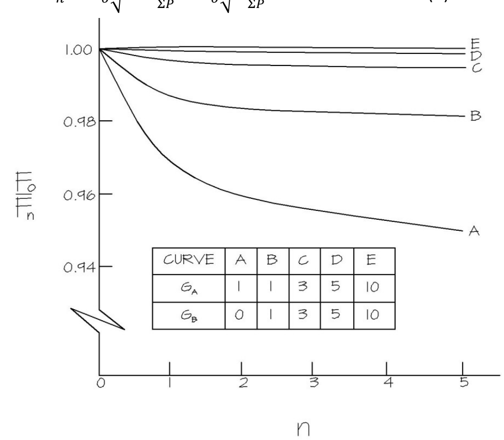
K_o = the effective length factor that would be determined from the nomograph, which does not account for the leaning column

B. **Lim & McNamara**

1. This approach was developed for columns of unbraced tube buildings and is based on the assumption that all columns in a rigid frame buckle in a simultaneous side sway mode.
2. This method accounts for leaner columns through the use of an effective length factor, see Equations (A) and (B).

$$K_n = K_o\sqrt{1+n\frac{F_o}{F_n}} \quad \text{(A)}$$

$$K_n = K_o\sqrt{1+\frac{\Sigma Q}{\Sigma P}} = K_o\sqrt{\frac{\Sigma P+\Sigma Q}{\Sigma P}} \quad \text{(B)}$$



F_o/F_n vs. n for Lim and McNamara Approach

C. **LeMessurier**

1. This is a more complex but accurate method, which is illustrated on the next slide, and also involves a modification of the effective length factors.
2. Where the two above approaches determine a constant value for a given story by which the effective length factor is modified, this approach determines a constant value for a story which is then used to multiply the individual column moment of inertia divided by the column load (I/P) for each column.
3. This method allows for the contribution of each column to the lateral resistance of the frame to be accounted for individually. See Equation (C).
4. Two modified LeMessurier equations are presented in the second edition of the commentary to the AISC/LRFD specifications.
5. It is interesting to note that for a structure in which only one column can be considered to provide lateral stability, the simplified equation in the LRFD specifications based on the assumption that no reduction in column stiffness occurs due to the presence of axial load, reduces to the same equation provided for the modified Yura and Lim & McNamara approaches indicated above.

Note: F_o = Eigenvalue solution for a frame without a leaning columns; F_n = Eigenvalue solution for a frame with leaning columns. Assume a F_o/F_n = 1.0 for a conservative K factor. n= ΣQ/ΣP.

$$K_i^2 = \frac{I_i}{P_i}\pi^2 \frac{\Sigma P + \Sigma Q + \Sigma(C_L PP}{\Sigma(\beta I)} \qquad (c)$$

where

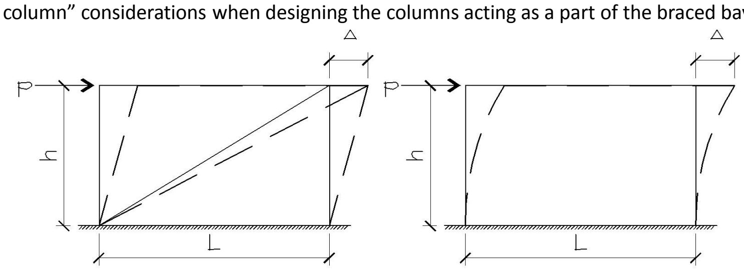
$$\beta = \frac{6(G_A+G_B)+36}{2(G_A+G_B)+G_A G_B+3}$$

$$C_L = \frac{\beta K_o^2}{\pi^2} - 1$$

C_L = 0 for leaning columns.
K_o = the column effective length based on the nomograph.
G_A, G_B = Column end conditions as defined for use with the nomograph.
P_i = load on column, i
I_i = moment of inertia for column, i
ΣP = load on the restraining columns in a story
ΣQ = load on the leaning columns in a story
$\Sigma(C_L P)$ = sum of (C_LP) for each column in the story
$\Sigma(\beta I)$ = sum of (βI) for each column participating in lateral sway resistance.

SPECIAL CONSIDERATIONS AT BRACED FRAMES:

A frame is braced when sidesway buckling and instability cannot occur. The AISC specification indicates that lateral stability of a frame is provided by attachment to diagonal bracing, shear walls or other similar structures having adequate lateral stability. The AISC specifications do not indicate however the amount of stiffness required to prevent sidesway buckling. The following information is presented to provide assistance in making the necessary engineering judgment regarding the strength required to create a braced frame adequate enough to prevent sidesway buckling and therefore avoid the need to consider "leaner column" considerations when designing the columns acting as a part of the braced bay or frame.



(a) Cross-braced frame.

$$P = R_B\Delta$$

$$P = \left[\frac{E}{\Sigma\frac{SUL_1}{A}}\right]\Delta$$

where:

- S = stress in each member caused by the load P
- U = stress in each member caused by unit load at P
- L_1 = Length of each member
- A = Area of each member

(b) Shear wall.

$$P = R_S\Delta$$

$$P = \left[\frac{E}{h\left(\frac{h^2}{3I}+\frac{3}{A}\right)}\right]\Delta$$

where:

- t = thickness of wall
- I = $\frac{tL^3}{12}$
- A = tL

Assumption:

The shear modulus is 40% of elastic modulus.

For an idealized bracing arrangement (See Figure 5) the assumptions listed on the next slide are made:

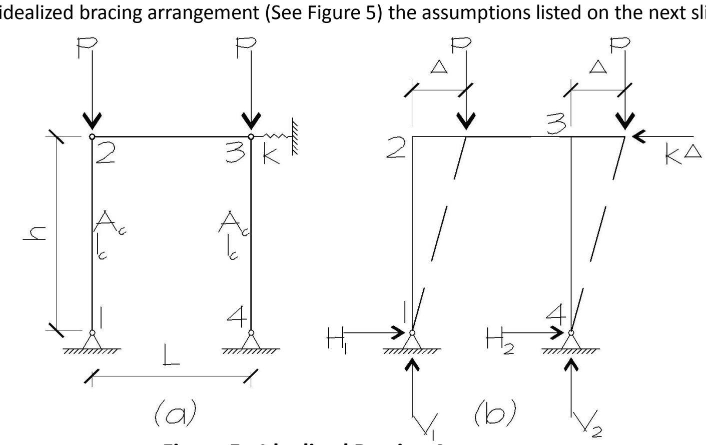


Figure 5 - Idealized Bracing Arrangement

For a simple rectangular frame as shown in the figure above, $k = \left(\frac{2P_{cr}}{h}\right)$. If the buckling load ($P_{cr}$), is assumed to be twice (i.e. factor of safety of 2.0) the service load (P), then $k = \left(\frac{4P}{h}\right)$. For a frame with several bays $\left(\frac{\Delta}{h}\right)(\Sigma P - kh) = 0$, where ΣP = summation of loads causing buckling. Where the factor of safety (FS) is applied and service loads (P) are used then $k_{required} = \frac{(FS)(\Sigma P)}{h}$. This assumes that only one of the bays in a multi-bay structure is braced or that $k_{required}$ is the combined stiffness of all bracing. ΣP includes design loads in all columns for which the particular brace is to provide support.

1. Column does not participate in resisting sidesway.
2. Columns are hinged at ends.
3. Bracing acts independently as a spring at the top of the columns.