PDHonline Course S222 (5 PDH)

Framed, Two-Way, Conventionally Reinforced, Concrete Flat Slab and Flat Plate Construction and Design

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Framed, Two-Way, Conventionally Reinforced, Concrete Flat Slab and Flat Plate Construction and Design

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COURSE CONTENT

Flat Plate and Flat Slab Systems

Flat Plates:

Slab Thickness

A flat plate floor system is a two-way concrete slab of uniform depth without interior beams, drop panels or column capitals supported directly on columns with reinforcement in two orthogonal directions (see Figure 1). This system includes the advantages of simple construction and formwork, and a flat ceiling, the latter of which reduces ceiling finishing costs, since the architectural finish can be applied directly to the underside of the slab. Even more significant are the cost savings associated with the low story heights made possible by this shallow floor system. Therefore using a flat plate will result in the accommodation of more stories within a given building height as opposed to a deeper one-way system of slabs, joists and beams.

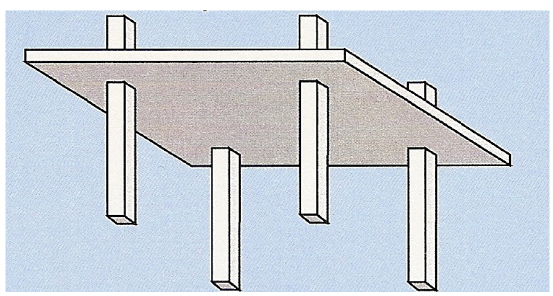


FIGURE 1

Spandrel beams at the edges of a flat plate are sometimes required for the support of perimeter cladding (such as brick) in order to help control deflections, and if the size of the exterior column is limited in order to avoid punching shear limitations. Spandrel beam and exterior columns can also be utilized to as rigid frames for resisting imposed lateral loads. Cantilever edges of the slab can be structurally advantageous as the extended slab helps to both minimize the positive moment at the first interior span and reduce the need for a spandrel beam because of the increased punching shear slab perimeter realized.

The minimum thickness requirements for flat plates are given in Section 9.5.3 of ACI 318-05 and in Table 9.5(c) for slabs without interior beams. Section 9.5.3.2 specifies the absolute minimum slab thickness for a flat plate as 5 inches. If the minimum slab thickness specified by ACI is not used, then as was the case with one-way slabs, joists and beams, the actual immediate and long-term deflections must be calculated per the requirements of Section 9.5.2.3 (using I_e). In addition, the actual deflections calculated must comply with the permissible deflections allowed by Table 9.5(b). It should also be noted that when calculating the deflection of a two-way system it is necessary to account for the combined deflection of each orthogonal design strip. This is generally accomplished by using a crossing-beam analogy in which the average deflection of the midspan of the column strips in one direction are added to the midspan deflections of the

perpendicular middle strip as shown in Figure 13-73 page 715 of *Reinforced Concrete Mechanics and Design*, 4th Edition.
For live loads of 50 PSF or less, the thickness of a flat plate will usually be controlled by deflection requirements. In addition, at this same live load capacity the required flexural reinforcement at the critical sections in the column and middle strips will typically satisfy the minimum requirements prescribed in Section 13.3, however, as indicated before it is my personal preference to use the minimum reinforcing requirements of Section 10.5 (flexural minimum) rather Section 7.12 (shrinkage/temperature minimum), which is the minimum reinforcing requirements referenced by Section 13.3. Section 13.3 also specifies the maximum spacing of the minimum reinforcement, which is two times the slab thickness or 18 inches (per Section 7.12.2.2).

Therefore, using a slab thickness greater than the minimum allowable is not considered economical, since a thicker slab will increase the concrete quantity and not reduce the reinforcement quantity. In addition, the minimum thickness requirements are independent of the concrete compressive strength. Therefore, specifying a compressive strength greater than 4,000 psi will increase the cost of the concrete without any allowable reduction in slab thickness. Therefore, for live loads of 50 PSF or less, the most economical flat plate floor system is one with a minimum thickness obtained from Table 9.5(c) and a concrete compressive strength of 4,000 psi. The practical span length range of a flat plate supporting 50 PSF live load is approximately 15 to 30 feet.

For live loads of 100 PSF or more, the thickness of the slab will be more than likely be controlled by shear stresses at the critical section around the columns and bending moments in the slab, and not be deflection criteria. Slabs thicker than that required by Table 9.5(c) are generally required to resist the larger punching shear stresses associated with these larger live loads. Although a thicker slab may result in a decrease in the required amount of flexural reinforcement, the reduction in the cost of reinforcement will not offset the increase in the cost of concrete. In addition, using a higher strength concrete is not the most effective way of increasing the nominal moment or shear strength provided by the concrete at the critical section around the columns. Therefore, for live loads of 100 PSF, the most cost-effective solution is to use a slab thickness equal to the minimum required for strength and a concrete compressive strength equal to 4,000 psi. The practical span length range of a flat plate supporting 100 PSF live load is approximately 15 to 25 feet.

It should also be noted that as was the case with one-way joists, beams and columns, it is also possible to reduce the live loads on two-way slab systems (both flat plates and flat slabs) per the previsions of ASCE 7-05. In addition, it should also be noted that for column cumulative load or "take-down" analysis, combining the loads from 2 orthogonal panels at the same time will result in more axial dead and live load on the column than really exists. Therefore, column loads should be based on tributary area and not design strip reactions.

Column Dimensions

The height and cross-sectional dimensions of the columns above and below the floor slab affect the bending moments and shear forces transmitted to the slab. The magnitudes of these reactions depend on the relative stiffness of the columns and slab. By definition the stiffness of a column is E_cI/L; where E is the modulus of elasticity of the concrete, I is the moment of inertia of the column cross-section and L is the height of the column. Since the stiffness is inversely proportional to L, a longer column is more flexible (i.e. less stiff). A more flexible column allows greater rotation at the slab-column joint, resulting in larger bending moments in the slab.

The cross-sectional column dimensions also have an effect on the bending moments in the slab. This is because the column dimensions parallel to the direction of analysis establishes the clear span lengths, which in turn are used in determining the bending moments for some of the methods of analysis discussed later in this lecture. The dimensions of the column also directly affect the magnitude of I in the stiffness equation. Because the properties of the critical shear section are also related to the cross-sectional dimensions of the column, the shear stresses are also impacted by the column size. Therefore, a larger column width or depth results in a larger nominal shear strength provided by the concrete.

Aspect Ratio

The aspect ratio of a slab panel is defined as the larger dimension of the panel divided by the smaller dimension of the panel, measured center-to-center of the supports. If the aspect ratio exceeds 2, the slab will act primarily as a one-way slab spanning the short direction. For an aspect ratio other than 1, the longer span will dictate the slab thickness, resulting in a loss of economy. Therefore, unless column layout is dictated by architectural or other functional requirements, square bays should be used, since they provide the most economical layout.

Flat Slabs:

Slab and Drop Panel Thickness

A flat slab floor system is similar to a flat plate floor system, with the exception that the flat slab has thickened portions around the columns called drop panels (see Figure 2). The drop panels are sometimes also supported by a flared column top, which is referred to as a column capital, however, the use of column capitals is not as common as it once was because of the increased cost of forming these same elements. The primary purpose of drop panels is to increase the nominal shear strength of the concrete at the critical section around the columns. Drop panels also increase the relative stiffness of the slab which in turn reduces the unbalanced moments that are transferred to the column. The flat slab system has advantages that are similar to the flat plate, with the exception that additional forming costs are associated with the construction of the drop panels.

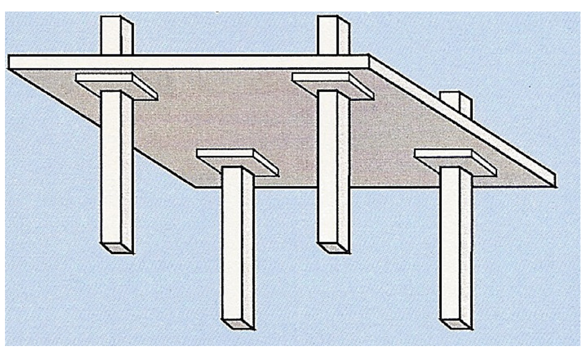


FIGURE 2

The minimum thickness requirements for flat slabs are also given in Sect. 9.5.3 of ACI 318-05. The minimum thickness of flat slabs is 10% less than that required for flat plates. Minimum dimensions for drop panels are given in Sect. 13.2.5 (see Figure 3). The drop panel shall extend in each direction from the centerline of the support a distance not less than one-sixth of the span length measured from center-to-center of supports in the same direction. Also, the projection of the drop panel below the slab shall be at least one-quarter of the slab thickness. The minimum slab thickness must be increased by 10% if drop panel dimensions provided do not conform to these provisions. Drop panel dimensions are also controlled by formwork considerations (see Figure 23, Lecture 6).

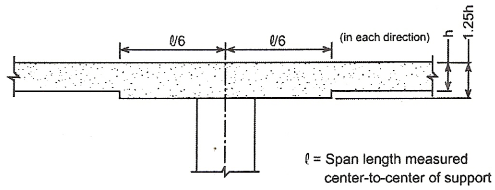


FIGURE 3

Typically in the preliminary design stage, a slab thickness is chosen based on the minimum thickness requirements of Sect. 9.5.3. The plan dimensions of the drop panels are then determined based on the minimum lengths specified in Section 13.2.5. The shear stresses at the critical section around the column are then checked based on the minimum drop panel depths conforming to Section 13.2.5, taking into account the formwork details referenced above. If this proves to be inadequate, the next larger drop panel depth is used until the shear strength requirements are satisfied. In addition, per the requirements of Section 11.12.1.2, the shear stresses must also be checked at the critical section around the drop panels.

For a live load of 50 PSF or less, flat slabs are cost-effective for span lengths between 30 and 35 feet. The economical range is 25 to 35 feet for a live load of 100 PSF.

Column Dimensions

The impact of the column dimensions for a flat slab are similar to that described for a flat plate system. In addition, flat slabs with column capitals at the drop panels can also impact the shear capacity and flexural stiffness of the system.

Aspect Ratio

As with flat plates, square bay sizes with an aspect ratio equal to 1 represent the most economical floor layout for flat slabs with drop panels.

Other Two-Way Systems:

Although two-way concrete framing systems other than flat plates and flat slabs will not be discussed as a part of this lecture, it is important to recognize that at least two other two-way systems are used occasionally. These systems are a two-way joist system (commonly referred to as a waffle slab) and a two way beam system. An illustration of a waffle slab was provided in Figure 19 of Lecture 6. An illustration of a two-way beam system is shown in Figure 4 below.

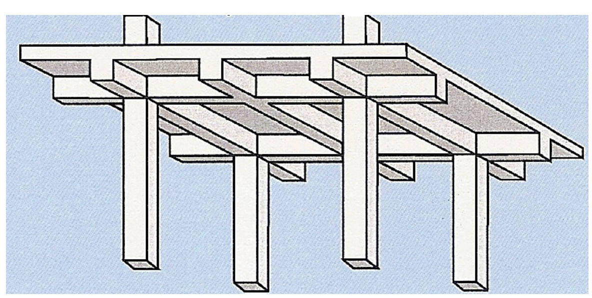


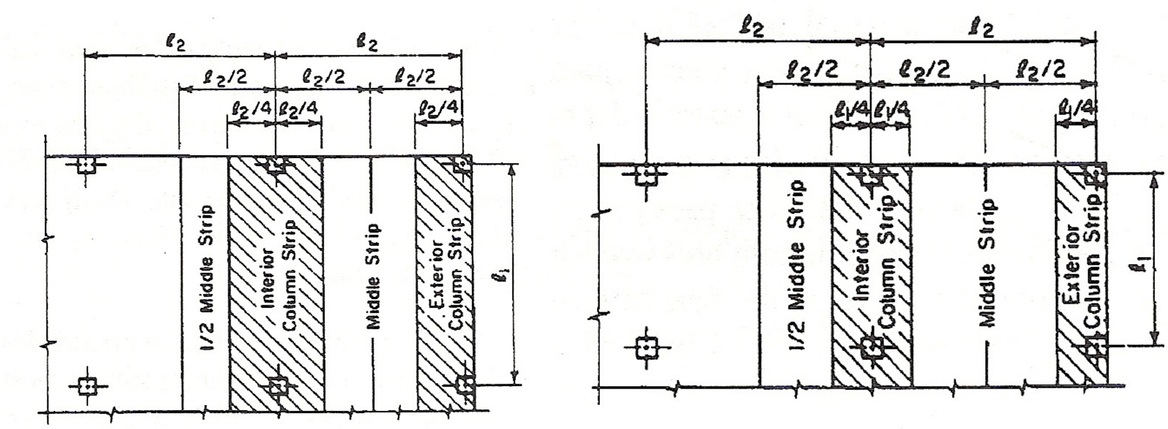
FIGURE 4

Methods of Analysis

General Discussion:

Design Strips and Gravity & Lateral Load Analysis

The analysis of a two-way slab system can be accomplished by either the Direct Design Method (Section 13.6) or the Equivalent Frame Method (Section 13.7). Both methods of analysis involve dividing the slab-beam (or panel) into design strips consisting of a column strip and two half middle strips as defined in Section 13.2.1 and 13.2.2, respectively (see Figure 5a and 5b). The column strip is defined as having a width equal to one-half the transverse (ℓ_2) or longitudinal span (ℓ_1), whichever is smaller. The middle strip is bounded by two adjacent column strips. Some judgment is required in applying the definitions given in Section 13.2.1 for column strips when varying span lengths occur along the design strip. The reason for specifying that the column strip width be based on the shorter of ℓ_1 or ℓ_2 is to account for the tendency for moment to concentrate about the column line when the span length of the design strip is less than its width.



(a) Column Strip for $\ell_2 \le \ell_1$ (b) Column Strip for $\ell_2 > \ell_1$

FIGURE 5a & b

Both of the methods of analysis prescribed by the ACI Code are intended for gravity load analysis only, however, the Equivalent Frame Method (EFM) may be used for lateral load analysis if the stiffnesses of the frame members are modified to account for cracking and other relevant factors. The other relevant factors that affect the stiffness of the slab include the parameters; ℓ_2/ℓ_1, c_1/ℓ_1 and c_2/c_1, and the concentration of reinforcement in the slab width defined in Section 13.5.3.2 for unbalanced moment transfer by flexure. This added concentration of reinforcement increases stiffness by preventing premature yielding and softening in the slab near the column supports. Consideration of the actual stiffness due to these factors, in addition to the effective cracked inertia, is important for lateral load analysis because the magnitude of lateral displacement can significantly affect the moments in the columns. The initial assumption of the cracked slab inertia (with minimum reinforcement at all locations) should be based on the stiffness for a slab-beam equal to one fourth of the gross area of concrete (i.e. $K_{sb}/4$). Although a two-way flat plate or slab can be used as a rigid moment frame for buildings 4 to 6 stories in height, it is more common to use shearwalls as the lateral resisting element for these types of structures.

The Direct Design Method (DDM) is an approximate method that uses moment coefficients. The EFM is a more complicated and more exact method than the DDM. Both methods are also limited in application to buildings with columns and/or walls laid out on a basically orthogonal grid (i.e. longitudinal and transverse column lines throughout the building are mutually perpendicular). Both methods are applicable to slabs with or without beams between supports. However, it is also important to note that neither method is applicable to slab systems with beams spanning between other beams. Therefore any beams accounted for by these two methods must be located along the column lines and supported by the columns or other essentially non-deflecting supports at the corners of the slab panels (as defined by Section 13.2.3). It should be noted that in this lecture the term "panel" is sometimes used interchangeably with the term design strip (i.e. a column strip plus two adjacent half middle strips) due to the fact that my interpretation of the terminology is based on a personal opinion that the term "panel" as defined by Section 13.2.3 should be replaced by the term "bay".

In addition to both methods of analysis dividing the slab-beam into design strips consisting of a column strip and middle strips, both methods of analysis also distribute gravity load moments (and lateral load moments for the EFM) at critical sections of the slab-beam in accordance with Section 13.6.4 and 13.6.6 to the column and middle strips, respectively.

Shear Transfer

When two-way slabs are supported directly by columns (as is typically the case), shear around the columns is of critical importance. Shear strength at an exterior slab-column connection (without edge beams) is especially critical because the total exterior negative slab moment, in addition to the applied shear, must be transferred directly to the column via the slab. This aspect of two-way slab design should not be taken lightly by the designer. An example of what can happen when this component of two-way design is not closely monitored can be found an article entitled *Communication Breakdown, It's Always the Same* from the 2007 November issue of STRUCTURE Magazine.

Two types of shear must be considered in the design of flat plates or flat slabs supported directly on columns. The first is the familiar one-way or beam-type shear, which may be critical in long narrow slabs. Analysis for beam shear assumes the slab acts as a wide beam spanning between the columns. The critical section is taken at a distance d from the face of the column. The design check for beam shear consists of satisfying the requirements illustrated in Figure 6a. Beam shear in slabs is seldom a critical factor in design, as the shear force is usually well below the shear strength of the concrete.

Two-way or "punching" shear is generally the more critical of the two types of shear in slab systems supported directly on columns. Punching shear considers failure along the surface of a truncated cone or pyramid around the column. The critical section is taken perpendicular to the slab at a distance d/2 from the perimeter of a column (see Figure 6b). For all practical purposes, only direct shear (uniformly distributed around the perimeter b_o) occurs around interior slab-column supports where no (or insignificant) moment is to be transferred from the slab to the column. Significant moments may, however, have to be transferred at interior supports when unbalanced gravity loads on either side of the column or horizontal loading due to lateral forces must be transferred from the slab to the column. At exterior slab-column supports, the total exterior slab moment from gravity loads (plus any lateral load moments due to wind or earthquake) must be transferred directly to the column, in addition to the direct shear, via the critical sections of the slab as defined by the Code.

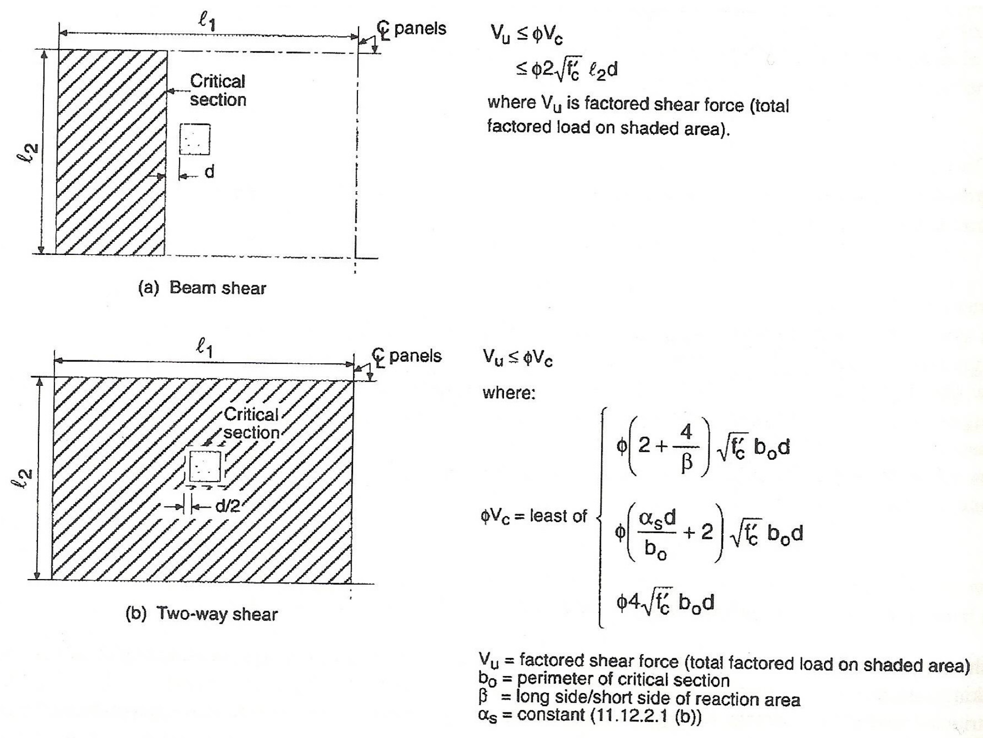


FIGURE 6a & b

It should be noted that the tributary areas used for the calculation of V_u in the DDM for edge panels in a two-way slab with no edge beams should be based on that indicated in Figure 7 below. The tributary area from the center of the exterior column for flat plates with edge beams should be based on 0.45 ℓ while the tributary area of all interior columns should be based on 0.50ℓ from the center of the column.

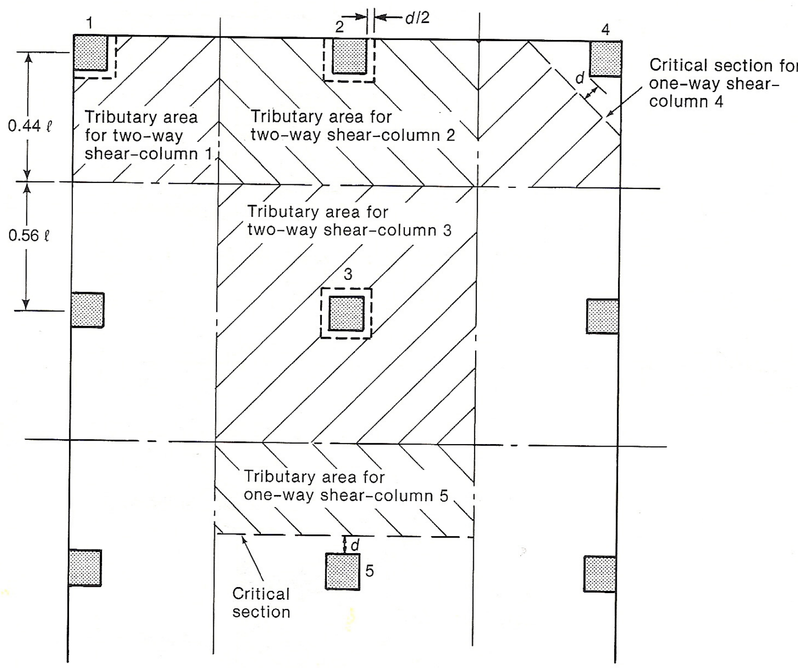


FIGURE 7

Moment Transfer

Transfer of moments between a slab and a column takes place by a combination of flexure (Section 13.5.3) and eccentricity of shear (Section 11.12.6.). Shear due to moment transfer is assumed to act on a critical section at a distance d/2 from the face of the column (the same critical section around the column as that used for direct shear transfer, see Figure 6b). The portion of the moment transferred by flexure is assumed to be resisted by a width of slab equal to the transverse column width (c_2) plus 1.5 times the slab thickness on either side of the column. A concentration of negative reinforcement is used to resist moment within this same effective slab width. The combined shear stress due to direct shear and moment transfer often governs the design of two-way slabs, especially at the exterior slab-column supports.

The portions of the total unbalanced moment (M_u) to be transferred by eccentricity of shear and by flexure are given by Equations 11-39 and 13-1, respectively, where $\gamma_v M_u$ is considered transferred by eccentricity of shear and $\gamma_f M_u$ is considered transferred by flexure. At an interior square column (where $b_1 = b_2$) 40% of the moment is transferred by eccentricity of shear ($\gamma_v M_u = 0.40M_u$) and 60% by flexure ($\gamma_f M_u = 0.60M_u$). The moment (M_u) at the exterior slab-column support will generally not be computed at the centroid of the crucial transfer section. In the EFM, moments are computed at the column centerline. In the DDM, moments are computed at the face of support.

Therefore, based on all of the above, the factored shear on the critical transfer section at the column-slab interface is the sum of the direct shear and the shear caused by a portion of the transfer moment such that;

$$v_u = (V_u/A_c) + ((\gamma_v M_u c)/J) \leq \Phi 4(f'_c)^{1/2}$$

Where: A_c = Area of critical section

c = Distance from centroid of critical section to face of section where stress is being computed

J = Property of critical section analogous to polar moment of inertia

$\gamma_v = 1 - \gamma_f$; and $\gamma_f = 1/((1 + ((2/3)(b_1/b_2)^{½})))$; see Figure 16-12 on the next page for an illustration of b_1 and b_2

It should be noted that in the case of flat slabs (with drop panels), two different critical sections need to be considered in the punching shear calculations, as shown in Figure 8.

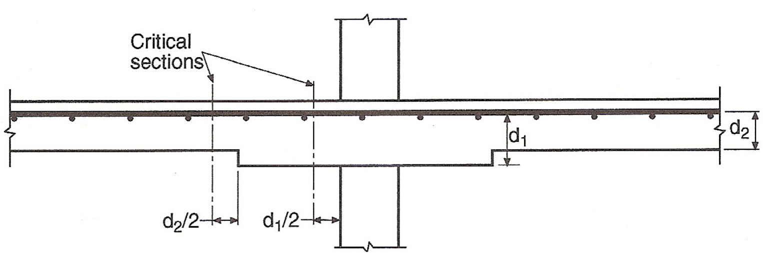


FIGURE 8

Unbalanced moment transfer between slab and an edge column (without spandrel beams) requires special consideration when slabs are analyzed by the DDM for gravity loads per the requirements of Section 13.6.3.6. See a further discussion of this topic below. Also, per Section 13.5.3.3, at exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear ($\gamma_v M_u$) may be reduced to zero provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75% of the shear strength (ΦV_c) defined in Section 11.12.2.1 for edge columns or 50% for corner columns. It should be noted that as a part of this Code provision, as $\gamma_v M_u$ is decreased, $\gamma_f M_u$ is increased.

For interior supports, the unbalanced moment transferred by flexure is permitted to be increased up to 25% provided that the factored shear (excluding the shear caused by moment transfer) at the interior support does not exceed 40% of the shear strength (ΦV_c) as defined in Section 11.12.2.1.

It should be noted that the above modifications are permitted only when the reinforcement ratio within the effective slab width defined in Section 13.5.3.2 is less than or equal to $0.375\rho_b$. This provision is intended to improve ductile behavior of the column-slab joint.

Methods for calculating the transfer of moment (and shear) in slab-column connections of two-way slabs as required by Section 11.12.6 are provided below. It should be noted that it is not necessary to perform a combined biaxial analysis of the transfer moment critical section. This is because tests have shown that it is not necessary. In addition, for reasons similar to that described above for column cumulative load or "take-down" analysis, combining the loads from 2 orthogonal panels at the same time is overly conservative.

11.12.6.2 Shear Stresses and Strength Computation

Assuming that shear stress resulting from moment transfer by eccentricity of shear varies linearly about the centroid of the critical section defined in 11.12.1.2, the factored shear stresses at the faces of the critical section due to the direct shear V_u and the unbalanced moment transferred by eccentricity of shear $\gamma_v M_u$ are (see Fig. 16-12, and R11.12.6.2):

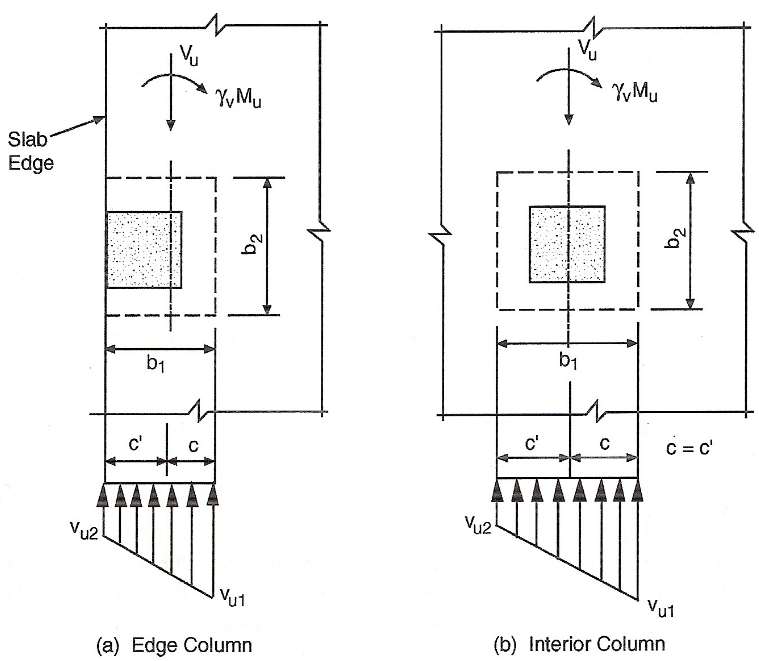


Fig. 16-12 Shear Stress Distribution due to Moment-Shear Transfer at Slab-Column Connection

$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$ *Eq. (1)*

$$v_{u2} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c'}{J}$$ *Eq. (2)*

where: A_c = area of concrete section resisting shear transfer, equal to the perimeter b_o multiplied by the effective depth d

J = property of critical section analogous to polar moment of inertia of segments forming area A_c.

c and c' = distances from centroidal axis of critical section to the perimeter of the critical section in the direction of analysis

Expressions for A_c, c, c', J/c, and J/c', are contained in Fig. 16-13 for rectangular columns and Fig. 16-14 for circular interior columns.

The maximum shear stress v_{u1} computed from Eq. (1) shall not exceed ϕv_n, where ϕv_n is determined from the following (11.12.6.2):

a. For slabs without shear reinforcement: $\phi v_n = \phi v_c$, where ϕv_n is the minimum of:

$$\phi v_c = \phi\left(2 + \frac{4}{\beta}\right)\sqrt{f'_c}$$ *Eq. (11-33)*

$$\phi v_c = \phi\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f'_c}$$ *Eq. (11-34)*

$$\phi v_c = \phi 4\sqrt{f'_c}$$ *Eq. (11-35)*

b. For slabs with shear reinforcement other than shearheads, ϕv_n is computed from (11.12.3):

$$\phi v_n = \phi\left(2\sqrt{f'_c} + \frac{A_v f_y}{b_o s}\right) \le \phi 6\sqrt{f'_c}$$ *Eqs. (11-15), (11.12.3.1), and (11.12.3.2)*

where A_v is the total area of shear reinforcement provided on the column sides and b_o is the perimeter of the critical section located at d / 2 distance away from the column perimeter, as defined by 11.12.1.2 (a). Due to the variation in shear stresses, as illustrated in Fig. 16-12, the computed area of shear reinforcement, if required, may be different from one column side to the other. The required area of shear reinforcement due to shear stress v_{u1} at its respective column side is:

$$A_v = (v_{u1} - \phi v_c)\frac{(c + d)s}{\phi f_y}$$ *Eq. (3)*

where $(c+d)$ is an effective "beam" width and $v_c = 2\sqrt{f'_c}$. However, R11.12.3 recommends symmetrical placement if shear reinforcement on all column sides. Thus, with symmetrical shear reinforcement assumed on all sides of the column, the required area A_v may be computed from:

$$A_v = (v_{u1} - \phi v_c)\frac{b_o s}{\phi f_y} \qquad \textit{Eq. (4)}$$

where A_v is the total area of required shear reinforcement to be extended from the sides of the column, and b_o is the perimeter of the critical section located at d/2 from the column perimeter. With symmetrical reinforcement on all column sides, the reinforcement extending from the column sides with lesscomputed shear stress provides torsional resistance in the strip of slab perpendicular to the direction of analysis.

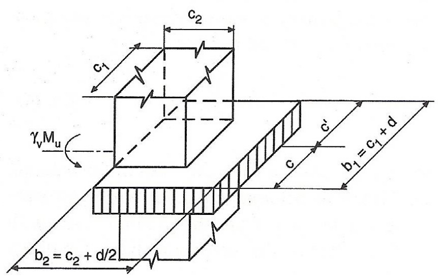
c. For slabs with shearheads as shear reinforcement, ϕv_n is computed from:

$$\phi v_n = \phi 4\sqrt{f'_c} \geq v_{u1} \qquad \textit{11.12.6.3}$$

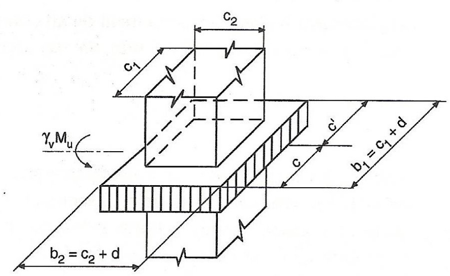
$$v_{u1} = \frac{V_u}{b_o d} + \frac{\gamma_v M_u c}{J} \leq \phi 4\sqrt{f'_c} \qquad \textit{Eq. (1)}$$

where b_o is the perimeter of the critical section defined in 11.12.4.7, c and J are section properties of the critical section located at d / 2 from the column perimeter (11.12.6.3), V_u is the direct shear force acting on the critical section defined in 11.12.4.7, and $\gamma_v M_u$ is the unbalanced moment transferred by eccentricity of shear acting about the centroid of the critical section defined in 11.12.1.2(a). Note that this seemingly inconsistent summation of shear stresses occurring at two different critical shear sections is conservative and justified by tests (see R11.12.6.3). At the critical section located d / 2 from the column perimeter, v_u shall not exceed $\phi 7\sqrt{f'_c}$ (11.12.4.8); see Fig. 16-5.

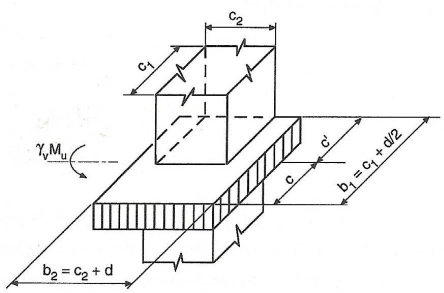
Case A: Edge Column (Bending parallel to edge)



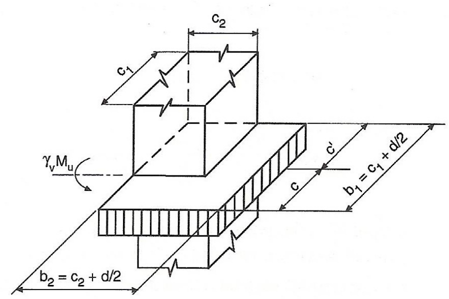
Case B: Interior Column



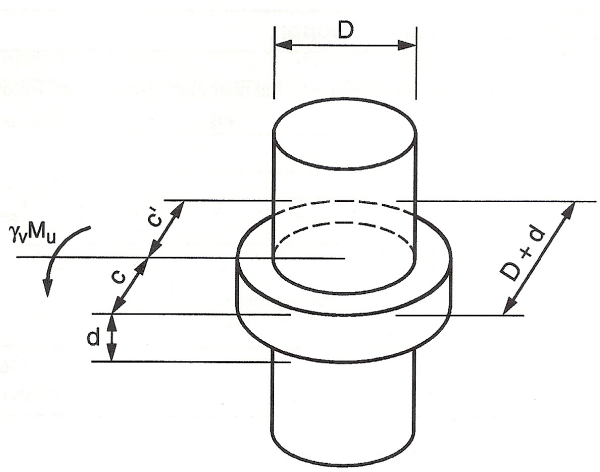
Case C: Edge Column (Bending perpendicular to edge)



Case D: Corner Column



Case	Area of critical section, A_c	Modulus of critical section J/c	Modulus of critical section J/c'	c	c'
A	$(b_1+2b_2)d$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
B	$2(b_1+b_2)d$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
C	$(2b_1+b_2)d$	$\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6b_1}$	$\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6(b_1+b_2)}$	$\frac{b_1^2}{2b_1+b_2}$	$\frac{b_1(b_1+b_2)}{2b_1+b_2}$
D	$(b_1+b_2)d$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6b_1}$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6(b_1+2b_2)}$	$\frac{b_1^2}{2(b_1+b_2)}$	$\frac{b_1(b_1+2b_2)}{2(b_1+b_2)}$



$$A_c = \pi(D+d)d$$

$$c = c' = \frac{D+d}{2}$$

$$\frac{J}{c} = \pi d\left(\frac{D+d}{2}\right)^2 + \frac{d^3}{3}$$

Fig. 16-14 Section Properties for Shear Stress Computations – Circular Interior Column

Direct Design Method:

The Direct Design Method (DDM) is an approximate procedure for analyzing two-way slab systems for gravity loads only. All design moments used in the DDM are determined directly from moment coefficients provided by the Code. The DDM does not include a live load pattern analysis as restricted by Item #5 below. The method is limited to slab systems meeting the restrictions specified in Section 13.6.1. Two-way slab systems not meeting these limitations must be analyzed by more accurate procedures such as the Equivalent Frame Method.

The limitations of the DDM include:

1. There must be 3 or more spans continuous in each direction.

2. Slab panels must be rectangular with a ratio of longer to shorter span (based on centerline-to-centerline of supports) not greater than 2.5.

3. Successive span lengths (based on centerline-to-centerline of supports) must not differ by more than 1/3 of the longer span.

4. Columns must not be offset more than 10% of the span (in the direction of offset) from either axis between centerlines of successive columns.

5. Loads must be uniformly distributed with the unfactored or service live load not more than 2 times the unfactored or service dead load (i.e. $L/D \leq 2.0$). It should be noted that it has been found that this provision satisfies the equivalent design requirements of a 2000# concentrated load on a 20 feet square and larger panel.

6. For two-way beam-supported slabs, the relative stiffness of the beams in two perpendicular directions must satisfy the minimum and maximum requirements provided in Section 13.6.1.6.

7. Redistribution of moments per Section 8.4 is not permitted.

Once a preliminary or trial slab thickness has been selected, the DDM involves essentially a three-step analysis procedure, which includes:

1. Determining the total factored static moment for each span.

2. Dividing the total factored static moment between negative and positive moments within each span.

3. Distributing the negative and positive moment to the column and the middle strips in the transverse direction.

Factored Moments

The total static design moment (M_o) for a span of the design strip = $(q_u \ell_2 \ell_n^2)/8$; where q_u = the combined factored dead and live load = $(1.2w_d + 1.6w_l)$ or $(1.4w_d + 1.7w_l)$ per Appendix B and C; ℓ_n is the clear span in the direction of analysis; and ℓ_2 is the centerline-to-centerline span of the transverse panel. It should be noted that when computing ℓ_n for circular columns or circular column capitals, the equivalent square column size that can be used is $0.886d_c$, where d_c is the diameter of the column. This same equivalent square column size

is also used when calculating the critical perimeter section properties for shear and moment transfer to the column for both the DDM and EFM.
For either a flat plate or flat slab without spandrels, the total static moment for a span is divided into negative and positive design moments as shown in Figure 9. For other end span conditions, the total static moment (M_o) is distributed as shown in Table 1.

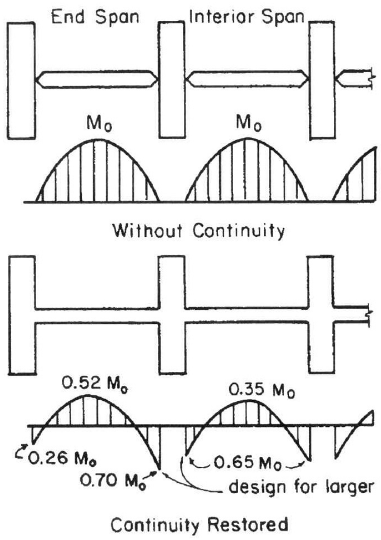


FIGURE 9

Factored Moment	(1) Slab Simply Supported on Concrete or Masonry Wall	(2) Two-Way Beam-Supported Slabs	(3) Flat Plates and Flat Slabs: Without Edge Beam	(4) Flat Plates and Flat Slabs: With Edge Beam	(5) Slab Monolithic with Concrete Wall
Interior Negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior Negative	0	0.16	0.26	0.30	0.65

TABLE 1

It should be noted that Section 13.6.3.6 requires that the minimum fraction of the design moment that must be used for the moment transferred by the eccentricity of shear is 0.30 M_o for a beamless slab edge at an exterior column. This fraction of the design moment is then factored by γ_v. Although it may seem inconsistent that the moment transfer coefficient required for this aspect of the design is greater than that required for the actual design of the slab (0.26 per Figure 9 and Table 1), this approach is required in order to assure that shear failures at this most critical support condition do not occur.

The negative and positive design moments are further distributed to the column and middle strips of the design panel. However, the column strip at the exterior of an end span is required to resist the total factored negative moment in the design panel unless edge beams are provided. Also, when the transverse width of a support is equal to or greater than three quarters (3/4) of the design panel width, Section 13.6.4.3 requires that the negative factored moment be uniformly distributed across the entire design strip width (ℓ_2).

The percentage of total negative and positive factored moments to be resisted by a column strip can be determined from the tables in Sections 13.6.4.1 (interior negative), 13.6.4.2 (exterior negative) and 13.6.4.4 (positive), or from the following expressions shown below. It should be noted that the effect of βt in Section 13.6.4.2, as already indicated above, is to assign all of the exterior negative factored moment to the column strip and none to the middle strip unless an edge spandrel beam is present.

Percentage of negative factored moment at interior support to be resisted by column strip

$$= 75 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$$

Percentage of negative factored moment at exterior support to be resisted by column strip

$$= 100 - 10\beta_t + 12\beta_t\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$$

Percentage of positive factored moment to be resisted by column strip

$$= 60 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right)$$

Note: When $\alpha_{f1}\ell_2/\ell_1 > 1.0$, use 1.0 in above equations. When $\beta_t > 2.5$, use 2.5 in Eq. (2) above.

For slabs without beams between supports ($\alpha_{f1} = 0$) and without edge beams ($\beta_t = 0$) the distribution of the negative moments to column strips is simply 75 and 100 percent for interior and exterior supports, respectively, and the distribution of total positive moment is 60%. An illustration of the transfer of the negative moment at the exterior support for a slab without beams is shown in Figure 10 below.

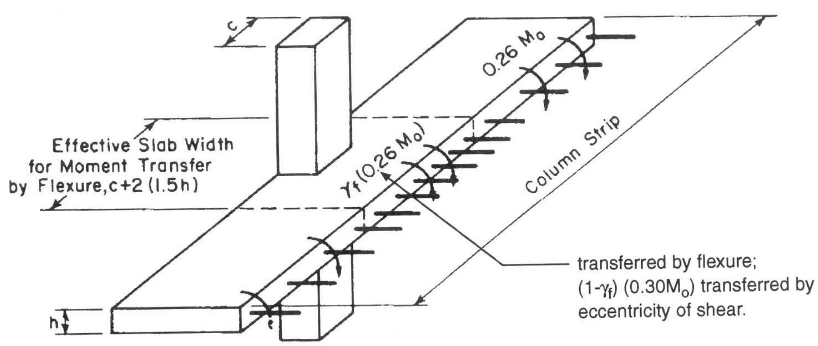
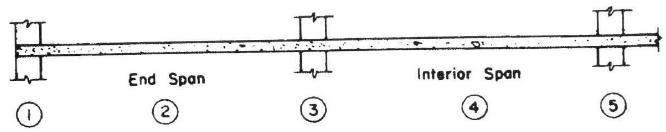


FIGURE 10

Factored moments not assigned to the column strips must be resisted by the two half-middle strips that comprising the remainder of the design strip. An exception to this is a middle strip adjacent to and parallel with an edge supported by a wall. At this condition, the moment to be resisted is twice the factored moment assigned to the half middle strip corresponding to the first row of interior supports.

As indicated above, the distribution of the total moment (M_o) into negative and positive moments, and then into column and middle strip moments, involves direct application of moment coefficients to the total moment M_o. The moment coefficients are a function of the location of span (interior or end), slab support conditions, and type of two-way slab system. For design convenience, moment coefficients for typical two-way slab systems are provided below in relation to the support conditions illustrated in the corresponding diagrams.

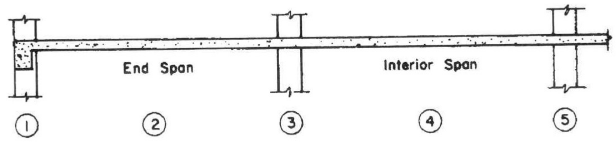
Design Moment Coefficients for Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.26M_o$	$0.52M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.26M_o$	$0.31M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.21M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

Design Moment Coefficients for Flat Plate or Flat Slab with Edge Beams

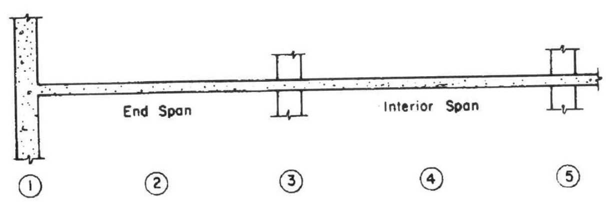


Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.30M_o$	$0.50M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.23M_o$	$0.30M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.07M_o$	$0.20M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of edge beam is such that $\beta_t \geq 2.5$. For values of β_t less than 2.5, exterior negative column strip moment increases to $(0.30 - 0.03\,\beta_t)M_o$.

The coefficients in table above are valid for $\beta_t \geq 2.5$

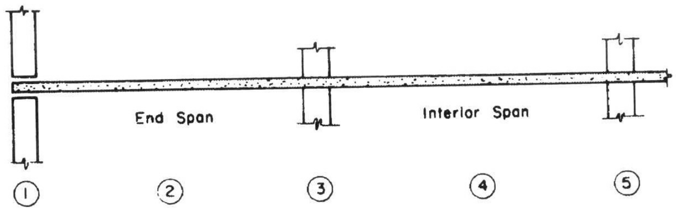
Design Moment Coefficients for Flat Plate or Flat Slab with End Span Integral with Wall



Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.65M_o$	$0.35M_o$	$0.65M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.49M_o$	$0.21M_o$	$0.49M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.16M_o$	$0.14M_o$	$0.16M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

Design Moment Coefficients for Flat Plate or Flat Slab with End Span Simply Supported on Wall



Slab Moments	End Span			Interior Span	
	(1)	(2)	(3)	(4)	(5)
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	0	$0.63M_o$	$0.75M_o$	$0.35M_o$	$0.65M_o$
Column Strip	0	$0.38M_o$	$0.56M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.25M_o$	$0.19M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

Equivalent Frame Method:

The Equivalent Frame Method (EFM) of analysis converts a three-dimensional frame system with two-way slabs into a series of two-dimensional frames (slab-beams and columns), with each frame extending the full height of the building, as illustrated in Figure 11. The width of each equivalent frame extends to mid-span between column centerlines. The EFM of elastic analysis is applicable for buildings with columns laid out on a basically orthogonal grid, with column lines extending longitudinally and transversely through the building. The analysis method is applicable to slabs with or without beams between supports. Also, as previously indicated above, the EFM may be used for lateral load analysis if the stiffnesses of frame members are modified to account for cracking and other relevant factors. It should also be noted that there are no limitations on the maximum ratio of; live load to dead load, longitudinal to transverse span, or adjacent successive spans, as is the case with the DDM.

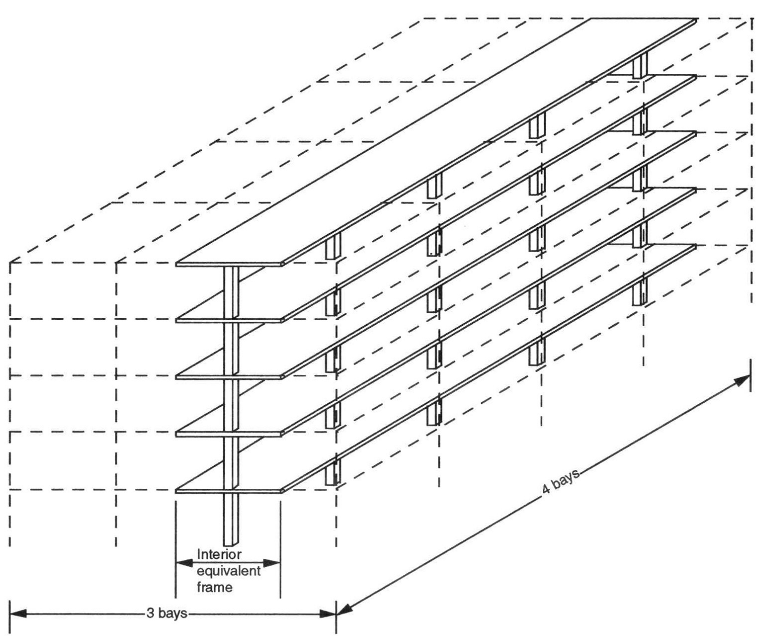


FIGURE 11

As is the case with the DDM, and in reference to Figure 5a and 5b above, some judgment is required in applying the definitions given in Section 13.2.1 for slab systems with varying span lengths along the design strip. Members of the equivalent frame are slab-beams and torsional members (transverse horizontal members) supported by columns (vertical members). The torsional members provide moment transfer between the slab-beams and the columns. The initial step in the frame analysis requires that the flexural stiffness of these same equivalent frame members be determined. The equivalent frame members are illustrated in Figure 12.

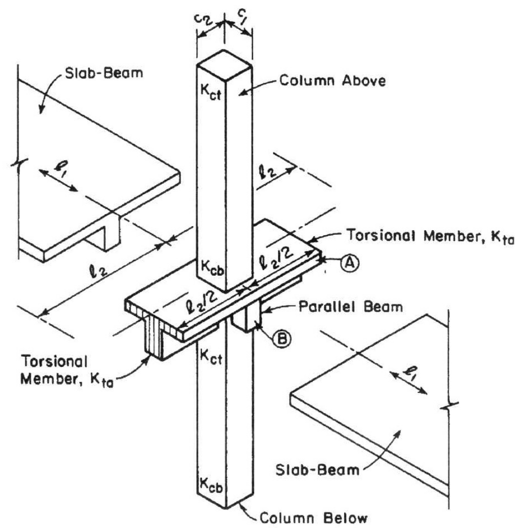


FIGURE 12

Slab-Beams

Common types of slab systems with and without beams between supports are illustrated in Figures 13 and 14. Cross-sections for determining the stiffness of the slab-beam members (K_{sb}) between support centerlines are shown for each type of slab-beam illustrated in Figures 13 and 14. These equivalent slab-beam stiffness diagrams may be used to determine moment distribution constants and fixed-end moments for equivalent frame analysis.

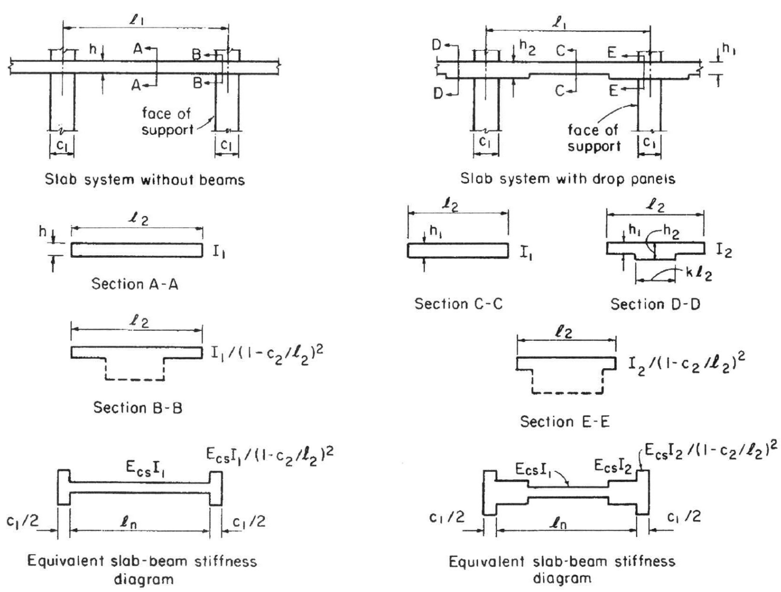


FIGURE 13

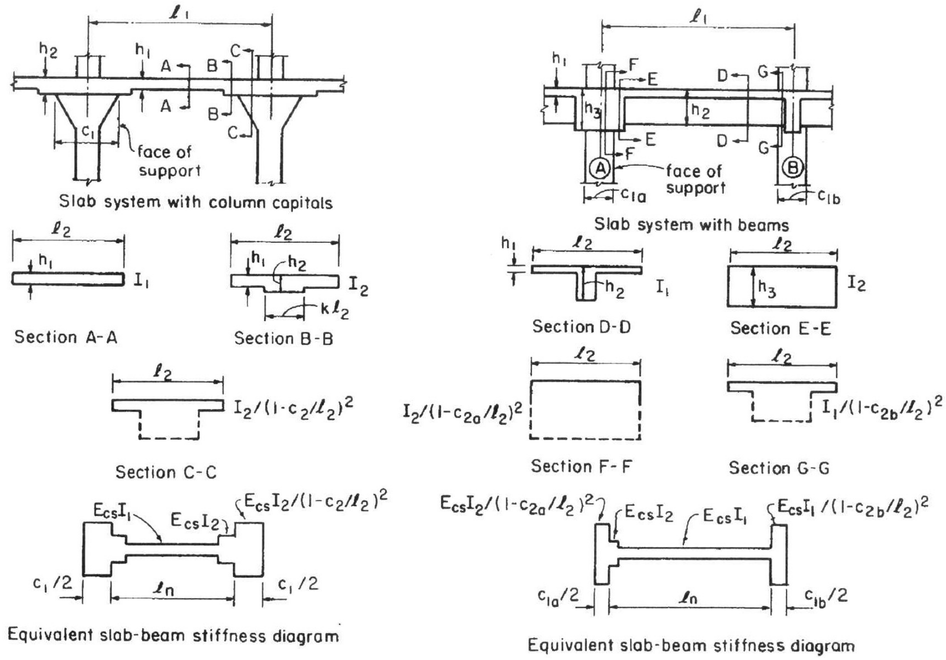


FIGURE 14

Stiffness calculations for the above slab-beam elements are based on the following considerations:

a. The moment of inertia of the slab-beam between the face of the supports is based on the gross cross-sectional area of the concrete. Variation in the moment of inertia along the axis of the slab-beam between supports must be taken into account (see Section 13.7.3.2).

b. A support is defined as a column, capital, bracket or wall. Note that a beam is not considered a supporting member for the equivalent frame (see Section R13.7.3.3).

c. The moment of inertia of the slab-beam from the face of support to the centerline of support is assumed equal to the moment of inertia of the slab-beam at the face of support, divided by the quantity $(1 - (c_2/\ell_2))^2$ (see Section 13.7.3.3).

The magnification factor $(1 - (c_2/\ell_2))^2$ applied to the moment of inertia between support face and support centerline, in effect, makes each slab-beam at least a haunched member within its length. Consequently, stiffness, carryover factors and fixed-end moments based on the usual assumptions of uniform prismatic members cannot be applied to the slab-beam members.

Columns

Common types of column end support conditions for slab systems are illustrated in Figure 15. The column stiffness is based on a height of column (ℓ_c) measured from the mid-depth of the slab above to the mid-depth of the slab below.

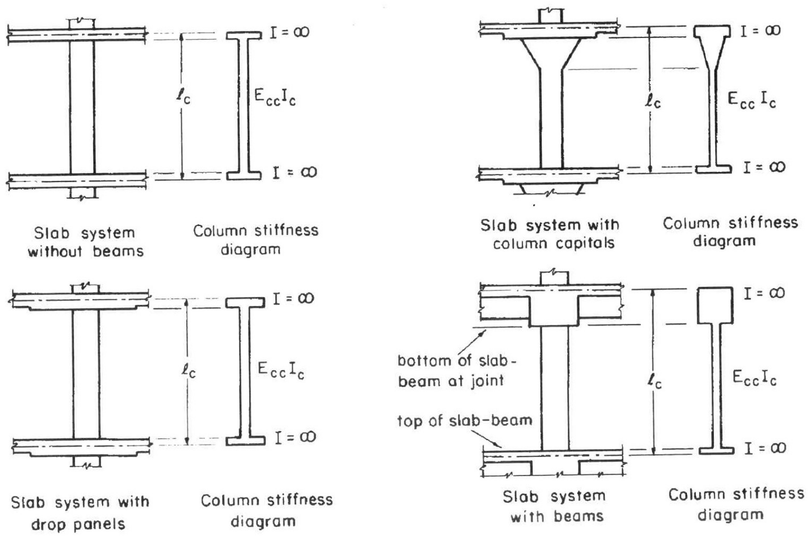


FIGURE 15

The stiffness diagrams above are based on the following considerations:

a. The moment of inertia of the column outside the slab-beam joint is based on the gross cross-sectional area of the concrete. Variation in the moment of inertia along the axis of the column between slab-beam joints is taken into account. For columns with capitals, the moment of inertia is assumed to vary linearly from the base of the capital to the bottom of the slab-beam (see Section 13.7.4.1 and 13.7.4.2).

b. The moment of inertia is assumed infinite from the top to the bottom of the slab-beam at the joint. As with the slab-beam members, the stiffness factor K_c of the columns cannot be based on the assumption of uniform prismatic members (see Section 13.7.4.3).

Torsional Members

Torsional members of common slab-beam joints are illustrated in Figure 16. The cross section of a torsional member is the largest of those defined by the three conditions given in Section 13.7.5.1 with the governing condition (a, b or c) indicated below each diagram shown in Figure 16.

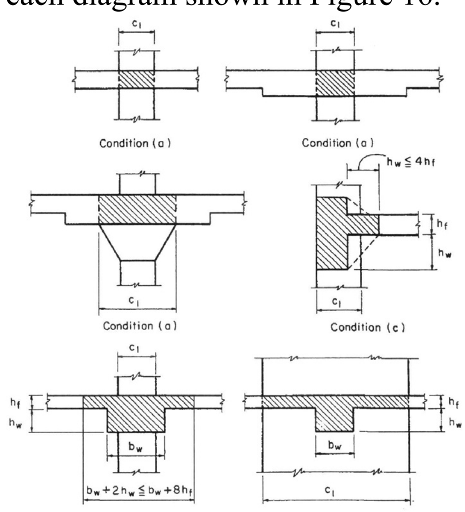


FIGURE 16

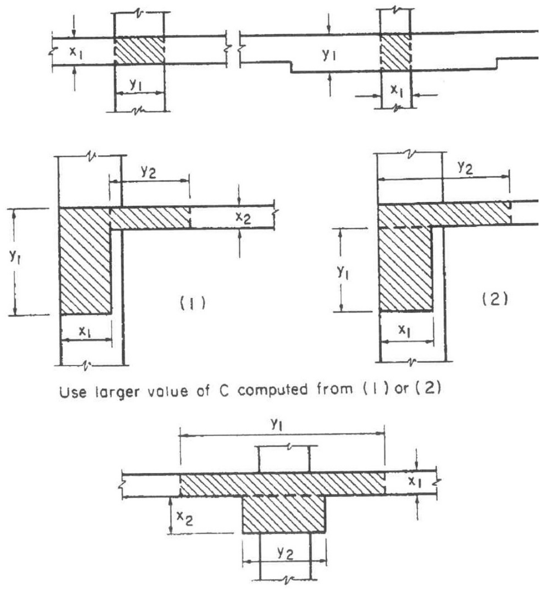
The stiffness (K_t) of a torsional member is calculated by the following expression:

$$Kt = \sum((9EcsC)/(\ell_2(1 - (c_2/\ell_2))^3))$$

The summation of Kt extends over two transverse torsional members for an interior frame and only one transverse torsional member for an end or exterior frame. The term C is a cross-sectional constant that defines the torsional properties of each torsional member framing into a joint where;

$$C = \sum((1 - 0.65(x/y))((x^3y)/3))$$

Where; x is the shorter dimension of the rectangular part and y is the longer dimension of a rectangular part. An illustration of the application of the expression C is shown in Figure 17.



$$C = \sum\left[\left(1 - 0.63\frac{x_1}{y_1}\right)\frac{x_1^3 y_1}{3}\right] + \left[\left(1 - 0.63\frac{x_2}{y_2}\right)\frac{x_2^3 y_2}{3}\right]$$

FIGURE 17

If beams frame into the support in the direction of the span under consideration, the torsional constant K_t given above must be increased as follows;

$$K_{ta} = (K_t I_{sb})/I_s$$

Where; I_s = the moment of inertia of a width of slab equal to the full width between panel centerlines (ℓ_2) excluding that portion of the beam stem extending above and below the slab (i.e. part A in Figure 12 above); and I_{sb} = the moment of inertia of the slab section specified for I_s including that portion of the beam stem extending above and below the slab.

Equivalent Columns

Although the current Code (see Section R13.7.4) still recognizes the concept of equivalent columns, the increasing use of computer programs for the purposes of the EFM has made this concept not as useful. The flexural stiffness for equivalent columns (K_{ec}) is provided below:

$$1/K_{ec} = (1/\sum K_c) + (1/\sum K_t)$$

It should be noted that the equivalent column stiffness K_{ec} at any joint of a two-way slab or plate will be less than the stiffness of the column itself, K_c, as is required to represent the fact that the attached torsional elements are not capable of transferring the design strip moment to the column that the column would otherwise be capable of receiving if the attached torsional elements were infinitively torsionally stiff.

Arrangement of Live load

In the case where the exact loading pattern is not known, the maximum factored moments are developed using loading conditions illustrated by the three-span partial frame in Figure 18 and described below:

a. When the service live load does not exceed three-quarters of the service dead load, only loading pattern (1) with full factored live load on all spans need be analyzed for negative and positive factored moments.

b. When the service live-to-dead load ratio exceeds three-quarters, the five loading patterns shown need to be analyzed to determine all factored moments in the slab beam members. Loading patterns (2) through (5) consider partial factored live loads for determining the worse case factored moments. However, with partial live loading, the factored moments cannot be taken less than those occurring with full factored live load on all spans, therefore load pattern (1) needs to be included in the analysis.

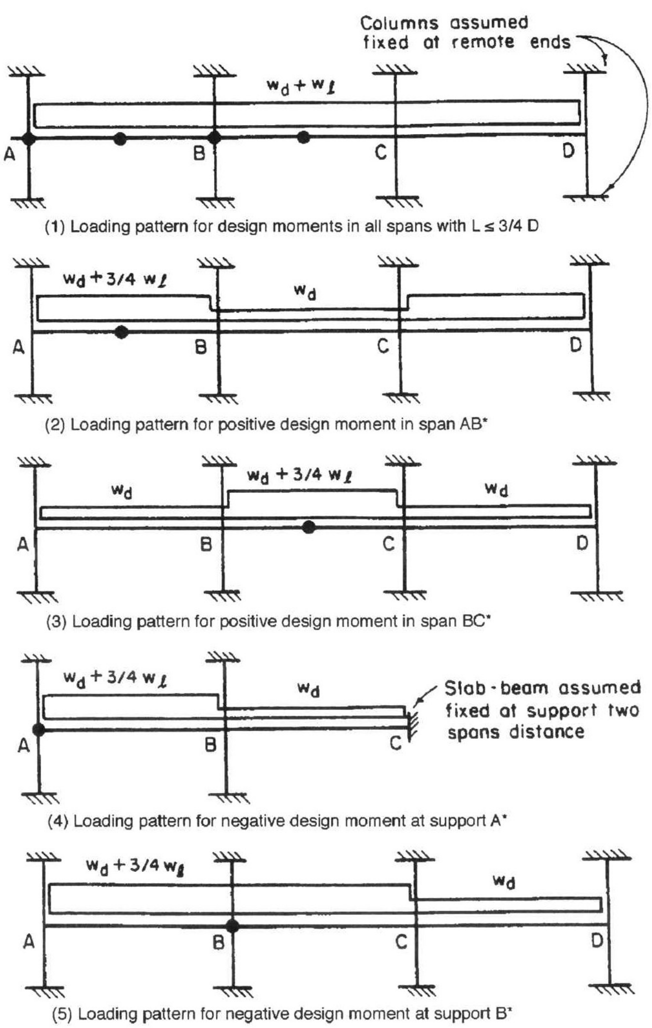


FIGURE 18

Factored Moments

Moment distribution is probably the most convenient hand calculation method for analyzing partial frames involving several continuous spans with the far ends of upper and lower columns fixed. The mechanics of the method will not be described here, except for a brief discussion of the following two points:

1. The use of the equivalent column concept to determine joint distribution factors and,

2. The proper procedure to distribute the equivalent column moment obtained in the frame analysis to the actual columns above and below the slab-beam joint.

Negative factored design moments must be taken at the faces of the supports in the EFM, but not at a distance greater than 0.175 ℓ_1 from the center of the support. The support member is defined as a column, capital, bracket or wall. It should be noted that since the EFM is no an approximate method like the DDM, moment redistribution allowed by Section 8.4 can be used, unless one is designing under the provisions of Appendix B and C.

Similar to the DDM, factored moments in the panel (or more precisely the design strip) for both negative and positive moments, may be distributed to the column strip and the two half-middle strips of the slab-beam in accordance with Sections 13.6.4, 13.6.5 and 13.6.6, provided that the requirements of Section 13.6.1.6 are satisfied.

Finally, it should be noted that the check of moment and shear transfer at an exterior column using the EFM differs from that performed for the DDM (in which the moment and shear are calculated at the center of the joint). This correction is accomplished by using the following formula to adjust the moment about the centroid of the shear perimeter when the shear acts through this same point.

$$M_u = M_{sj} - M_c - ((V_{slab} + V_{cant})e_{sc}); \text{ where } M_c = V_{cant}e_c$$

See Figure 14-15 (page 751) in of your textbook (*Reinforced Concrete Mechanics and Design*, 4th Edition) for an explanation of the other variables used in the above formula.

Other Methods:

Finite Element Analysis (FEA) Method

Two-way slab behavior can be modeled directly using FEA methods, which typically involves plate bending elements. The FEA approach is available in a number of proprietary software packages and is beyond the scope of this course.

Explicit Transverse Torsional Member Method

As already described, the behavior of two-way slab systems under gravity and lateral loads is complex. Unlike planar frames, in which beam moments are transferred directly to columns, slab moments are transferred indirectly, due to the torsional flexibility of the slab. Also, slab moments from gravity loads can "leak" from loaded to unloaded spans. All of these phenomenon must be accounted for in the analysis of a two-way slab. The need to model torsional flexibility and moment leakage has given rise to two main analysis approaches for two-way slabs; FEA (see Lecture 12), and equivalent frames (see above).

One of the components of the equivalent frame approach is the transverse torsional member procedure. The transverse torsional member procedure was developed following extensive testing of two-way slabs in the 1960's and 1970's. Those portions of the slab attached to the columns and transverse to the direction of the span (plus the transverse beams, if any) are assumed to act as transverse torsional members, which transfer the moments from the slabs to columns. These transverse members are assumed rigid except in torsion. Moment transfer is treated as occurring directly over the column width c_2 and along the torsional members. The rotational stiffness of the joint is determined as a function of the torsional stiffness of the transverse members on each side of the joint and of the flexural stiffness of the columns above and below the joint.

The transverse torsional member procedure accounts both for slab torsional flexibility and moment leakage and has been incorporated into the ACI equivalent frame method. Another method, termed the Explicit Transverse Torsional Member Method (ETTMM), was developed in the late 1980's at the University of Texas, Austin. In this method, as with the equivalent frame method, member actions are computed, distributed to column and middle strips and then used for the design of the slab. I have a personal preference for this approach because it is simple and analytically represents a real world physical model that can be constructed using any simple three-dimensional frame analysis program. Section 13.5.1 allows for the use of this method of analysis.

An illustration of an ETTMM model is shown in Figure 19. Figure 19 shows how conventional columns are connected indirectly to two conventional slab-beam elements (each with half the stiffness of the actual slab-beam) by the explicit torsional members. The indirect connections using explicit transverse torsional members permit the modeling of moment leakage as well as slab torsional flexibility using three-dimensional computer software. Because the transverse torsional members are present only in the analytical model, their lengths are arbitrary, provided that their torsional stiffnesses are consistently and properly defined.

The ETTMM has several advantages. Structural modeling is simple and direct, requiring very few hand computations. Also, computed member actions in the slab-beams and transverse torsional members can be used directly for design of slabs and spandrels, respectively. Finally this method can even be used for true three-dimensional analysis of slab systems under combined gravity and lateral loads.

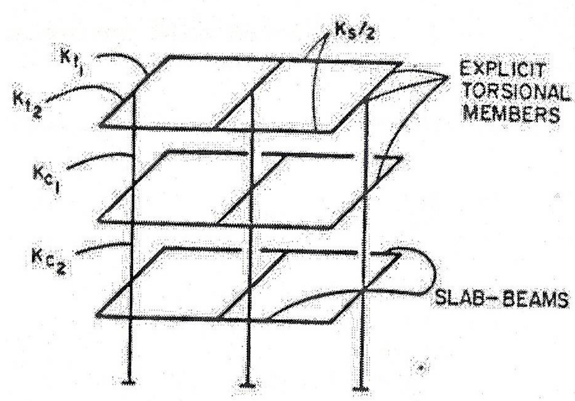


FIGURE 19

Gross member properties are used for slab-beams and columns for gravity analysis, however, when lateral loads are present, slab-beam cracking should be considered by multiplying the slab-beam moment of inertia by a reduction factor of .33 for flat slabs or flat plates for the initial lateral analysis. Area, moment of inertia, and shear area are all calculated conventionally. For computer input, the torsional stiffness (J) of the transverse torsional members is calculated by the following procedure. First the torsional stiffness (K_t) is calculated as:

$$K_t = (\Sigma 9E_cC/\ell_2)(1 - (c_2/\ell_2))^3$$

Where: c_2 = The size of the column measured transverse to the direction of span

ℓ_2 = The length of span transverse to the direction of span

C = Cross-sectional constant used to define torsional properties (Section 13.6.4.2)

Using an arbitrary length (L) for the transverse torsional members, the torsional stiffness (J_i) of each torsional member is calculated as:

$J_i = K_{ti}L/G$; where G = Shearing modulus of concrete

The analysis of shear and unbalanced moments at interior and exterior column supports for this method of design is similar to the ACI equivalent frame method.

Openings in Two-Way Concrete Floor Slabs

For new construction, the locations and sizes of openings in the slab are usually determined in the early stages of design and can be accommodated into the design of the slab in the majority of cases. However, it is also not unusual to be called upon to modify an existing structure for a new slab opening. In this later case the analysis and strengthening (if required) are typically more involved than for a similar opening in a new slab. This is because it is unlikely that the existing internal reinforcement was arranged and designed to account for any future openings.

For existing construction, when the location of small openings is carefully selected, it is often possible to accommodate the remedial work without strengthening the slab. However, it is more common that a new opening will require strengthening because the size and location of the opening is dictated by concerns other than the strength of the structure.

Openings in New Slabs:

Section 13.4.1 of ACI 318-05 permits openings of any size in any new slab system provided an analysis demonstrates both strength and serviceability requirements are satisfied. As an alternative to a detailed analysis for slabs with openings ACI 318-05 also provides the following guidelines for opening sizes in different locations for flat plates and flat slabs. These guidelines are illustrated in Figure 20 for slabs with $\ell_2 \geq \ell_1$:

- In the area common to intersecting middle strips, openings of any size are permitted (Section 13.4.2.1).

- In the area common to intersecting column strips, the maximum permitted opening size is 1/8 the width of the column strip in either span (Section 13.4.2.2).

- In the area common to one column strip and one middle strip, the maximum permitted opening size is limited such that only a maximum of ¼ of the slab reinforcement in either strip may be interrupted (Section 13.4.2.3).

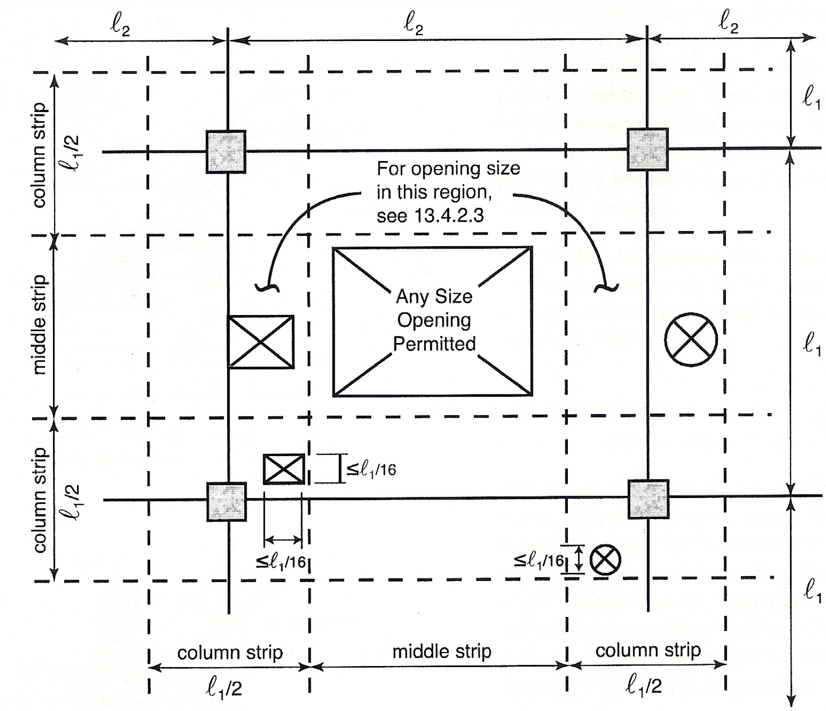


FIGURE 20

In order to use the above simplified approach ACI 318-05 also requires that the total amount of reinforcement calculated for the panel without openings in both directions must be maintained. Therefore, half of the reinforcement interrupted must be replaced on each side of the opening. In addition to the flexural reinforcement requirements, the reduction in slab shear strength must also be considered when the opening is located anywhere within a column strip of a flat slab or within 10 times the slab thickness from a concentrated load or reaction.

The effect of the slab opening on the shear capacity of the slab is evaluated by reducing the perimeter of the critical section (b_o) by a length equal to the projection of the opening enclosed by two lines extending from the centroid of the column and tangent to the opening, as shown in Figure 21a. For slabs with shearheads or other similar internal shear reinforcement the effect of the opening is reduced, and b_o is reduced by only half of the length enclosed by the tangential lines, as shown in Figure 21b.

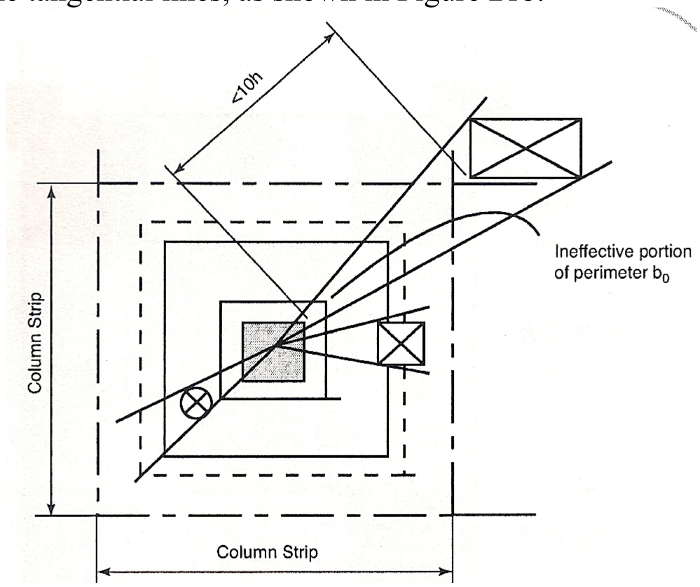


FIGURE 21a

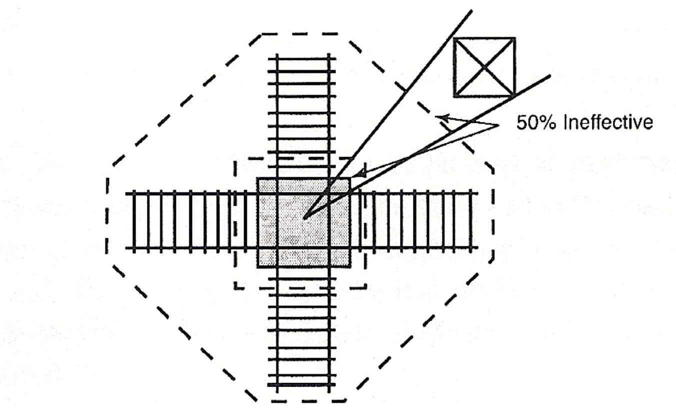


FIGURE 21b

Openings in Existing Slabs:

Small openings in existing slabs are usually core-drilled to the required diameter. Larger openings are cut with a circular saw or a concrete chain saw with plunge cutting capabilities. Because a saw makes a longer cut on the top of the slab than on the bottom, small cores or holes are first drilled at the corners to help avoid over-cutting the opening with the saw and to also prevent reentrant corner cracking. Cutting openings in existing slabs should be approached with caution and avoided if possible. When cutting an opening in an existing slab, the effect on the structural integrity of the slab must be analyzed first. The following guidelines can assist in making preliminary decisions concerning new openings in existing slabs as required to avoid, if possible, having to reinforce the slab.

Because the punching shear capacity of the slab around the columns typically governs the thickness of flat plates, any openings at the intersection of column strips (see Area 3 in Figure 22) should be avoided as much as possible. This is especially critical near corner and edge columns where the shear stresses in the slab are typically highest. If openings must be made in Area 3, to install for example a drainage pipe, the size of the opening should be no larger than 8 inches in diameter. Because openings cut in this area reduce the critical section for resisting punching shear (see Section 11.12.5 of ACI 318-05), any openings cut in this same area should be evaluated carefully. One exception to this guideline is when a column capital is encountered, which is common in older structures.

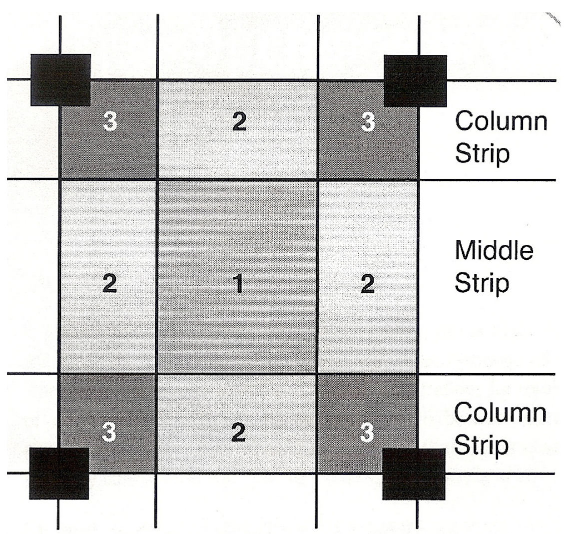


Figure 22

Openings in Area 2, located at the intersection of column and middle strips, are less critical than in Area 3. Small openings having a width less than 15% of the span length can often be made in Area 2 without any reinforcement being required. The most favorable location for openings, from a structural point of view, is the intersection of the two middle strips (see Area 1). Unfortunately, this area is normally the least favorable location from an architectural point of view because it is the most disruptive to the function of the space in relation to the support columns.

For two-way beam systems, because the shear is transferred to the column through the beams, the total width of openings in Area 3 can often be up to ¼ of the span, as long as the beams are left intact. Openings in Area 2 can be more problematic because they may intersect the portion of the slab used as the compression flange of a T-Beam section. Area 1 is the least desirable location. However, openings with minimum dimensions up to 1/8 of the span can often be located in the intersection of the two middle strips. When removing an entire panel of slab between beams, it is often advantageous to leave enough of a slab overhang to allow for the development of the top reinforcing bars from the adjacent slab spans. In this later case it is also important that the beams be checked for torsion because the balancing moments from the portion of the slab that was removed will no longer be present.

Openings in new structures can often be accommodated by the proper detailing and introduction of additional reinforcing steel in the slab, or thickening of portions of the slab around the opening. However, when it is determined that an existing structure can't accommodate a new openings there are several common strengthening methods that can be used to make the installation of the opening possible.

One of the most common methods for increasing moment capacity in an existing slab detrimentally impacted by a new opening is to add steel plates to the surface of a slab using through-bolts, post-installed anchors or epoxy. The installation process is fairly simple, however, because the plates and through-bolts can interfere with finish floor surfaces, plates are normally installed on the bottom of the slab only. Also, because overlapping of the plates is difficult, this method works best when strengthening is required in only one direction.

A similar method of external reinforcing involves the installation of fiber-reinforced or steel-reinforced polymer strips to strengthen the slab. The strips can be overlapped at the corners of the opening, making strengthening in two directions simpler. In addition, this type of strengthening does not typically interfere with the finish floor surface as much as anchored steel plates. The installation of composite materials for the purposes of strengthening, however, requires more highly skilled labor that that associated with external steel plates.

Another method of strengthening to offset the effects of moment and shear reduction on an existing slab with new openings involves the installation of steel beams that span between the main support columns. Shims or non-shrink grout should be installed between the top flange of the steel beam and the bottom of the slab to assure continuous uniform bearing support between the beam and the existing structure. When only shear strengthening is required around columns, another common solution is to install steel or concrete collars around the columns to increase the perimeter of the critical section for punching shear.

It is important to remember that exposed reinforcing systems may require fire protection. Systems that incorporate epoxy adhesives must be carefully evaluated, as they can lose strength rapidly at elevated temperatures. For higher levels of strengthening, special coatings (intumescent paint or spray on fireproofing) may be required to achieve a specific fire rating.

Reinforcement Placement

The minimum extensions for reinforcement in slabs without beams (flat plates and flat slabs) are illustrated in Figure 13.3.8 (see Section 13.3.8.1 of ACI 318-05). It is important to note that the reinforcement details shown in Figure 13.3.8 do not apply to two-way slabs with beams between supports or to slabs in non-sway or sway frames resisting lateral loads. For these types of slabs, a general analysis must be made according to Chapter 12 of the Code to determine bar lengths based on the moment variation, however, the bar extensions cannot be less than those prescribed in Figure 13.3.8.

In slabs without beams, all bottom bars in the column strip shall be continuous or spliced with Class A splices or with mechanical or welded splices (satisfying Section 12.14.3) in order to provide some capacity for the slab to span to an adjacent support in the event a single support is damaged. The splices shall be located as shown in Figure 13.3.8. Additionally, at least two of these continuous bottom bars shall pass through the column and be anchored at exterior supports. For slabs with shearhead reinforcement, inadequate clearance may make it impractical to pass the column strip bottom reinforcing bars through the column. In this case, two continuous bonded bottom bars in each direction shall pass as close to the column as possible or through holes in the shearhead arms.

Similar to that which was discussed in Lecture 6 for one-way construction, where the bars of each two-way orthogonal panel intersect and overlap (in both the positive and negative moment regions for two-way flat slab or flat plate construction), in order to account for the reduction in depth from the compression face of the slab or plate to the center of gravity of the tension reinforcement, it is necessary to reduce the "d" for at least one of the panel elements assumed in the design of the same. In this situation it is common to reduce the d for the shortest design strip span in a building with rectangular bays, so that the design strip or panel with the longest span can take advantage of the greatest d possible.

It is possible to supplement the shear strength of a two-way flat plate or slab using internal reinforcement. Shear reinforcement can include stirrups, shearheads (structural steel sections) and proprietary systems. I have never seen conventional stirrups used in a two-way slab and I haven't seen a shearhead used in two-way construction since the late 1970's. The most common type of internal shear reinforcement used today is the proprietary systems. The proprietary systems include both vertical headed studs mechanically anchored to plates (sometimes referred to as rails) or articulated "zigzag" bars such as the Lenton Steel Fortress.