DEVELOPMENT OF A METHOD TO ANALYZE STRUCTURAL INSULATED PANELS UNDER TRANSVERSE LOADING

By

HEMING ZHANG ALWIN

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To the faculty of Washington State University:

The members of my Committee appointed to examine the thesis of HEMING ZHANG ALWIN find it satisfactory and recommend that it be accepted.

Chair

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Abstract

By Heming Zhang Alwin, M.S.
Washington State University
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Chair: John C. Hermanson

Structural insulated panel (SIP) use in residential building began in the 1950s. Over the last two decades, greater SIPs usage has been encouraged by many factors. ICBO ES provides “Acceptance Criteria for Sandwich Panels AC04” for sandwich panels recognition. The criteria require that full-scale panels be tested in the laboratory. The criteria also allow the use of rational analysis to obtain full-scale panel mechanical properties. APA-The Engineered Wood Association (APA) published the design specifications for plywood sandwich panels. Yet, recent research showed that the design specifications provided by APA are inaccurate and incomplete. The goal of this research was to understand the limitations of APA design specifications and develop a better understanding of SIPs mechanical behavior to guide future simplified design equations.

Mechanical tests were conducted on expanded polystyrene (EPS) core and oriented strand board (OSB) sheathing properties were obtained from the literature. The EPS property

values obtained from the tests were consistent with the published values. The stress-strain relationship of EPS foam in compression, tension, and shear were fit to material empirical models. Mechanical properties of the OSB and EPS empirical models were input to finite element models of four-point flexure testing. The results were compared to the corresponding mechanical tests.

The load-displacement curves generated by the hyperfoam and bilinear models and the curves obtained from beam bending testing did not match. However, the hyperbolic tangent model matched the data quite well.

Both experimental data and analytical modeling showed that the SIPs behavior is governed by compression and shear of EPS. A multi-span flexure test can be used to obtain an initial shear modulus and compression strength can be used as the shear strength. Future design equations for SIPs must incorporate checks for shear and bearing capacity.

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Chapter 1

INTRODUCTION

Structural insulated panels (SIPs) are a sandwich system constructed with an insulating core between two structural sheathings. They can be used in walls, roofing, and flooring. The core provides insulation and shear rigidity, and sheathings provide flexural stiffness and durability. Expanded polystyrene (EPS), extruded polystyrene (XPS), and polyurethane are the most common core materials. Sheathings typically are made of oriented strand board (OSB) or plywood. The research reported here focuses on SIPs that consist of OSB sheathings and EPS core since they are the most commonly used in residential applications.

STATEMENT OF PROBLEM

The use of structural insulated panel in residential building began in the 1950s. Since then, SIP manufacturers have continued to develop the manufacturing process and the product [1]. Fluctuating lumber prices, lumber quality, greater concern for energy conservation, ease of construction, and economy have encouraged greater SIP use over the last two decades.

The International Conference of Building Officials Evaluation Service, Inc. (ICBO ES) does technical evaluations of building products, components, methods, and materials. ICBO ES acceptance criteria are documents for evaluating a type of product, and establishing conditions of acceptance. Acceptance Criteria for Sandwich Panels AC04 [2] provides a guideline for recognition of sandwich panels under the Uniform Building Code (UBC), the International Building Code (IBC), and the International Residential Code (IRC). The criteria require that full-scale panels be tested for their specific use. Allowable loads may be interpolated for smaller scale panels, but extrapolation to larger panels is not permitted. Obviously, it is expensive and time consuming to test large panels. The criteria also permit the use of rational analysis to obtain mechanical properties of full-scale panel based on each component's mechanical properties.

APA-The Engineered Wood Association (APA) published the design specifications for plywood sandwich panels [3]. Those equations in the APA publication are based upon classical laminated beam theory that only includes a deflection check and stress checks due to bending and shear. Esvelt [4] conducted laboratory tests of full-size panels in 1999 and found that the SIPs failed either in shear at a wire chase or bearing at a support, with one exception in bending. Yet, the published APA design equations do not include bearing checks, nor predict the correct deflections. The APA calculations deflections differed by 2 to 12 standard deviations from those observed in Esvelt's testing. Esvelt concluded from her research that the APA's design equations are inaccurate and incomplete.

Many computational models based on rational analysis have been developed for predicting the response of sandwich panels [5]. However most have been developed only to model sandwich panels with metal honeycomb core and metal or synthetic composite sheathing in aeronautical applications. Expanded polystyrene (EPS) has very different mechanical properties than metal honeycomb. The applicability of these computational models for sandwich panels is critically dependent on the core's properties. To date, no reliable computational models have been developed for SIPs with OSB sheathing and EPS core.

Classical beam theories assume that there is no transverse flexibility of the core. Obviously, the assumption is not applicable for SIPs, for which deflection of the top and bottom sheathings are not equal due to deformation of the compressible core. Frostig et al. [6] used high-order theories in the analysis of sandwich beams with a transversely flexural core, ie, through the depth of the beam. High-order theories include the non-linearity of the longitudinal and the transverse deformations of the core through the depth and incorporate appropriate boundary conditions at the interface between core and sheathing. They can be used in analyzing SIPs that consist of various sheathing material and dimensions and core material of foam or honeycomb. These theories are applicable to all types of loading and boundary conditions [6, 7].

High-order theories proved to be a more accurate predictor of a composite beam's mechanical response to loading, but their use is far too complicated for a design equation. Accurate and simplified design equations for SIPs are needed. An understanding of the behavior of SIPs under transverse loading is prerequisite to generating those simplified design equations.

OBJECTIVES

The hypothesis for this research is that the mechanical response of SIPs in flexure can be predicted from the mechanical responses of the individual components. The research objective for the work described here is to obtain the mechanical response of the individual components, EPS foam and OSB sheathing, and of the flexure response of SIP beams and to use these responses to model, within finite element analysis, the response of the observed SIP. Such a model would help to identify the critical point and failure mode in SIPs which can lead to future research to developing simplified design equations.

Chapter 2
LITERATURE REVIEW

The International Conference of Building Officials Evaluation Service, Inc. (ICBO ES), does technical evaluations of building products, components, methods, and materials. ICBO ES provides “Acceptance Criteria for Sandwich Panels AC04” [2] for sandwich panels’ evaluation. The criteria require that full-scale panels be tested for their specific use. Allowable loads may be interpolated for smaller scale panels, but extrapolation to larger is not permitted. However, the criteria also allow the use of rational analysis to obtain mechanical properties for full-scale panels based on each component’s properties. APA published the design specifications for plywood sandwich panels according to classical beam theory [3]. However, Esvelt found that the APA design specifications can not accurately predict SIPs’ behavior [4]. Noor, Burton, and Bert have published a literature review of computational models for sandwich panels and plates [5]. Included among their more than 800 references is Frostig’s “High-Order Theory in Analysis of Sandwich Beams with a Transversely Flexural Core” [6].

ABAQUS and ADINA are two finite element analysis programs that can be used for computational analysis of SIPs. Mechanical properties of OSB sheathings and EPS core are required inputs. For the OSB sheathing, necessary properties were provided by Bozo [8]. However, there are some difficulties in determining the mechanical properties of EPS foam [9]. EPS mechanical properties have been published by ASTM [10] and they appear on some EPS industry websites [11]. But those were limited to a single value of material properties. For ADINA’s user-supplied material, a stress-strain curve is required to describe every material property. Murphy found that a hyperbolic and linear equation can fit stress-strain data of woodfiber-plastic composites with four parameters [12].

ICBO ES

The ICBO ES oversees technical evaluations of building products, components, methods, and materials. In July 2001, they issued "Acceptance Criteria for Sandwich Panels[2]." The criteria, which is consistent with Uniform Building Code, International Building Code, and International Residential Code, provides a procedure for recognition of sandwich panels.

The criteria stipulate that full-scale panel tests must be performed to determine the allowable load. This may be interpolated for smaller scale panels, but extrapolation to larger ones is not permitted. According to panels' usage and load type, the following tests may need to be conducted: wall panels transverse load test, wall panels axial load test, wall panels racking shear tests, roof and floor panels uniform load test, and roof and floor concentrated load test. Three tests of each type are mandated with the results varying no more than 15 percent from the average of the three. A minimum factor of safety of three is applicable to the ultimate load according to the average test value. When tests are not conducted to failure, the highest load reached for each test will be assumed to be ultimate.

To provide flexibility of panel size, the criteria permit the use of rational analysis to obtain full-scale panels' mechanical properties based on each component's properties. Confirmatory tests on actual panels will only be necessary for verifying design assumptions and criteria.

APA'S DESIGN SPECIFICATIONS

APA-The Engineered Wood Association published "Design and Fabrication of Plywood Sandwich Panels" [3] based on rational analysis. This publication presents a method for design of sandwich panels under horizontal, vertical, or combined loading. It is assumed that a sandwich panel acts as a laminated beam. Axial forces and bending moments are resisted by the sheathings, shear forces and stability of the sheathings are carried by the core.

The dimensions of a structural sandwich panel used in the APA specifications are shown in Figure 2-1. Deflection and stresses for structural sandwich panels in these specifications are found as:

(1) Deflection due to uniform transverse loading only is

$$\Delta = \Delta_b + \Delta_s = \frac{5wL^4 \times 1728}{384EI} + \frac{wL^2}{4(h+c)G_c} \tag{1}$$

Total deflection including the effects of axial loads is approximately equal to

$$\Delta_{\max} = \frac{\Delta}{1 - P / P_{cr}} \tag{2}$$

$$P_{cr} = \frac{\pi^2 EI}{(12L)^2 \left[1 + \dfrac{\pi^2 EI}{(12L)^2 \times 6(h+c)G_c}\right]} \tag{3}$$

(2) Maximum bending stress

$$f_{b,\max} = \frac{1.5wL^2 + P\Delta_{\max}}{S_1} \tag{4}$$

(3) Maximum shear stress

$$f_v = \frac{wL}{12(h+c)} \tag{5}$$

Where,

A_1 = *area of upper sheathing (in.^{2}/ft)*
A_2 = *area of lower sheathing (in.^{2}/ft)*
c = *core thickness (in.)*
h = *panel thickness (in.)*
E = *modulus of elasticity of plywood (psi)*
G_c = *modulus of rigidity of core (shear modulus) in direction of span (psi)*
I = *panel moment of inertia (in.^{4} per foot of width)*

L = *span length (ft)*
P = *axial load (lb per foot of panel width)*
P_{cr} = *theoretical column buckling load (lb per foot of panel width)*
S = *section modulus of panel (in.^{3} per foot of width)*
S_1 = *section modulus with A_1 face in tension*
S_2 = *section modulus with A_2 face in tension*
w = *normal uniform load (psf)*
$\bar{y}$ = *distance from neutral axis to outmost fiber (in.)*
Δ = *deflection due to transverse loading (in.)*
Δ_b = *deflection due to bending (in.)*
Δ_s = *deflection due to shear (in.)*

$$I = \frac{A_1 A_2 (h+c)^2}{4(A_1 + A_2)} \tag{6}$$

$$S_1 = \frac{1}{h-\bar{y}}, \qquad S_2 = \frac{1}{\bar{y}} \tag{7}$$

$$\bar{y} = \frac{A_1 (h - \frac{t_1}{2}) + A_2 (\frac{t_2}{2})}{A_1 + A_2} \tag{8}$$

ESVELT'S RESEARCH

Esvelt [4] investigated the behavior of structural insulated panels under transverse loading. She determined the core and sheathing mechanical properties and modeled panels under various failure modes. In the initial step, Esvelt performed small-size testing of the EPS core in tension, compression, and shear. The modulus of elasticity, yield stress, maximum strength and strain-stress curve also were obtained from small-size testing of OSB sheathing in bending. Additionally, Esvelt tested full-scale panels with simple-span and multi-span under a uniform transverse load. Two common failure modes (shear failure for panel with wire chase and bearing failure for panel without wire chase), and one uncommon failure (flexural failure) were observed during testing. Loads that produced the mid-span deflection of L/360, L/240, and L/180 were recorded. Empirical data and calculated data obtained from the APA design equations then were compared. The APA design equation significantly under-predicted the actual load to deflect the panels at L/360, L/240, and L/180 by between 2 and 12 standard deviations.

Esvelt used the COSMOS finite element program for modeling SIPs. A plane strain, two-dimensional, four-node isoparametric element was used to analyze SIPs' non-linear behavior. For deflection models, the differences between the finite element model and laboratory results ranged from -5.6% to 31.1%, which showed that these models failed to predict panels' actual response. For the bearing failure model, she suggested modeling the core as a bilinear material. She determined that even a minor change of the core's shear modulus significantly affected the stiffness of SIPs.

NOOR, BURTON, AND BERT'S REVIEW

Noor, Burton, and Bert [5] performed an extensive literature review of sandwich panels. In that review, they classified the various computational models for predicting the response of sandwich panels and shells as ordinary, open-face, and multi-layer. The modeling method distinguished four categories: detailed models, three-dimensional continuum models, two-dimensional plate and shell models, and simplified models. Most studies they referenced focused on metallic and nonmetallic honeycomb cores. Knowing core properties is a prerequisite for modeling sandwich panels and shells with reliable response predictions. They grouped their citations on core properties' determination in three categories: experiments, analytical models, and finite element models.

The authors also reviewed the literature on miscellaneous problems of sandwich panels and shells. These were listed under ten categories: heat transfer; static thermomechanical stress; free vibrations and damping; transient dynamic response; bifurcation buckling, local buckling, face sheet wrinkling and core crimping; large deflection and post-buckling; effects of discontinuities and geometric changes; damage and failure of sandwich structures; experimental studies; and optimization and design studies.

In the thermomechanical stress analysis category, research has been performed in three general geometries: panels with rectangular cross-section, panels with circular cross-section, and cylindrical shells with circular cross-section.

FROSTIG'S HIGH-ORDER THEORY

Frostig, *et al* [6] pioneered the use of high-order theory in the analysis of sandwich beams with a transversely flexural core. The theory assumes sheathings to be ordinary thin beams, acting only longitudinally, and interconnected through equilibrium and compatibility at their interface with the core. The core is considered to be a two-dimensional elastic medium. All behavior equations, with given boundary and continuity conditions, for the entire beam can be derived from the horizontal and vertical deflections of the upper and lower sheathings and the shear stress in the core.

This high-order theory is based on the following four assumptions: 1) longitudinal stresses in the core are negligible; 2) height of the core and its plane section can deform in a nonlinear pattern; 3) stresses and deformation fields are uniform through the width; and 4) loads applied at the sheathings can be arbitrary. Figure 2-2 provides the information necessary to analyze sandwich panels using high-order theory.

The governing equations are:

$$EA_t u_{ot,xx} + \tau b = -n_t \tag{9}$$

$$EA_b u_{ob,xx} + \tau b = -n_b \tag{10}$$

$$EI_t w_{t,xxxx} + \frac{bE_c w_t}{c} - \frac{bE_c w_b}{c} - \frac{\tau_{,x} b(c+d_t)}{2} = q_t - m_{t,x} \tag{11}$$

$$EI_b w_{b,xxxx} - \frac{bE_c w_t}{c} + \frac{bE_c w_b}{c} - \frac{\tau_{,x} b(c+d_b)}{2} = q_b - m_{b,x} \tag{12}$$

$$u_{ot} b - u_{ob} b - \frac{w_{t,x} b(c+d_t)}{2} - \frac{w_{b,x} b(c+d_b)}{2} - \frac{\tau_{,xx} bc}{12E_c} + \frac{\tau bc}{G_c} = 0 \tag{13}$$

Where,

EA_t = *axial rigidities of top sheathing*
EA_b = *axial rigidities of bottom sheathing*
u_{ot} = *longitudinal displacement of centroid of the top sheathing*
u_{ob} = *longitudinal displacement of centroid of the bottom sheathing*
τ = *shear stress in core*
b = *width of beam*
n_t = *distributed horizontal stress resulted from external loads in top sheathing*
n_b = *distributed horizontal stress resulted from external loads in bottom sheathing*
EI_t = *flexural rigidities of top sheathing*
EI_b = *flexural rigidities of top sheathing*
w_t = *vertical displacement of centroid of top sheathing*
w_b = *vertical displacement of centroid of bottom sheathing*
E_c = *elastic modulus of core*
c = *height of core*
d_t = *thickness of top sheathing*
d_b = *thickness of bottom sheathing*
q_t = *distributed vertical stress resulted from external loads in top sheathing*
q_b = *the distributed vertical stress resulted from external loads in bottom sheathing*
m_t = *bending moments resulted from external load in top sheathing*
m_b = *bending moments resulted from external load in bottom sheathing*
G_c = *shear modulus of core*

The order of the equivalent differential equation that replaces this set of equations (9) to (13) is 14. Under certain boundary conditions, those five equations above can be solved for vertical and horizontal displacements in the top and bottom sheathings, and shear stress in the core. The normal stresses at the upper sheathing and lower sheathing are shown in Eq. (14) and (15).

$$\sigma_{zz}(x, z = 0) = \frac{E_c(w_b - w_t)}{c} + \frac{\tau_{,x}c}{2} \tag{14}$$

$$\sigma_{zz}(x, z = c) = \frac{E_c(w_b - w_t)}{c} - \frac{\tau_{,x}c}{2} \tag{15}$$

BOZO'S RESULTS

Bozo [8] conducted mechanical testing of OSB with three different nominal densities of 450 kg/m^3, 550 kg/m^3, and 650 kg/m^3, respectively, with tolerance limit of ± 25 kg/m^3. The mechanical properties studied in his research were the modulus of elasticity and maximum values for compression, tension, and shear. Compression and tension tests were performed according to ASTM D1037. Shear tests were conducted based on ASTM D5379/D5379M-93. The crosshead displacement speed in his tension tests was controlled to be 4.0 mm/minute, while in compression and shear tests, the speed was 0.36 mm/minute. Mechanical properties for OSB with a density of 650 kg/m^3 are shown in Table 2-1.

OSB	Max Stress (Psi)	E or G (Psi)
Compression (E)	1700	594000
Tension (E)	1800	790000
Shear (G)	1330	200000

Table 2-1 OSB Mechanical Properties with a Nominal Density of 650 Kg/m^3

RUSMEE AND DEVRIES' RESEARCH ON EPS FOAM

P. Rusmee and K. L. DeVries[9]' research showed that the size, loading rate, and loading configuration have significant influences on the apparent material properties of EPS foam. From three groups of mechanical tests of EPS with different size, loading rate, and loading configuration, they found that the modulus in compression they obtained from the 13 mm thick foam specimens was 0.9 MPa and 2.8 MPa for 50 mm thick foam. The modulus for the 50 mm thick foam increased to 3.3 MPa as the loading rate increased from 0.042 mm/s to 4.2 mm/s. In a dynamic test, the value of the modulus for 50.8 mm thick foam was about 390% of the value of the quasi-static modulus. They drew the conclusion that when using EPS foam in design, one needs to determine the usage condition, such as its lateral dimensions, thickness, and rate of loading.

PUBLISHED MECHANICAL PROPERTIES FOR EPS

ASTM C578 *Standard Specifications for Rigid, Cellular Polystyrene Thermal Insulation* [10] provides the EPS physical property requirements of thermal insulation based on EPS type. The strength properties for EPS are shown in Table 2-2.

Properties	EPS Type			
	Type I	Type VIII	Type II	Type VI
Density, minimum (pcf)	0.90	1.15	1.35	1.80
Compressive 10% Deformation (psi)	10	13	15	25

Table 2-2 EPS Properties Published by ASTM Standard

For other material characteristics that are not required by the standards, but are very important, were modified by the Huntsman Corporation [11]. Their modified EPS typical physical properties at 1 lb/ft^3 is listed in Table 2-3.

Property	Value
Tensile Strength (psi)	28
Shear Strength (psi)	16
Shear Modulus (psi)	440

Table 2-3 EPS Properties Published by Huntsman Corporation

HYPERBOLIC AND LINEAR FUNCTION

Nonlinear materials, such as woodfiber-plastic composites, behave differently compared to wood and wood products. The Engineering Mechanics Laboratory of the USDA Forest Service generated a four parameter hyperbolic and linear function to fit load-displacement data for paper, joint slip, steel, and woodfiber-plastic composites. Using these four known parameters, one can determine the theoretical load-displacement curve as well as its initial slope.

The hyperbolic and linear function is

$$p = c_1 Tanh(c_2(x - c_4)) + c_3(x - c_4) \tag{16}$$

and the slope at x is

$$\frac{dp}{dx} = c_1 c_2 Sech^2(c_2(x - c_4)) + c_3 \tag{17}$$

The initial slope at x = zero is

$$\frac{dp}{dx} = c_1 c_2 + c_3 \tag{18}$$

The parameters are estimated using standard nonlinear least-squares techniques. The intercept of the curve at the x-axis is c_4, and the slope of the second straight line is c_3 as shown in Figure 2-3.

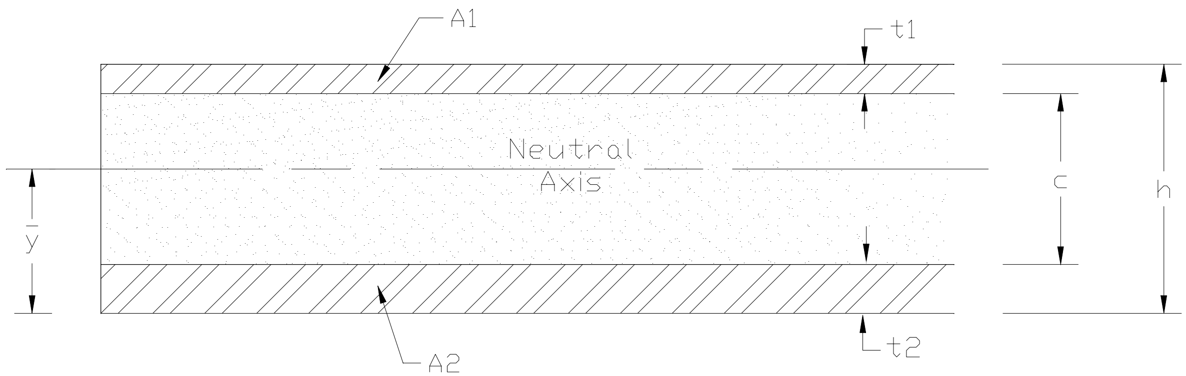

Fig. 2-1 Dimensions of Structural Sandwich Panel Used in the APA's Design Equations

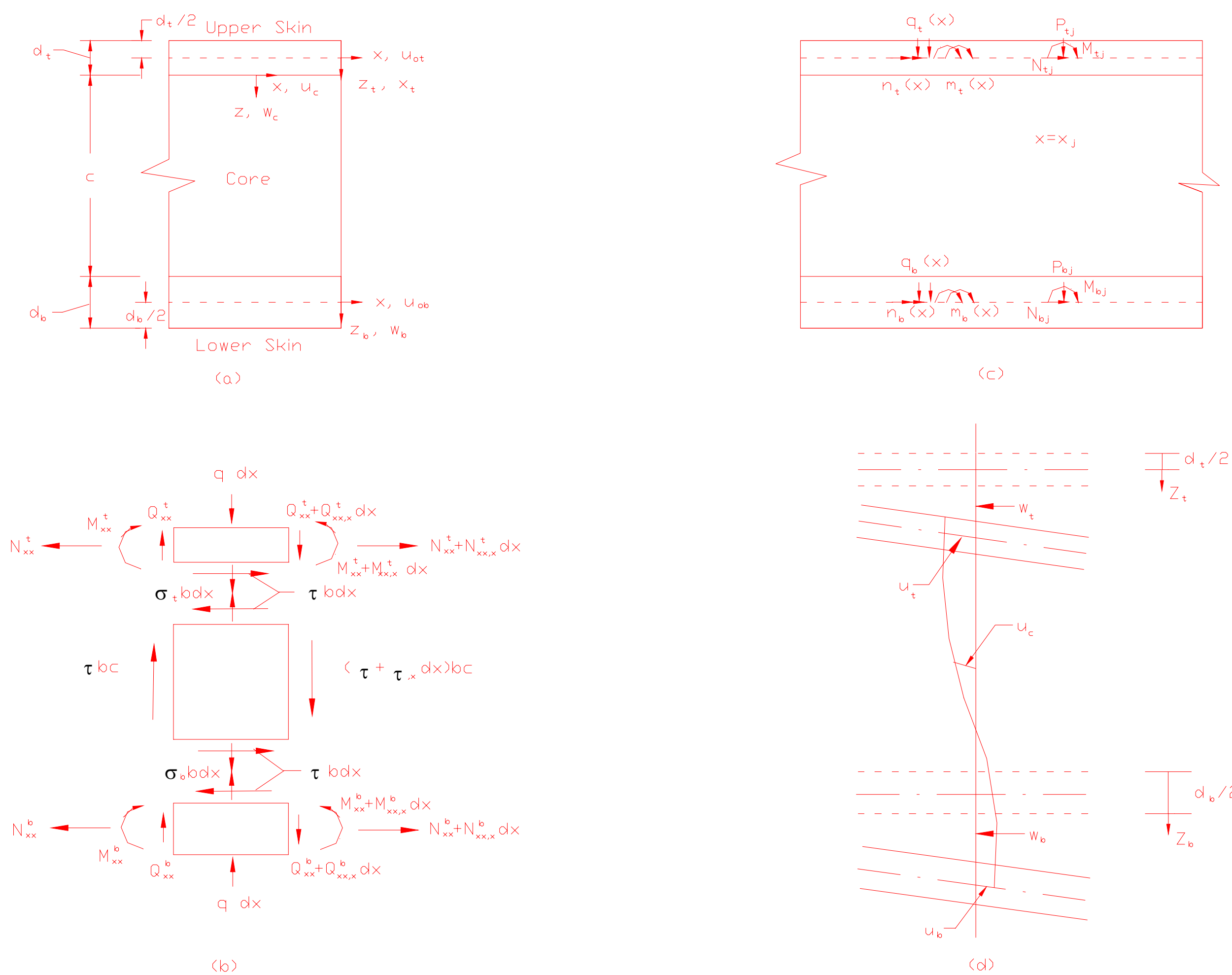

Fig. 2-2 Geometry, Load, Internal Results and Deformation: (a) geometry; (b) internal Resultants and stresses; (c) external loads; (d) deformation pattern

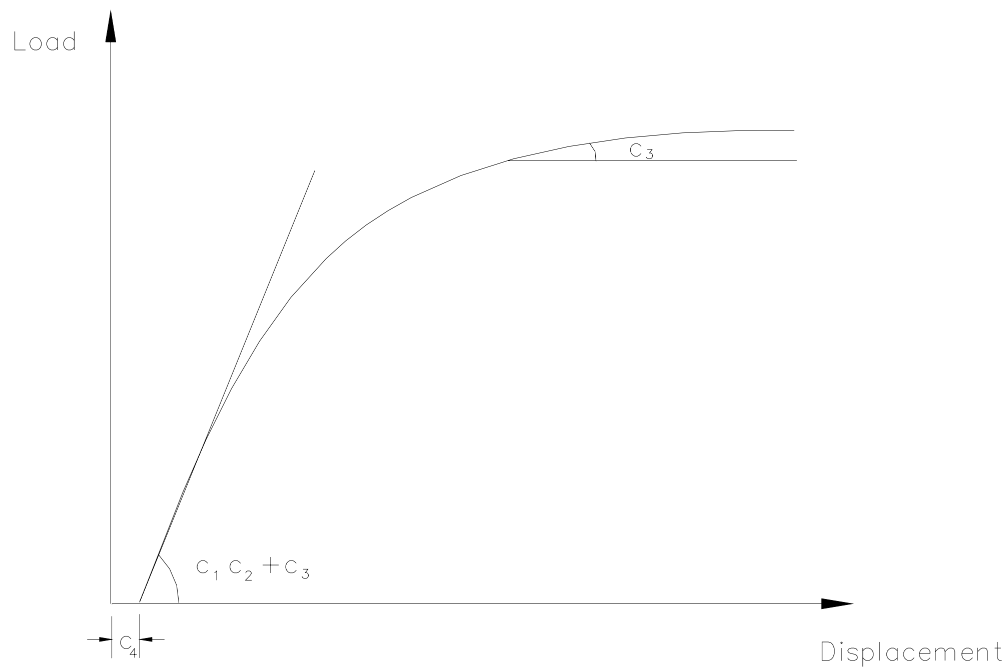

Fig. 2-3 Hyperbolic and Linear Function

Chapter 3
RESEARCH METHODS

Modeling of SIPs was accomplished in three steps. First, the mechanical properties of panel components, OSB sheathing and EPS core were collected. Compression, tension, and shear properties of OSB sheathing were obtained from Bozo [8]. The compression, tension, and shear properties of the EPS core were obtained from mechanical testing as described below. Also, EPS density in this research was determined for the comparison of shear modulus between calculated and published values [10]. Second, the material properties obtained from above were used with three different material models within finite element programs. Lastly, sections of SIP panels were tested in flexure to validate the finite element analysis.

MECHANICAL TESTING

Four structural insulated panels used in this research were obtained from R-Control Group of Excelsior, MN. Each of the four panels measured 4 ft by 8 ft. OSB sheathing had a thickness of 7/16 in. and the EPS core had a thickness of 3-5/8 in. as shown in Figure 3-1. Using a band saw, SIPs were cut into 3.5 in. wide beams for flexural testing. Left-over ends from cutting beams were used for compression, tension, and shear specimens.

EPS COMPRESSION TESTS

Compression testing specimens were fabricated from SIP strip ends as shown in Figure 3-2. First, two-inch-wide specimens were cut from the edge of left-over SIP ends. Then they were cut to make 2.00 in. by 2.00 in. by 4.50 in. blocks. Seven of these blocks were used for testing. Specimen dimensions are listed in Table 3-1.

Compression	Width (in.)	Thickness (in.)
C1	2.05	1.97
C2	2.05	1.99
C3	2.05	2.01
C4	2.08	1.99
C5	2.05	2.03
C6	2.07	2.00
C7	2.06	2.02

Table 3-1 SIP Block Dimensions for Compression Tests

The compression test setup is shown in Figure 3-3. Specimens were tested at a speed of 0.15 in./min. with an Instron universal test machine (model number: 4466) having a load range of ± 2 kips. A 1-inch MTS extensometer (Model number: 634.12E-24) with strain range from –10% to + 50% was centered midway on the EPS core height. The EPS foam was wrapped with thick paper to eliminate the damage resulting from fastening the extensometer. Data was collected at 2 samples per second.

EPS TENSION TESTS

Fabrication of tension testing specimens followed the same procedures used to make those for compression, but with cross-section of 1.80 in. by 2.00 in. as shown in Figure 3-4. Blocks then were refined to dog-bone shape. Dimensions of the dog-bone cross-section for each specimen are listed in Table 3-2.

Tension	Width (in.)	Thickness (in.)
T1	1.80	1.06
T2	1.83	1.12
T3	1.79	1.12
T4	1.82	1.10
T5	1.79	1.05
T6	1.84	1.08
T7	1.80	1.10

Table 3-2 SIP Block Dimensions for Tension Tests

Tension tests were performed as shown in Figure 3-5. They were tested at a speed of

0.05 in./min. with an Instron universal test machine (model number: 4466) having a load range of ± 2 kips. A 1-inch MTS extensometer (model number: 634.12E-24) with strain range from –10% to +50% was centered midway on the EPS core height. Again, the EPS foam was wrapped with thick paper to eliminate the damage resulted from fastening the extensometer. Data was collected at 2 samples per second.

EPS SHEAR TESTS

Shear testing specimens were cut as shown in Figure 3-6. Top and bottom OSB sheathings were originally attached to EPS 1 and 2, which were glued to OSB 1. In order to ensure that top and bottom OSB sheathings remain vertical when applying load to OSB 1, using Arrow hot melt glue, OSB 2 and OSB 3 were glued to both ends of top and bottom OSB sheathings. OSB 4 was then glued to top and bottom OSB sheathings. Sixteen 1.00 in. by 2.00 in. by 3.00 in. EPS blocks were glued to eight specimens. EPS shear dimensions are listed in Table 3-3.

Shear	**Length (in.)**	**Thickness (in.)**
S1	2.99	2.03
S2	3.02	2.04
S3	2.98	2.03
S4	3.00	2.05
S5	2.97	1.98
S6	2.98	1.97
S7	2.48	2.04
S8	2.47	1.97

Table 3-3 SIP Block Dimensions for Shear Tests

The shear test setup is as shown in Figure 3-7. They were tested at a speed of 0.02 in./min. with an Instron (model number: 4466) having a load range between ± 2 kips. A 1-inch MTS Extensometer (model number: 634.12E-24) with strain range from –10% to + 50% was centered midway on the left side of the EPS core. Data were collected at either 2 samples or 5 samples per second.

SIPS FLEXURAL TEST

Structural insulated panels were cut into 3.5 in. wide beams with lengths of 3 feet, 5 feet, 7 feet, and 8 feet. Beams were tested in five groups: 1) 3 foot long beam with 2 foot span; 2) 5 foot long beam with 4 foot span; 3) 7 foot long beam with 6 foot span; 4) 8 foot long beam with 6 foot span; and 5) 8 foot long beam with 8 foot span. Three beams were tested in groups 1 to 3 and one beam in groups 4 and 5. Each beam was subjected to concentrated loads at the third-points. They were tested with an Instron machine (model number: 1137) of load range between ±30 kips. A linear varying differential transducer with a range of ± 2 in. was used to determine the deflection of SIP beams at the mid-span. Data was collected at 5 points per second. The flexural test setup is shown in Figure 3-8.

Detailed set-up dimensions and test speed for each group are listed in Table 3-4.

Group	Distance a (in.)	Distance L (in.)	Test Speed (in./min.)
1	6	24	0.15
2	6	48	0.20
3	6	72	0.20
4	12	72	0.30
5	0	96	0.30

Table 3-4 Set-up Dimensions and Test Speed for Beam Bending Testing

SHEAR MODULUS VIA FLEXURAL TEST

ASTM D 198 [13] provides a formula for calculating shear modulus via flexural test. The elastic deflection of a prismatic beam under a single center point load is:

$$\Delta = \frac{PL^3}{48EI} + \frac{PL}{4GA'} \tag{19}$$

where,

Δ------deflection at mid-span,
P------applied load,
L------span,

E------modulus of elasticity,
I-------moment of inertia,
G-------shear modulus,
A'------modified shear area.

If the shear contribution is ignored, the relationship between deflection and "apparent" modulus of elasticity (E_f) is:

$$\Delta = \frac{PL^3}{48E_f I} \tag{20}$$

Eq. (19) then can be rewritten as Eq. (21)

$$\frac{1}{E_f} = \frac{1}{E} + \frac{1}{KG}(h/L)^2 \tag{21}$$

Where,

$K = \dfrac{10(1+\nu)}{12+11\nu}$ for rectangular cross-section

$K = \dfrac{6(1+\nu)}{7+6\nu}$ for circular cross-section

ν is Poisson's ration.

For each beam span, the "apparent" modulus of elasticity (E_f) can be calculated using Eq. (20). Plotting $1/E_f$ versus $(h/L)^2$ for each span produces a distribution of points that can be approximated by a straight line. Knowing the slope k of that line, shear modulus $G = \dfrac{1}{kK}$.

For a simply supported beam under double quarter-point loads, the deflection due to bending is $\Delta_1 = \dfrac{23PL^3}{1296EI}$, and the deflection due to shear is $\Delta_2 = \dfrac{M}{AG}$ [14],

Where,

P = load applied at the beam,

A = bh^2/c *(for sandwich beams),*

M = the moment at mid-pan.

Dimensions of the SIP beam's cross section is shown in Figure 3-9.

For a SIP beam,

$$EI = E_s \frac{bc_1^2}{6} + E_s \frac{bc_1 h^2}{2} + E_c \frac{bc^3}{12} \tag{22}$$

If $3\left(\frac{h}{c_1}\right)^2 > 100$, then the first term of Eq. (22) can be ignored,

and if $6\frac{E_s}{E_c}\frac{c_1}{c}\left(\frac{h}{c}\right)^2 > 100$, then the third term in Eq. (22) can be ignored.

For the SIP beams used in this research, $3\left(\frac{h}{c_1}\right)^2 = 3\left(\frac{4.0625}{0.4375}\right)^2 = 259 > 100$, and

$$6\frac{E_s}{E_c}\frac{c_1}{c}\left(\frac{h}{c}\right)^2 = 6\frac{456876}{854}\frac{0.4375}{3.625}\left(\frac{4.0625}{3.625}\right)^2 = 487 > 100,$$

So, Eq. (22) can be simplified as

$$EI = E_s \frac{bc_1 h^2}{2} \tag{23}$$

Eq. (19) can be rewritten again as

$$\begin{aligned} \Delta &= \frac{23PL^3}{1296E_s \frac{bc_1h^2}{2}} + \frac{PLc}{6bh^2G} \\ &= \frac{23PL^3}{648bc_1h^2E_s} + \frac{PLc}{6bh^2G} \end{aligned} \tag{24}$$

Also,

$$\Delta = \frac{23PL^3}{648bc_1h^2\left(E_s\right)_f} \tag{25}$$

Eq. (24) can be written as

$$
\begin{aligned}
\frac{1}{(E_s)_f} &= \frac{1}{E_s} + \frac{108}{23}\frac{1}{G_c}\frac{cc_1}{L^2} \\
&= \frac{1}{E_s} + \frac{108cc_1}{23G_c}\frac{1}{L^2}
\end{aligned} \tag{26}
$$

To remove the foam's crushing from the determination, $(E_s)_f$ was calculated according to the initial slope of the load-displacement curve of SIP beams bending. Equation (26) can be graphed as a line by letting y = $1/(E_s)_f$ and x = $(1/L)^2$. In the resulting graph, the slope k of the line is equal to $\frac{108cc_1}{23G_c}$. The shear modulus of EPS core is $G_c = \frac{108cc_1}{23k}$.

EPS DENSITY TESTS

Four EPS blocks with nominal dimensions of 2.00 in. by 1.50 in. by 4.00 in. were weighed. For EPS, its density is the average result of the four tests, with none of them varying more than 15% from the average. Specimen dimension are listed in Table 3-5.

Group	Weight (g)	Width (in.)	Thickness (in.)	Length (in.)
1	3.0	2.02	1.49	3.99
2	3.0	2.02	1.49	3.99
3	3.0	2.02	1.50	4.01
4	3.1	2.03	1.50	4.00

Table 3-5 EPS Dimensions for Density Tests

FINITE ELEMENT METHODS

HYPERFOAM MODEL FOR EPS CORE

The existing hyperfoam material model within ABAQUS was a logical choice to model the EPS foam core in a structural insulated panel. This theory for hyperfoam is a modified form of Hill's strain energy potential. In ABAQUS, test data are expressed as nominal-stress-nominal-strain data pairs of uniaxial test data, biaxial test data, simple shear test data, planar test data, or volumetric test data [15]. For each stress-strain data pair, ABAQUS generates an expression for the stress in terms of the stretches λ (which $\lambda = \frac{ds}{dS} = \sqrt{\frac{dx^T \cdot dx}{dX^T \cdot dX}}$) and the unknown hyperfoam constants. The strain and stress values obtained from compression and shear tests were substituted into the equations and solved or those constants. A review of the theory for this hyperfoam model follows.

UNIAXIAL COMPRESSION MODE

For the uniaxial compression mode, the nominal stress T_c is

$$T_c = \frac{\partial U}{\partial \lambda_j} = \frac{2}{\lambda_j} \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left[\lambda_j^{\alpha_i} - J^{-\alpha_i \beta_i} \right] \tag{27}$$

where U is the strain energy potential,

λ_j is the stretch in the primary displacement direction,

$$\lambda_1 = \lambda_U, \qquad \lambda_2 = \lambda_3,$$
$$J = \lambda_U \lambda_2^2, \qquad \lambda_U = 1 + \varepsilon_U$$

μ_i, α_i, β_i are hyperfoam constants.

Using MATHEMATICA, all parameters can be determined and Eq. (27) can be written as the relationship between nominal stress T_c and strain ε_u. Graphing the stress–strain curves obtained from the mechanical testing and generated by the relationship, one can determine how closely they match and then judge if the hyperfoam model is appropriate for EPS core.

SIMPLE SHEAR MODE

In addition to the uniaxial compression mode of the hyperfoam model, one can include simple shear data in the model. The simple shear deformation is described in terms of the deformation gradient,

$$F = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{28}$$

where γ is the shear strain. For this deformation, J = det (F) = 1. The nominal shear stress T_s is

$$Ts = \frac{\partial U}{\partial \gamma} = \sum_{j=1}^{2} \left\{ \frac{2\gamma}{2\left(\lambda_j^2 - 1\right) - \gamma^2} \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left(\lambda_j^{\alpha_i} - 1\right) \right\} \tag{29}$$

where λ_j are the principal stretches in the plane of shearing, related to the shear strain γ as follows,

$$\begin{aligned} \lambda_{1,2} &= \sqrt{1 + \frac{\gamma^2}{2} \pm \gamma \sqrt{1 + \frac{\gamma^2}{4}}} \\ \lambda_3 &= 1 \end{aligned} \tag{30}$$

Using MATHEMATICA, all parameters can be calculated and Eq. (29) can be written as the relationship between nominal stress T_s and strain γ. Again, by graphing the stress–strain curves obtained from the mechanical testing and generated by the relationship, one can evaluate how closely they match and then judge if the hyperfoam model is appropriate for EPS core.

BILINEAR MODEL FOR EPS CORE

The discontinuity point in bilinear stress-strain curve is as shown in Figure 3-10. In ADINA, this point is determined according to von Mises yield condition. The bilinear model can be used with the 2-D solid element. Required material constants for this model are Young's modulus (initial slope), initial yield stress (discontinuity point), and strain hardening modulus (secondary slope). They can be obtained from mechanical testing.

USER-SUPPLIED MODEL FOR EPS CORE

The last material model attempted was a user-supplied model that represented the observed compression, tension and shear via hyperbolic and linear functions of Equation (16). The nonlinear regression algorithm ("NonLinearFit [data, model, variables, parameters]") within MATHEMATICA was used to solve for the constants c_1 to c_3 in Equation (16). Constant c_4 was zero in all cases.

FINITE ELEMENT MODEL FOR A SIP BEAM

SIP beams were modeled with OSB as an elastic-orthotropic material and EPS as a bilinear material or a user-supplied material. Since the beam was symmetrical in terms of geometry and loading, only the left half of the beam was modeled. The configuration of the finite element model for SIP beams is shown in Figure 3-11.

Where,

a = distance between beam end and support,

b = 1/6 of the span,

c = 1/2 of the length of SIP beam.

In Figure 3-11, both top and bottom OSB sheathings were modeled as elastic-orthotropic material, with an OSB tensile modulus of 790000 psi and compressive modulus of 594000 psi [8]. EPS was modeled as either a bilinear material or a user-supplied material with inputs which will be described in the next chapter. Point 2 (P2) in the finite element model was fixed in the Z direction, and lines 3(L3), 4 (L4), and 5 (L5) were fixed in the Y direction. The shear modulus for both top and bottom OSB was assigned a value of 200000 psi as determined by Bozo [8]. Load was applied at point 7 (P7) with displacement control.

The input for the EPS core as a user-supplied material was divided into four groups. Groups 1 to 3 are constants which defined the EPS properties of compression, tension, and shear separately. In each group, there are three constants to describe the load-displacement curve of EPS under compression, tension, and shear. Group 4 is the value of Poisson's ratio.

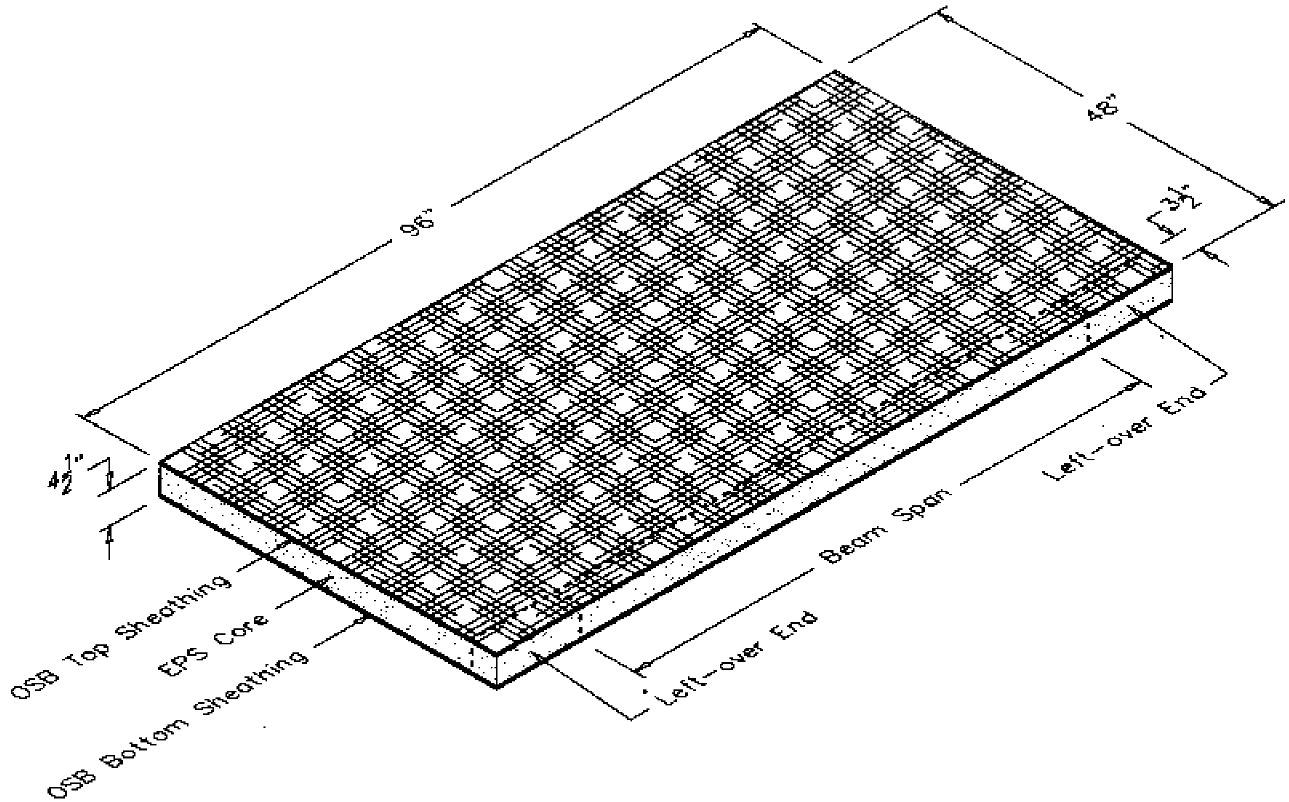

Fig. 3-1 Dimensions of Structural Insulated Panel

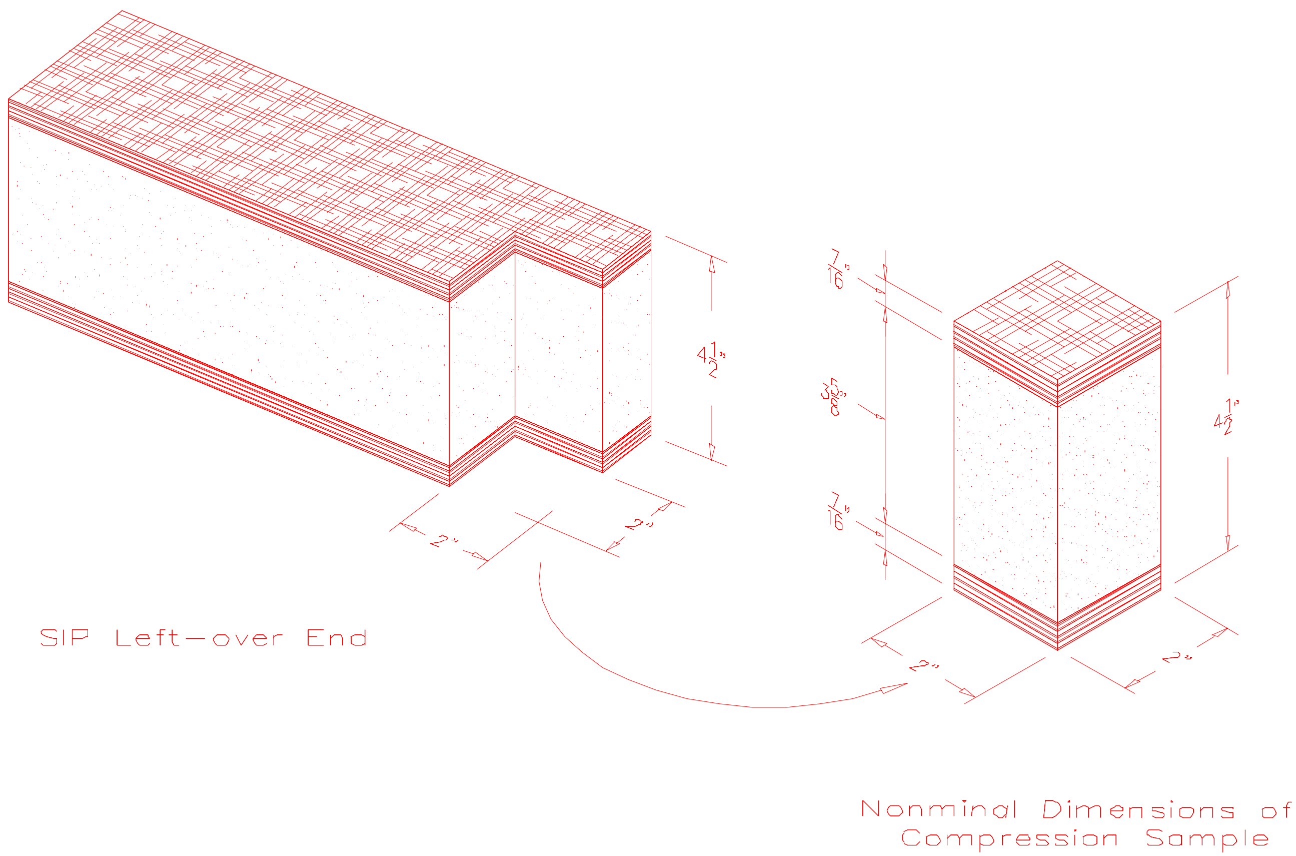

Fig. 3-2 Dimensions of Compression Sample

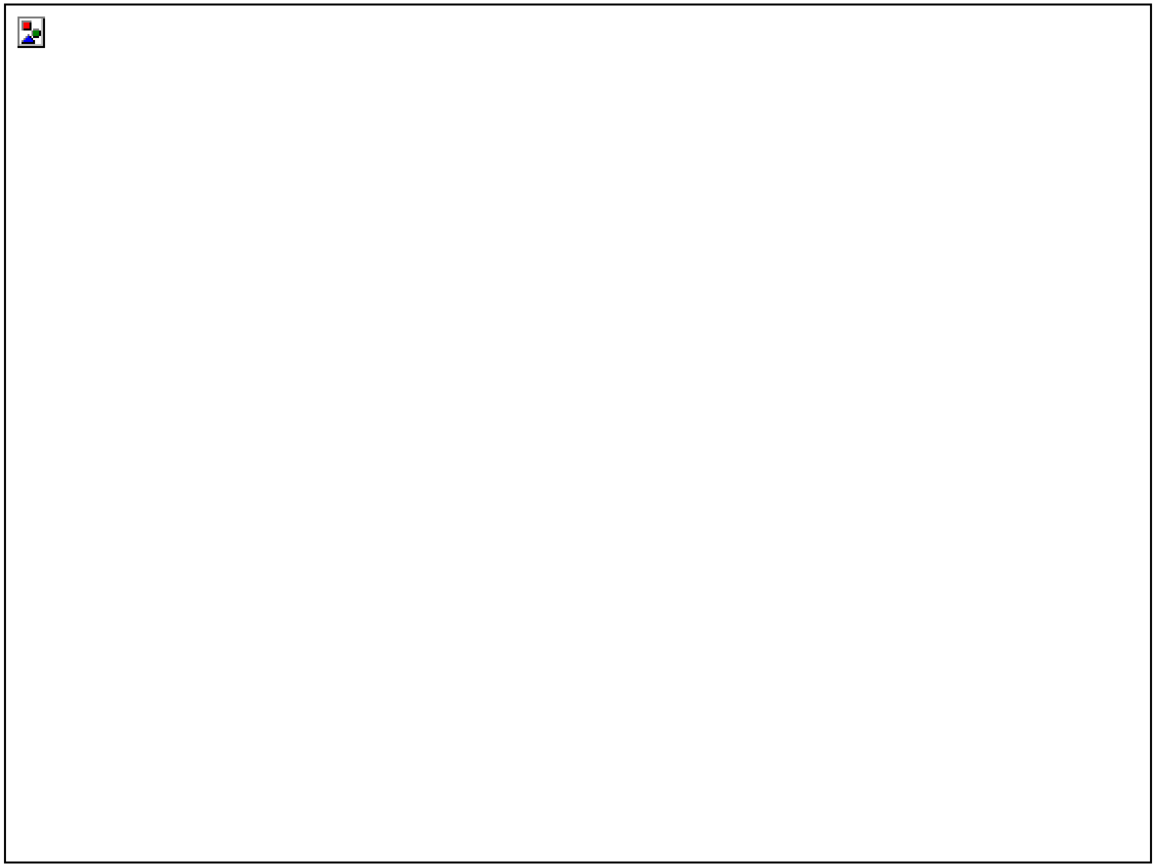

Fig. 3-3 Compression Test Setup

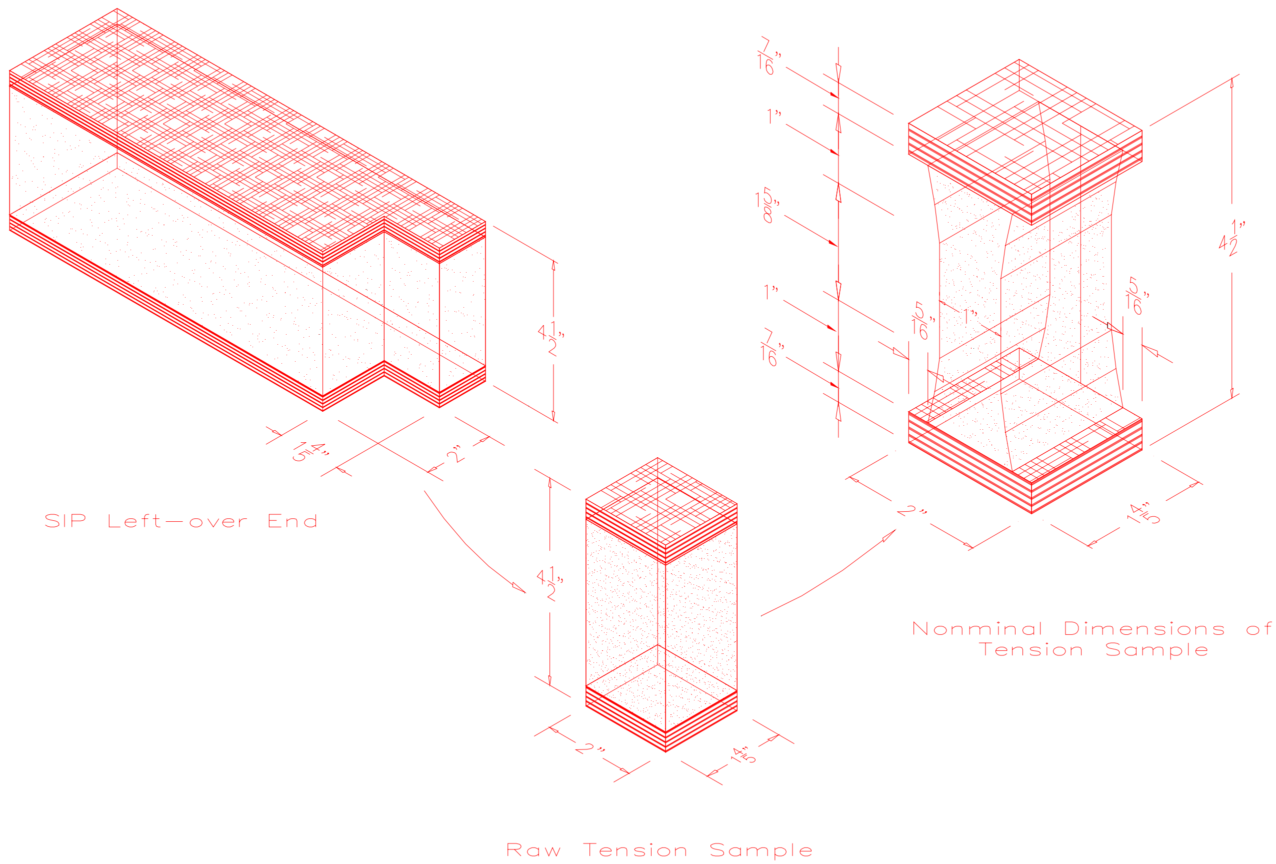

Fig. 3-4 Dimensions of Tension Sample

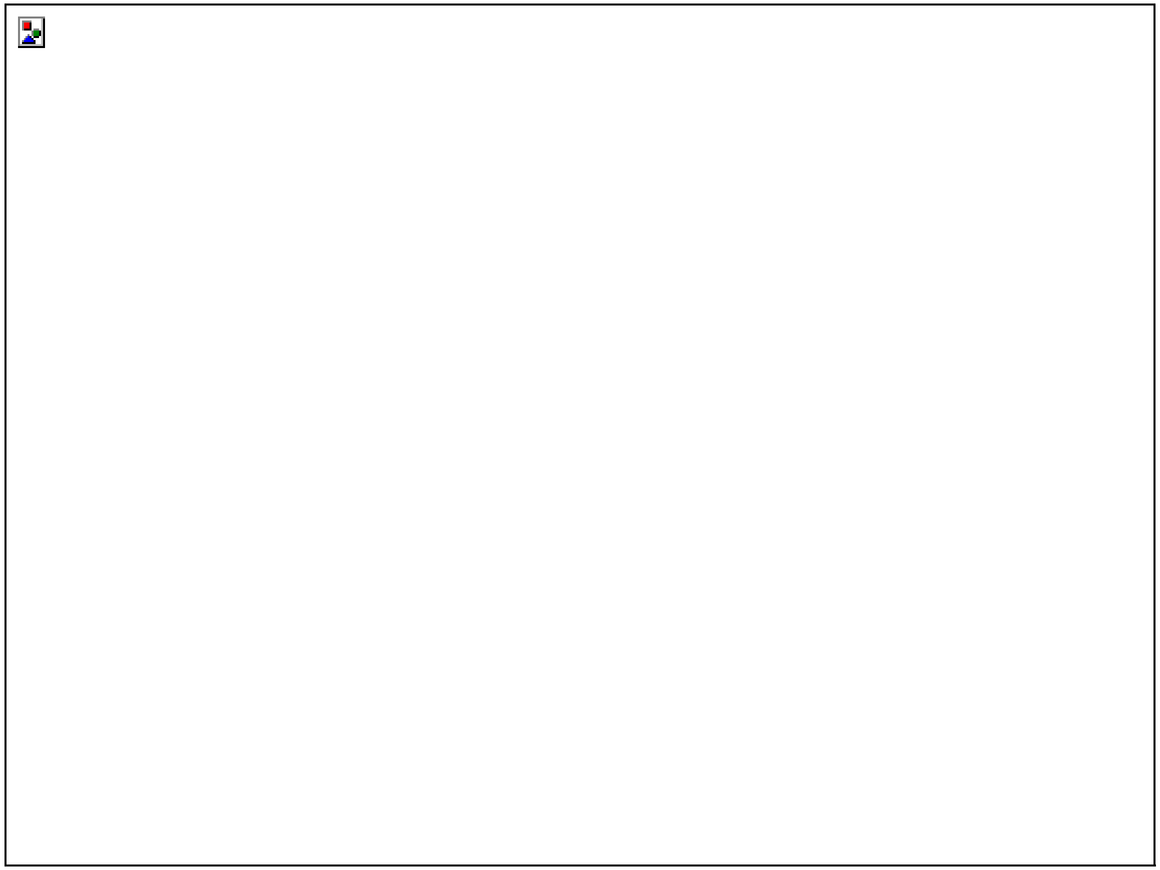

Fig. 3-5 Tension Test Setup

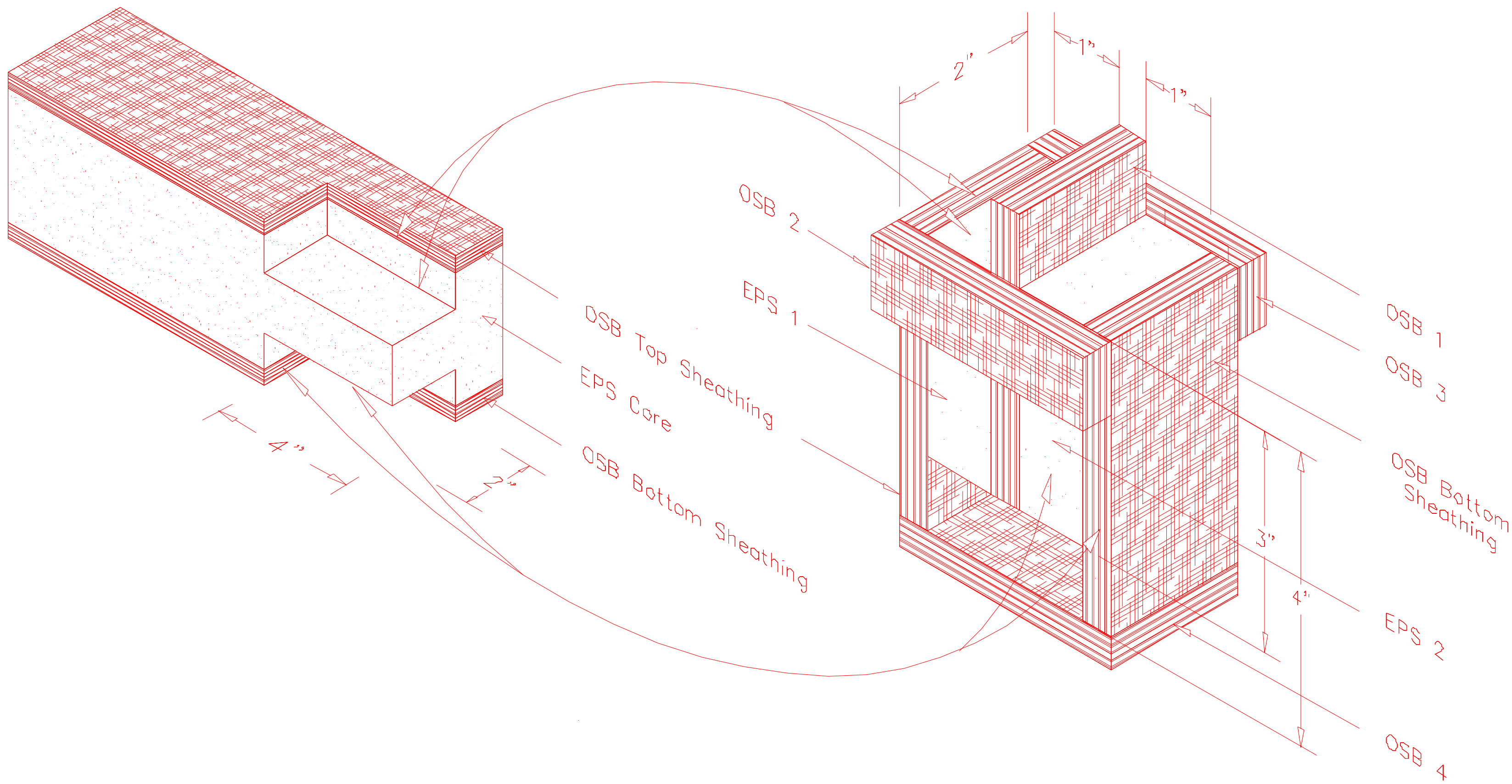

Fig. 3-6 Dimensions of Shear Sample

Fig. 3-7 Shear Test Setup

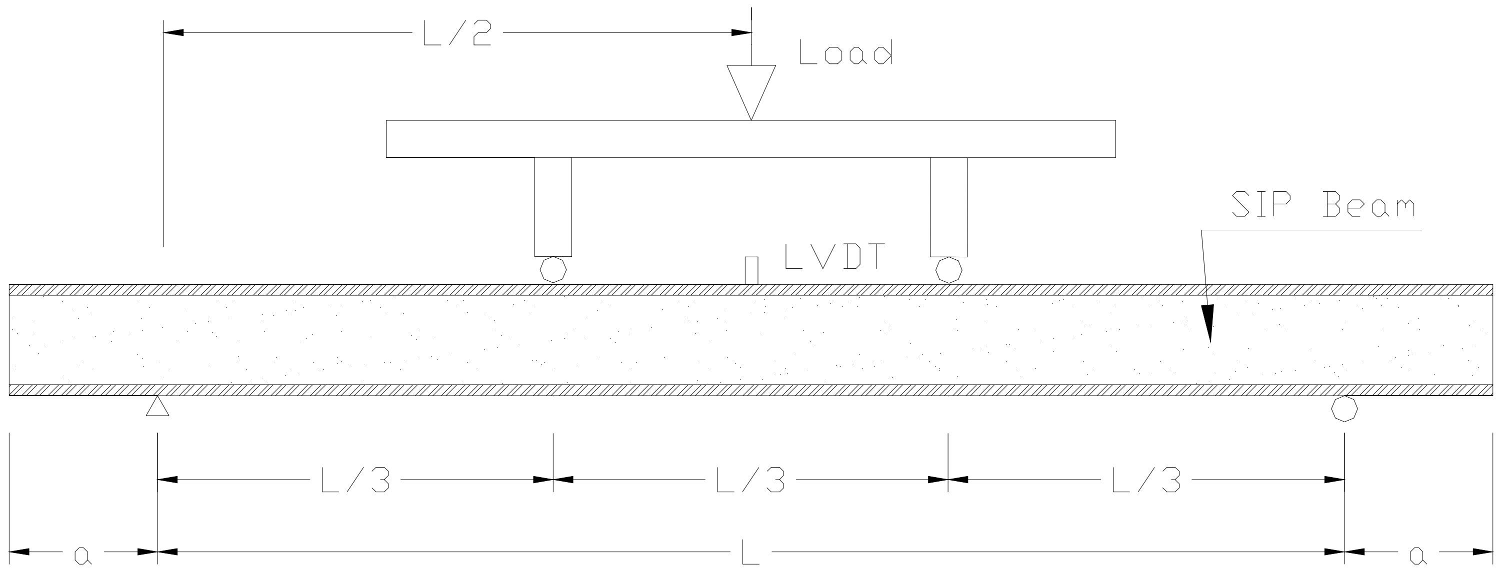

Fig. 3-8 Dimensions of SIPs Flexural Test

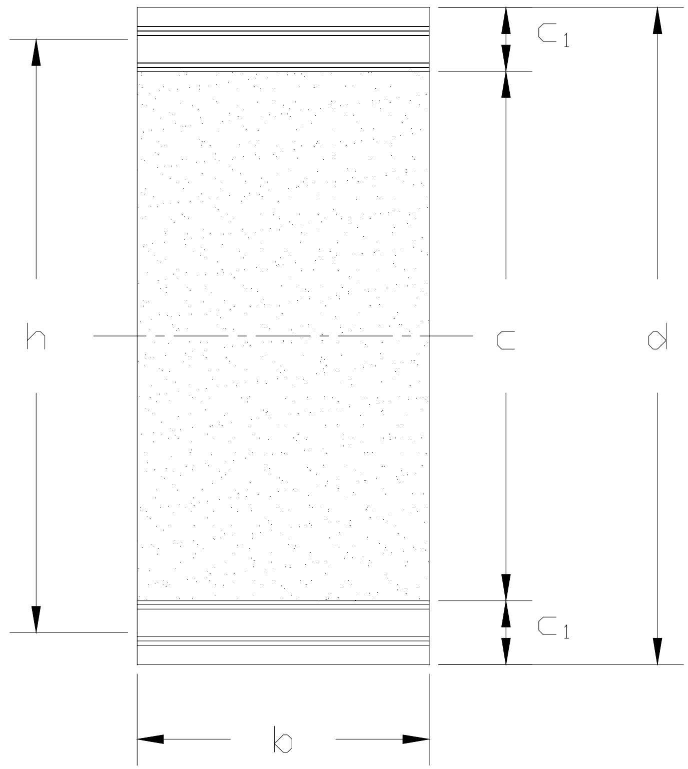

Fig. 3-9 Dimensions of SIP Beam's Cross Section

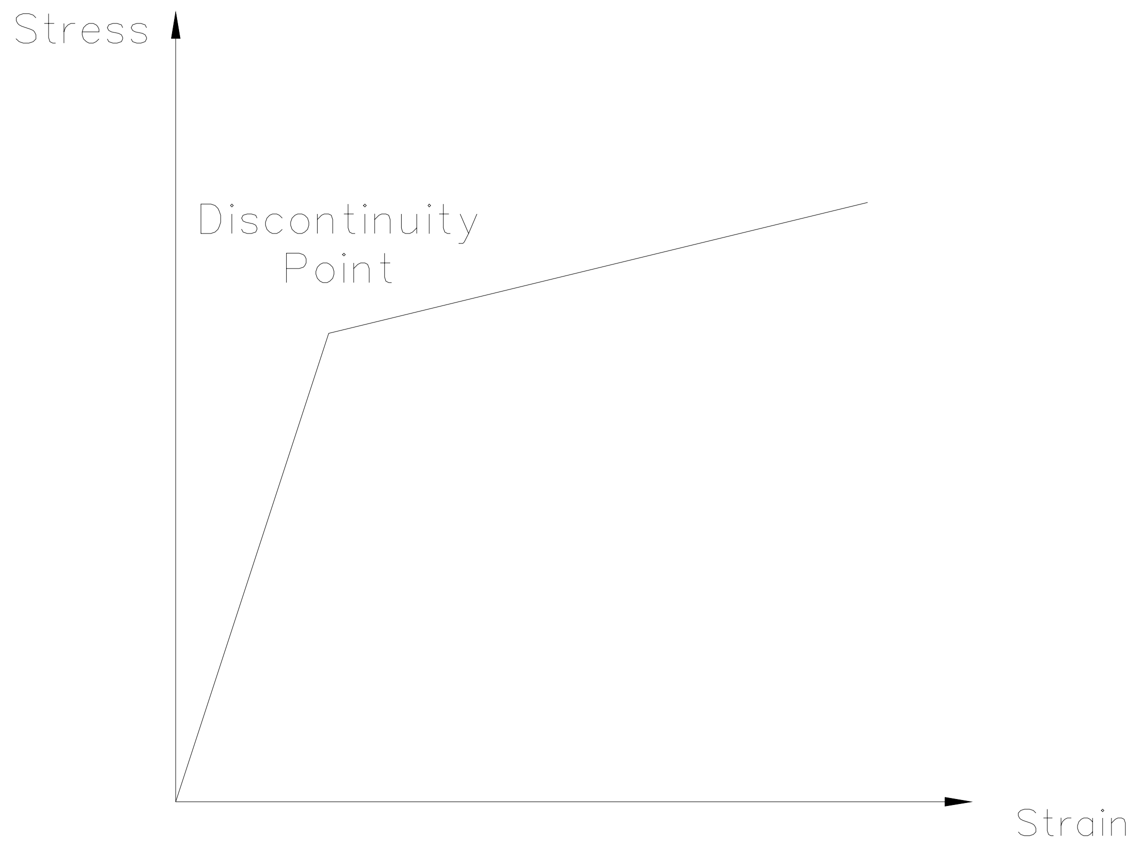

Fig. 3-10 Idealized Stress-Strain Curve for Bilinear Material

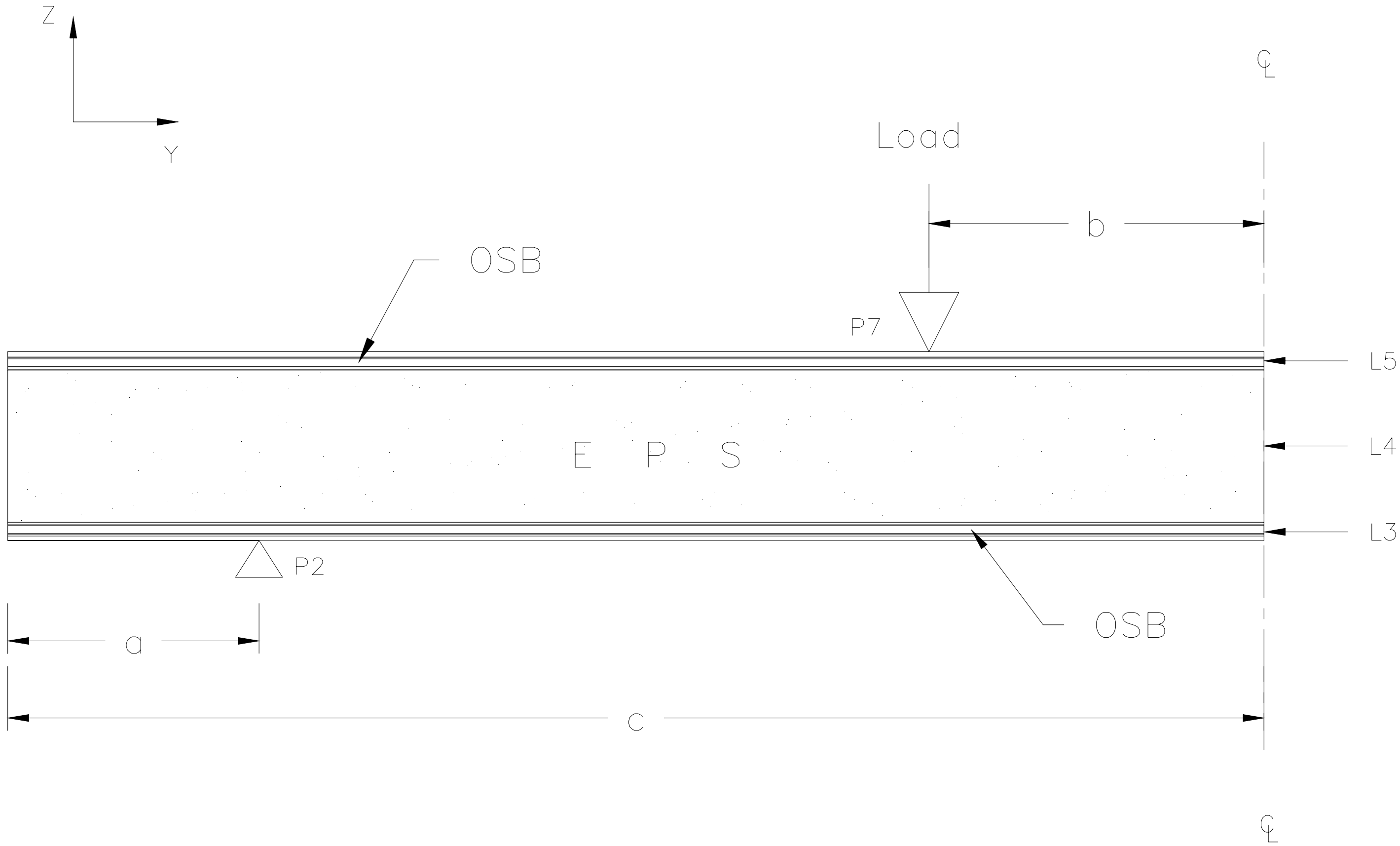

Fig. 3-11 Typical Model for SIP Beams

Chapter 4
RESEARCH RESULTS

As with the research methods, the results fall within the same two broad areas of mechanical testing and finite element analysis. Mechanical testing provided the necessary data to use for finite element analysis to generate a predictive model for SIP beams. Laboratory testing also made it possible to verify this new model.

MECHANICAL TESTING RESULTS

The mechanical testing determined three EPS material properties (compression, tension, and shear) and SIP beams' behavior under four-point bending. Beam bending with multiple spans also made it possible to calculate EPS shear modulus in flexure. EPS density in this research was determined for the comparison between calculated and published shear modulus which is listed based on EPS density.

EPS COMPRESSION TEST RESULTS

In compression tests, maximum loads occurred between 4 and 5 minutes. As shown in Figure 4-1, six out of seven tests were consistent. The generalized stress-strain curve for EPS in compression was generated by solving the constants in the hyperbolic tangent and linear function $p = c_1 Tanh(c_2 x) + c_3 x$ using known stress-strain curves from tests. Constants c_1 to c_3 were solved for as $c_1 = 9.65$, $c_2 = 86.4$, and $c_3 = 19.8$. Curves obtained from mechanical testing and generated by Eq. (16) with the solved constants are shown in Figure 4-2. The initial modulus of elasticity of EPS in compression is 854 psi.

DISCONTINUITY POINT

The discontinuity point of EPS in compression was defined as the intersection of line 1 and line 2 in Figure 4-3. Those two lines intersect at (0.0122 in./in., 9.87 psi).

The modulus of elasticity is the slope of the beginning portion of the stress-strain curve, the same as the slope of line 1,

$$E_1 = 809 \quad psi$$

The second modulus is the slope of the ending portion of the stress-strain curve, the same as the slope as line 2,

$$E_2 = 19.8 \quad psi$$

EPS TENSION TEST RESULTS

As shown in Figure 4-4, all tests produced consistent results except two. A generalized stress-strain curve for EPS in tension was generated based on results from the five consistent tests. Using Eq. (16), constants c_1, c_2, and c_3 are found to be:

$$\begin{aligned} c_1 &= 41.5 \\ c_2 &= 38.6 \\ c_3 &= -235 \end{aligned}$$

Curves obtained from mechanical testing and generated by Eq. (16) with the solved constants are shown in Figure 4-5. The initial modulus of elasticity of EPS in tension is 1370 psi.

DISCONTINUITY POINT

The discontinuity point of EPS in tension was defined as the intersection of line 1 and line 2 in Figure 4-6. Those two lines intersect at (0.0255 in./in., 27.1 psi).

Modulus of elasticity is the slope of the beginning portion of the stress-strain curve, the same as the slope of line 1,

$$E_1 = 1320 \quad psi$$

The second modulus is the slope of the ending portion of the stress-strain curve, the same as the slope as line 2,

$$E_2 = 84.1 \quad psi$$

EPS SHEAR TEST RESULTS

As shown in Figure 4-7, shear data are not as consistent as compression and tension data. The generalized stress-strain curve for EPS in shear was based on the results from tests S1, S2, S3, S4, S7, and S8. Using Eq. (16), constants c_1, c_2 , and c_3 are found to be:

$$c_1 = 12.8$$
$$c_2 = 32.7$$
$$c_3 = 0$$

Curves obtained from mechanical testing and generated by Eq. (16) with the solved constants are shown in Figure 4-8. The initial modulus of elasticity of EPS in shear is 419 psi.

SIP BENDING TEST RESULTS

CONSTANTS C_1 TO C_3 AND INITIAL SLOPES OF SIP BEAMS

Bending tests were performed in five groups: 1) 3 foot long beam with 2 foot span; 2) 5 foot long beam with 4 foot span; 3) 7 foot long beam with 6 foot span; 4) 8 foot long beam with 6 foot span; and 5) 8 foot long beam with 8 foot span. Each beam was subjected to concentrated loads at the four-point. Based on the load-displacement curves generated in beam testing, constants c_1 to c_3 for each beam can be solved using Eq. (16). Using Eq. (18), initial slope for each load-displacement curve can be calculated. All the results are listed in Table 4-1.

Beam	Shown in Figure	c_1	c_2	c_3	Initial Slope
3 Foot Long Beam with 2 Foot Span	4-9	334	3.55	110	1340
5 Foot Long Beam with 4 Foot Span	4-9	303	2.05	42.7	666
7 Foot Long Beam with 6 Foot Span	4-9	289	1.19	15.9	360
8 Foot Long Beam with 6 Foot Span	4-9	337	1.08	-0.57	364
8 Foot Long Beam with 8 Foot Span	4-9	271	0.75	7.85	212

Table 4-1 Constants c_1 to c_3 Values and Initial Slope for Beams

SHEAR MODULUS IN FLEXURE

As described in the last chapter, the "apparent" modulus of elasticity for OSB sheathings $(E_s)_f$ can be determined by Eq. (25). Values of x-y for calculating the EPS shear modulus are listed in Table 4-2:

	$x = (1/L)^2$	$(E_s)_f$	$y = 1/(E_s)_f$
3 foot long beam with 2 foot span	0.00174	252000	3.96E-05
5 foot long beam with 4 foot span	0.000434	103000	9.67E-06
7 foot long beam with 6 foot span	0.000193	189000	5.30E-06
8 foot long beam with 6 foot span	0.000193	191000	5.24E-06
8 foot long beam with 8 foot span	0.000109	263000	3.80E-06

Table 4-2 Values of x-y for Calculating EPS Shear Modulus

One can see that the slopes of load-displacement curves for a 7 foot long beam with 6 foot span and an 8 foot long beam with 6 foot span are very close. The value of a 7 foot long beam with 6 foot span and other three values of different spans were used for the four-point plotting. By graphing $y = 1/(E_s)_f$ and $x = (1/L)^2$ as shown in Figure 4-10, the slope for the line connecting the four points was determined as 0.0222, and G was calculated as follows:

$$G = \frac{108 \times c \times c_1}{23 \times slope} = \frac{108 \times 3.63 \times 0.438}{23 \times 0.0222} = 335 \quad psi$$

EPS DENSITY

The EPS density in this research is 0.954 pcf based on the four test results. Test result showed a very low variability and were listed in Table 4-3.

Density Test	Weight (g)	Volume (in^3)	Density (pcf)	Average (pcf)
1	3.0	12.01	0.952	0.954
2	3.0	12.01	0.952	
3	3.0	12.15	0.941	
4	3.1	12.18	0.970	

Table 4-3 EPS Density in This Research

FINITE ELEMENT RESULTS

Findings of finite element results are presented beginning with the modeling of EPS foam as a hyperfoam material. Results of the bilinear model for EPS follow. Lastly, the results of user-supplied material for EPS were reported.

HYPERFOAM MODEL FOR EPS CORE

The hyperfoam models for EPS core are explained in the uniaxial compression mode and the simple shear mode.

UNIAXIAL COMPRESSION MODE

Using MATHEMATICA, α_i and μ_i were determined to be (Appendix A):

$$\alpha_1 = -161 \qquad \mu_1 = -330$$
$$\alpha_2 = 82.1 \qquad \mu_2 = -182$$
$$\alpha_3 = 82.1 \qquad \mu_3 = -154$$

A comparison of strain-stress curves based on Eq. (27) with determined values as shown above and the compression test is shown in Figure 4-11. Obviously, these two curves are a poor match. This shows that the uniaxial compression mode of the hyperfoam model is not a valid material for modeling EPS foam core.

SIMPLE SHEAR MODE

Using MATHMATICA, α_i and μ_i were determined (Appendix B):

$$\alpha_1 = 0.000726 \qquad \mu_1 = 23.5$$
$$\alpha_2 = 0.00622 \qquad \mu_2 = 84.1$$
$$\alpha_3 = 5.03\times10^{-6} \qquad \mu_3 = 107$$

A comparison of stress-strain curves based on Eq. (29) with determined values as shown above and the shear test is shown in Figure 4-12. Again, the simple shear mode of the hyperfoam model for EPS core proved to be inappropriate.

BILINEAR MODEL FOR EPS CORE

The EPS core was modeled as a bilinear material for compression, tension, and shear loading. The diagrams showing the geometry used in defining the compression, tension, and shear models are shown in Figure 3-2, 3-4, and 3-6. In each case, a plane stress, two-dimensional element was employed for the analysis.

The comparison of load-displacement curves of EPS in compression and tension from mechanical test and bilinear model is shown in Figures 4-13 and 4-14. Load-displacement curves generated by the finite element model and obtained by mechanical beam testing with spans of 2 foot, 4 foot, 6 foot, and 8 foot are shown in Figures 4-15 to 4-19.

USER-SUPPLIED MODEL FOR EPS

SIP beams were modeled with OSB as an elastic-orthotropic material and EPS as a user-supplied material two times, each time with different shear properties. In the first model, shear properties were obtained from the shear tests as shown in Figure 3-3. For the second model, the shear modulus in flexure was generated from the SIP beam bending tests.

USER-SUPPLIED MATERIAL 1

The inputs for OSB are as described in Chapter 3 – Finite Element Model For a SIP Beam. For EPS, the inputs are shown in Table 4-4.

Group 1 -- Compression	Constant 1	9.65
	Constant 2	86.4
	Constant 3	19.8
Group 2 -- Tension	Constant 4	41.5
	Constant 5	38.6
	Constant 6	-235
Group 3 -- Shear	Constant 7	12.8
	Constant 8	32.7
	Constant 9	0
Group 4 -- Poisson's Ratio	Constant 10	0.05

Table 4-4 Inputs of User-Supplied Material 1 for EPS

SIP beams with spans of 2 foot, 4 foot, 6 foot, and 8 foot under four-point loading were modeled using above inputs. A comparison of load-displacement curves generated by the finite element model and obtained in mechanical testing are shown in Figures 4-20 to 4-24.

Figures 4-20 to 4-24 show that models with shear properties input obtained from the shear tests over predicted the results obtained from mechanical testing of the SIP beams. Yet, the two curves had the same overall behaviors.

USER-SUPPLIED MATERIAL 2

The inputs for OSB are as described in Chapter 3 – Finite Element Model For a SIP Beam. Figures 4-27 to 4-31 showed that SIP beams behaviors were over predicted. That might because the shear properties, which were obtained from non-pure tests, inputs for EPS were too high. In order to model SIPs behavior accurately, shear strength of EPS needs to be lower down. Constant 7 was chosen to be equal to the EPS compression strength, constant 8 was set to 34.7 to match the shear modulus obtained from SIP beam bending tests (335/9.65=34.7). And the new inputs for EPS are shown in Table 4-5.

Group 1 -- Compression	Constant 1	9.65
	Constant 2	86.4
	Constant 3	19.8
Group 2 -- Tension	Constant 4	41.5
	Constant 5	38.6
	Constant 6	-235
Group 3 -- Shear	Constant 7	9.65
	Constant 8	34.7
	Constant 9	0
Group 4 -- Poisson's Ratio	Constant 10	0.05

Table 4-5 Inputs of User-Supplied Material 2 for EPS

Substituting these new shear property constants in the finite element model, new load-displacement curves were generated for each span. Pairings of those and the curves obtained from mechanical tests are shown in Figures 4-25 to 4-26.

Curve pairs in Figures 4-25 to 4-26 exhibit close behaviors except the one obtained from 3 foot long beam with 2 foot span, suggesting that the inputs for user-supplied material are valid and the models are appropriate for predicting SIP beams' behavior. For a 2 foot short span beam, it failed at OSB sheathing

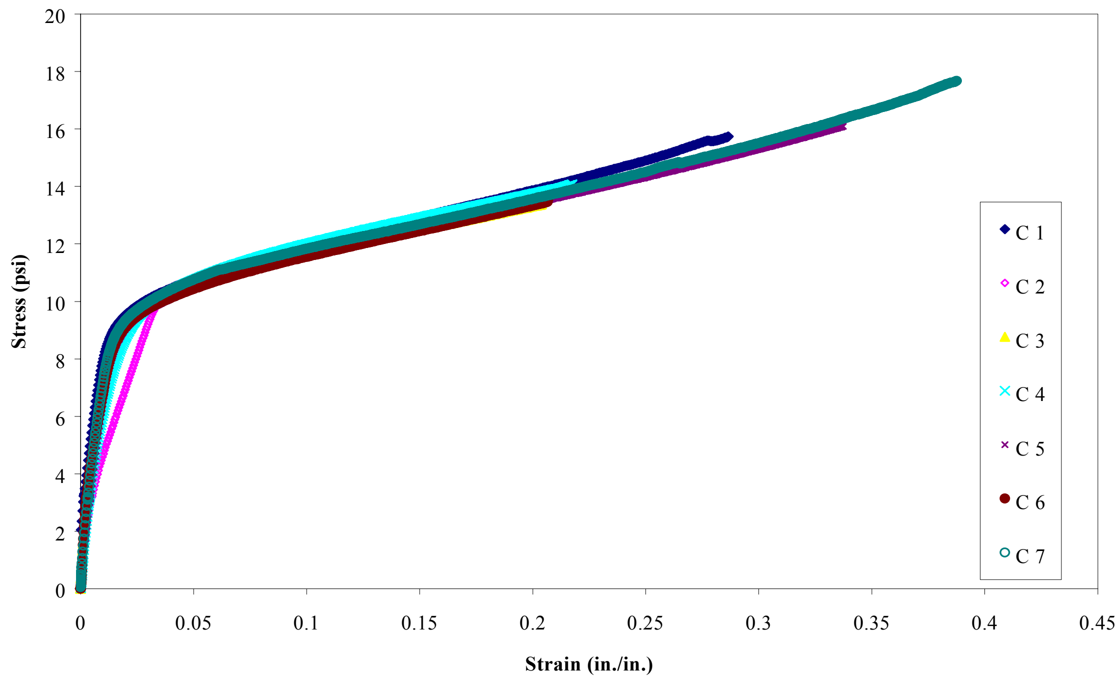

Fig. 4-1 Stress-Strain Curves for EPS in Compression

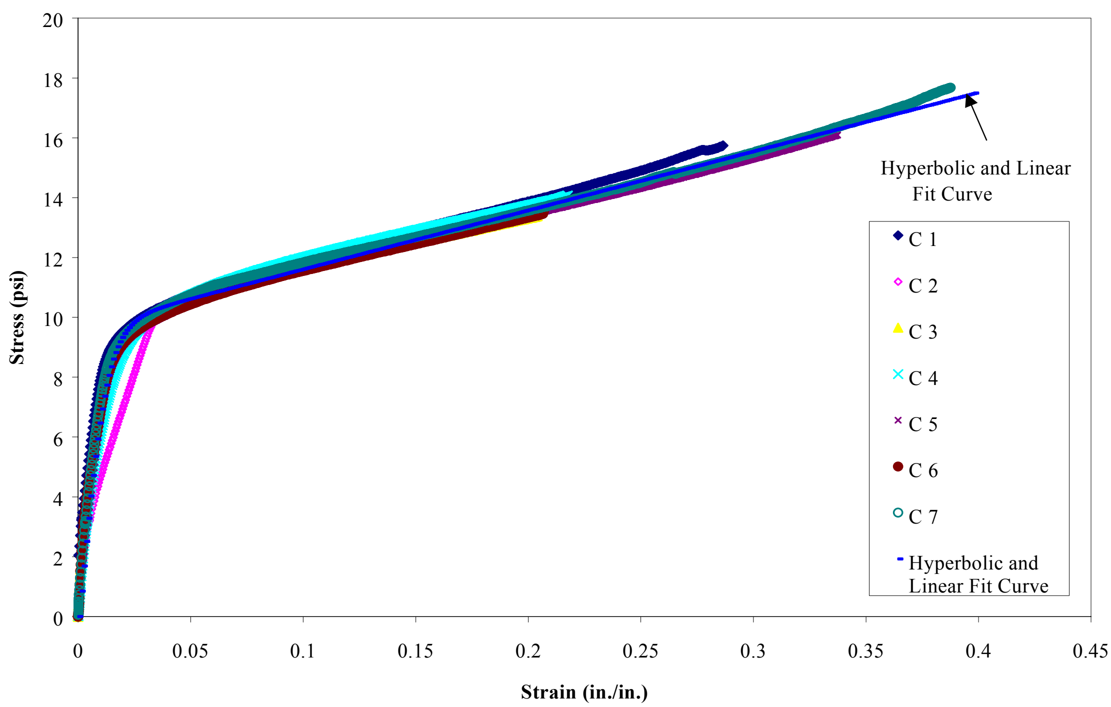

Fig. 4-2 Stress-Strain Curves for EPS in Compression between Testing & Hyperbolic and Linear Function Fit

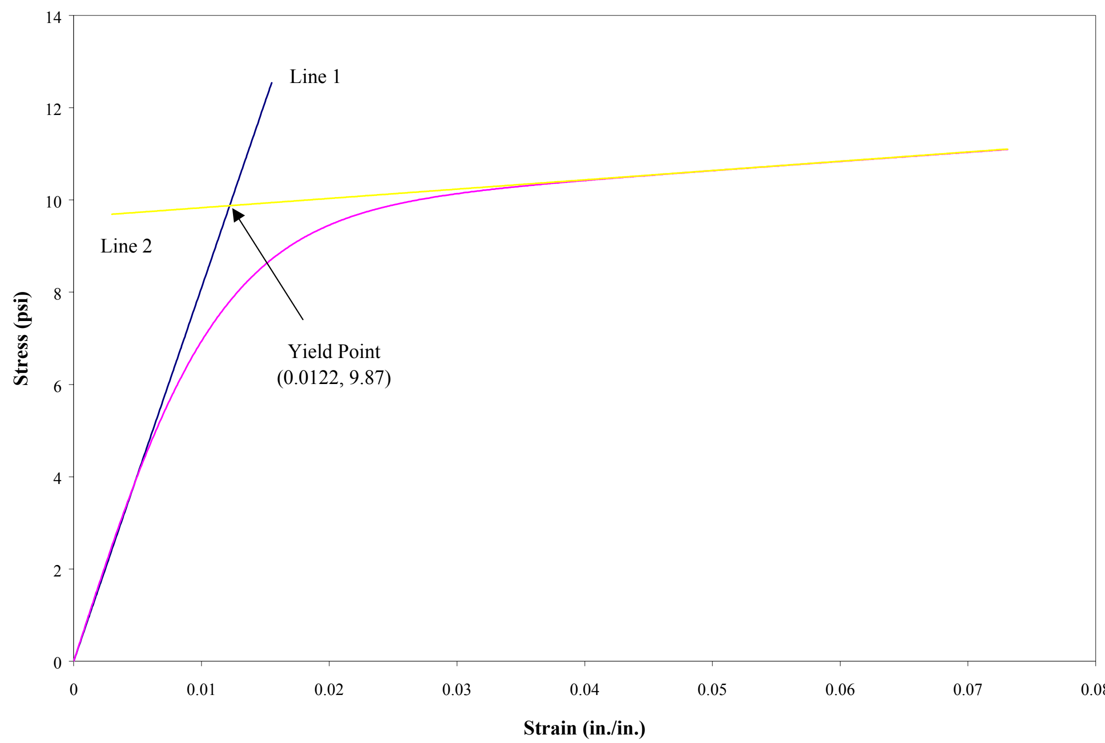

Fig. 4-3 EPS Continuous Point for Compression

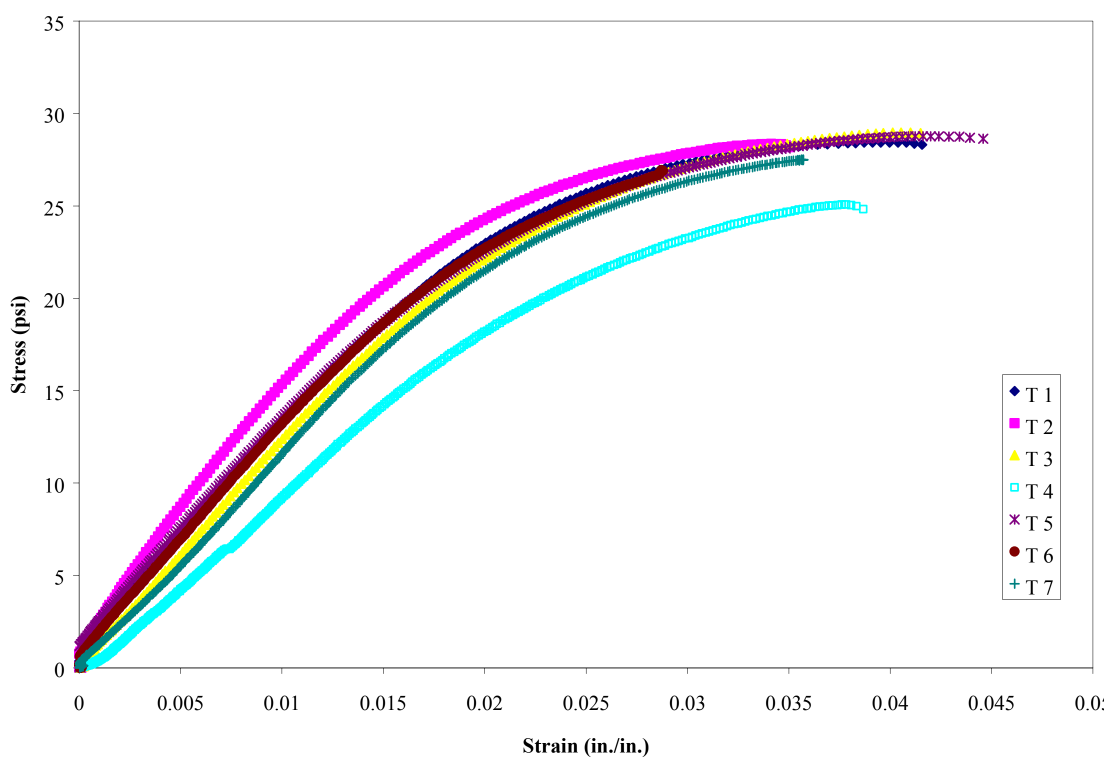

Fig. 4-4 Stress-Strain Curves for EPS in Tension

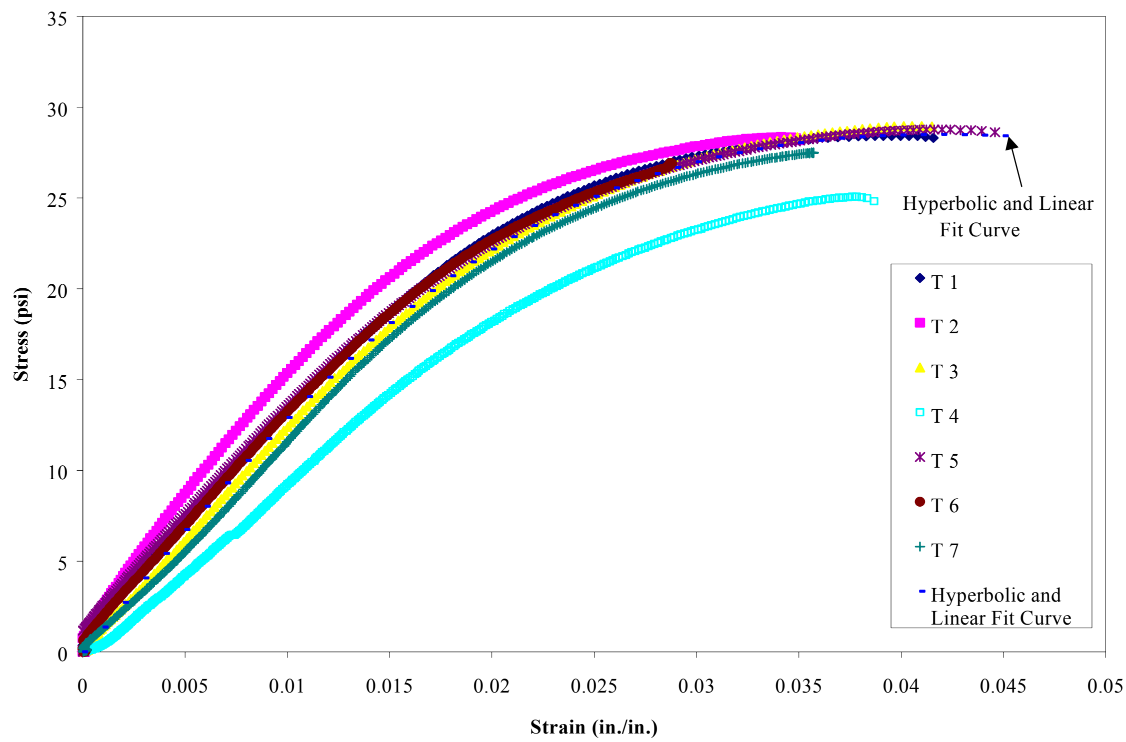

Fig. 4-5 Stress-Strain Curves for EPS in Tension between Testing & Hyperbolic and Linear Function Fit

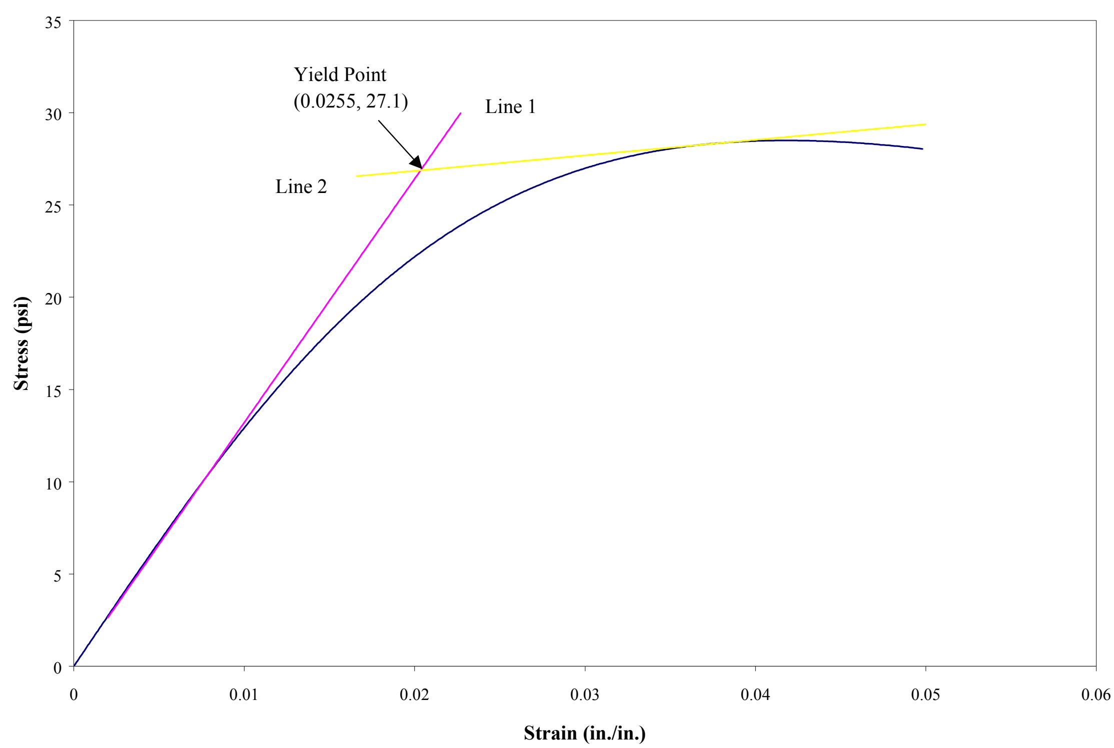

Fig. 4-6 EPS Discontinuous Point for Tension

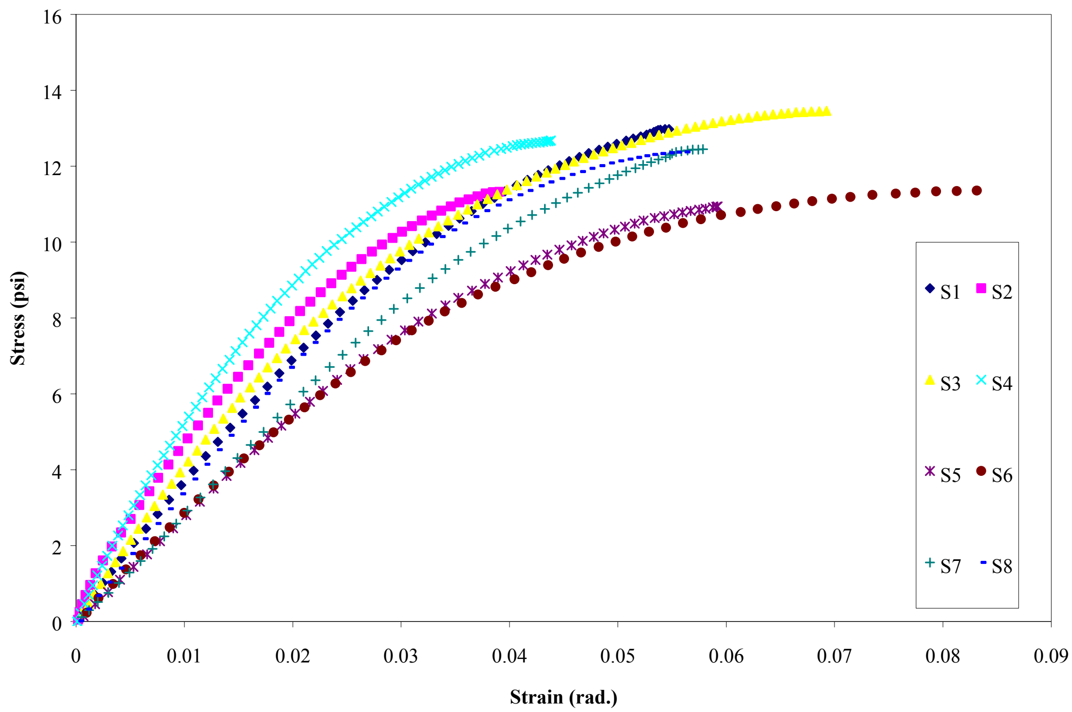

Fig. 4-7 Stress-Strain Curves for EPS in Shear

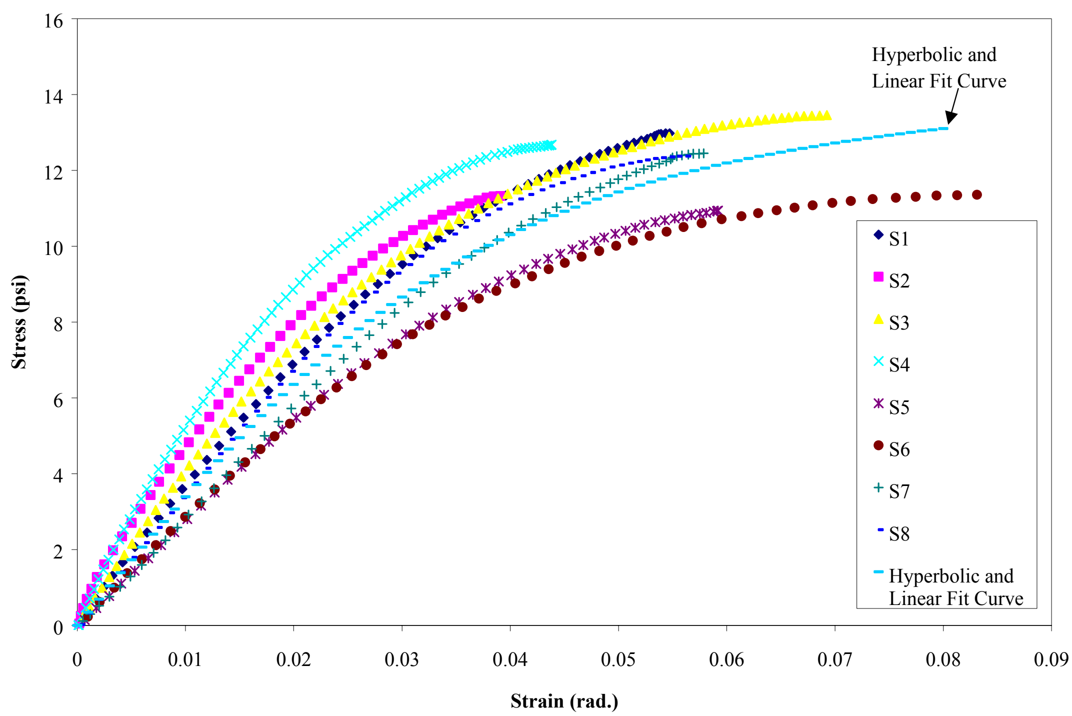

Fig. 4-8 Stress-Strain Curves for EPS in Shear between Testing
& Hyperbolic and Linear Function Fit

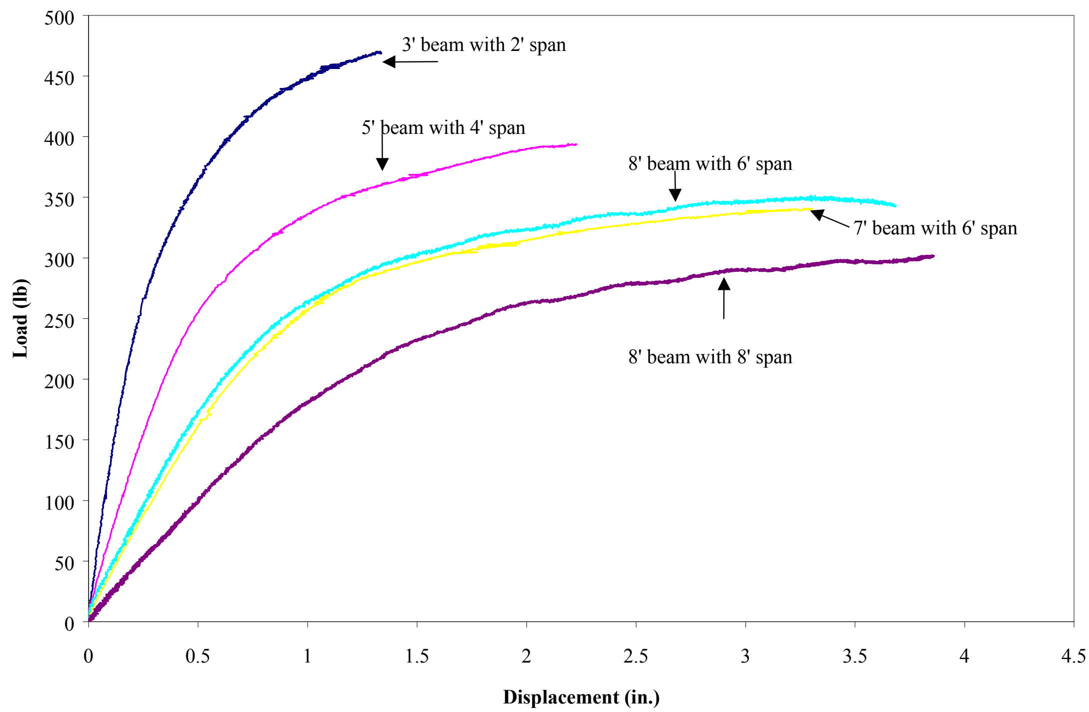

Fig. 4-9 Load-Displacement Curve for SIP Beams with Various Span

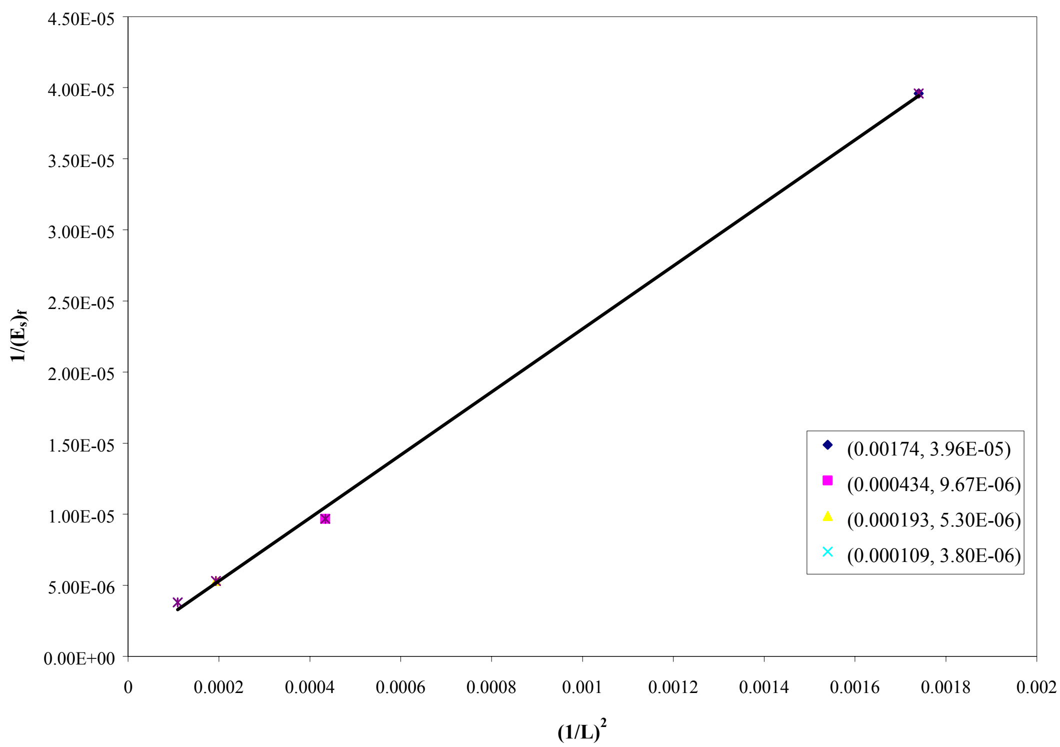

Fig. 4-10 Determination of the Slope by Using Initial Slopes of Load-Displacement Curves

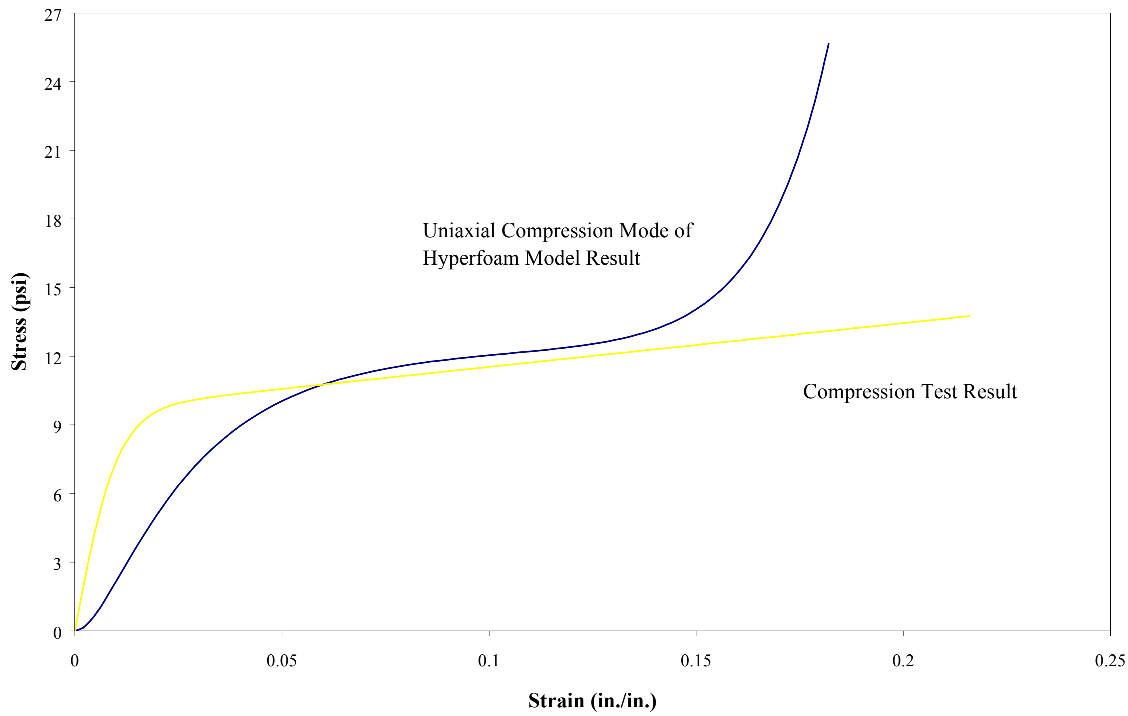

Fig. 4-11 Comparison of Stress-Strain Curves from Compression Test and Uniaxial Compression Mode of Hyperfoam Model

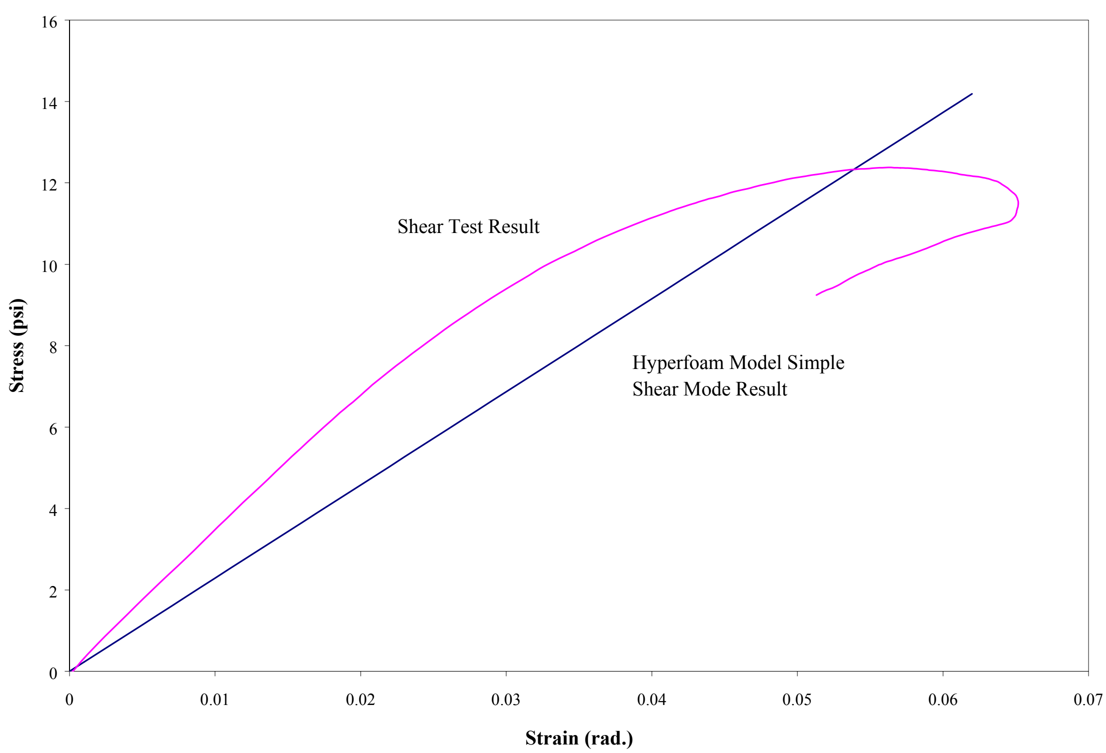

Fig. 4-12 Comparison of Stress-Strain Curves from Compression Test and Simple Shear Mode of Hyperfoam Model

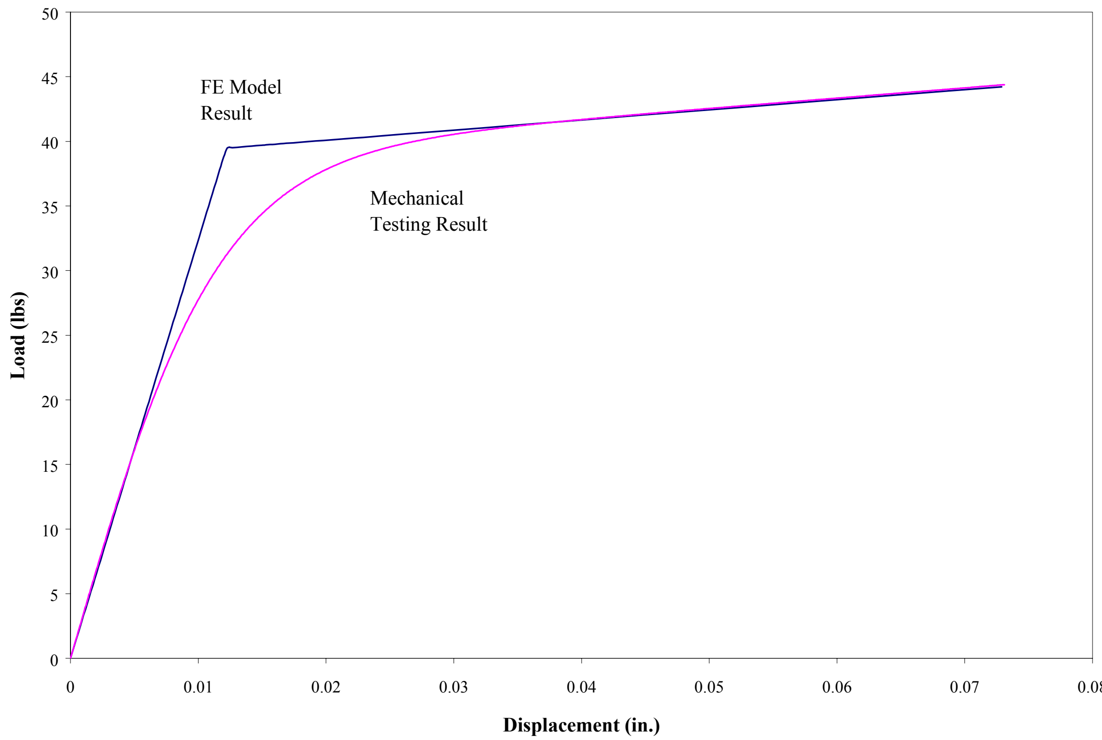

Fig. 4-13 Comparison of Load-displacement Curves of EPS in Compression from Mechanical Test and Bilinear Model

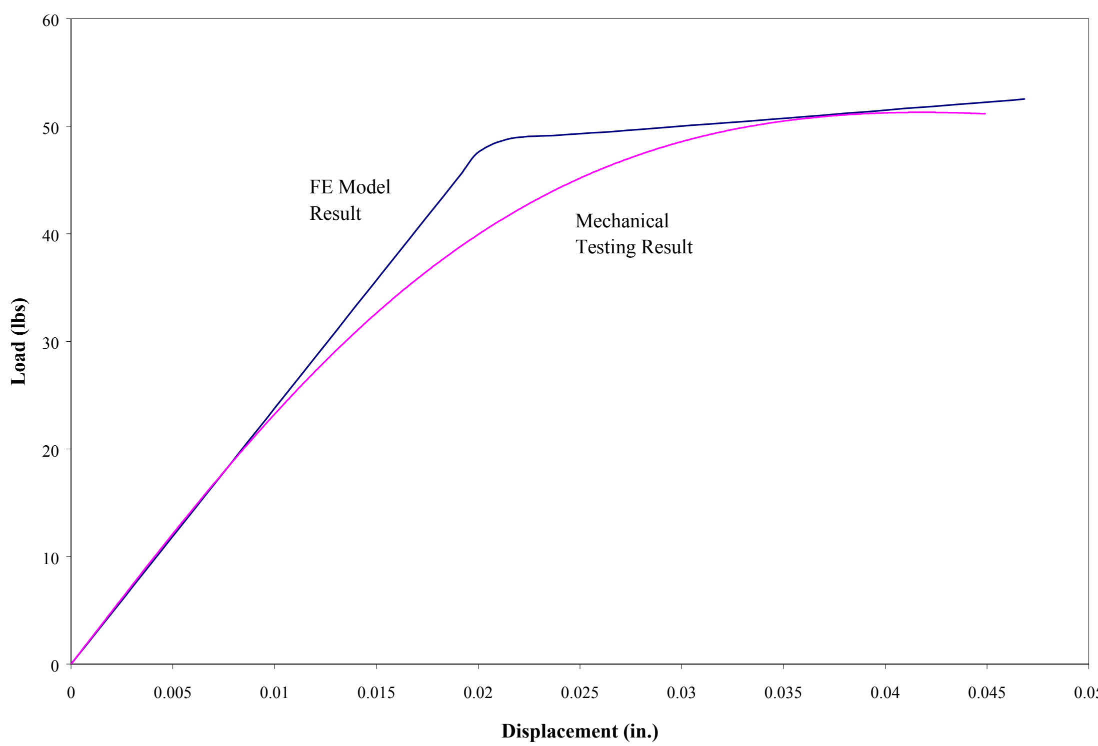

Fig. 4-14 Comparison of Load-displacement Curves of EPS in Tension from Mechanical Test and Bilinear Model

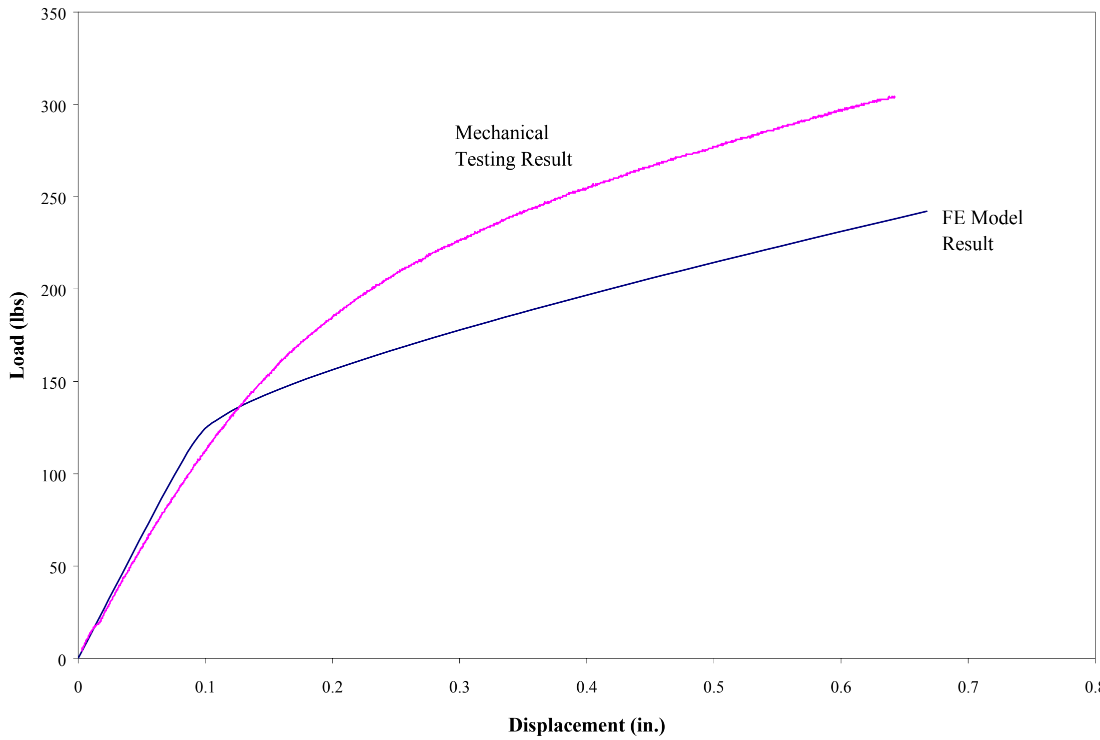

Fig. 4-15 Comparison of Load-Displacement Curves for a 3 Foot Long Beam with 2 Foot Span and Load Applied at 1/3 of the Span (Bilinear Model)

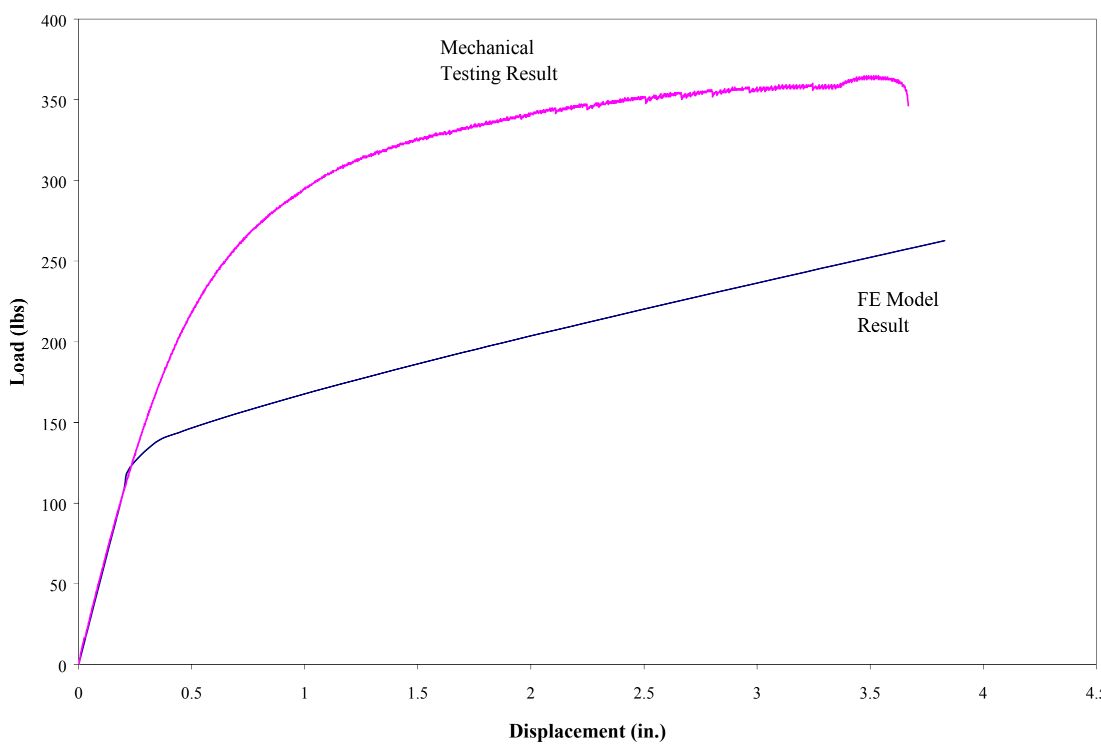

Fig. 4-16 Load-Displacement Curves for a 5 Foot Long Beam with 4 Foot Span and Load Applied at 1/3 of the Span (Bilinear Model)

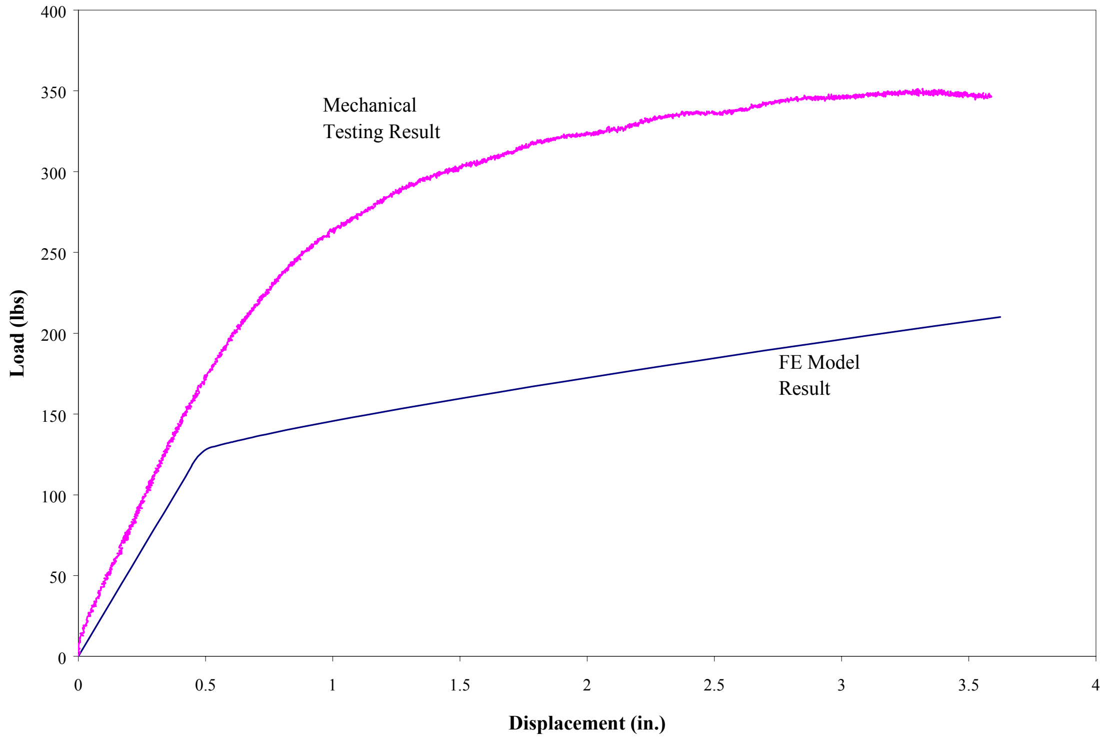

Fig 4-17 Load-Displacement Curves for a 7 Foot Long Beam with 6 Foot Span and Load Applied at 1/3 of the Span (Bilinear Model)

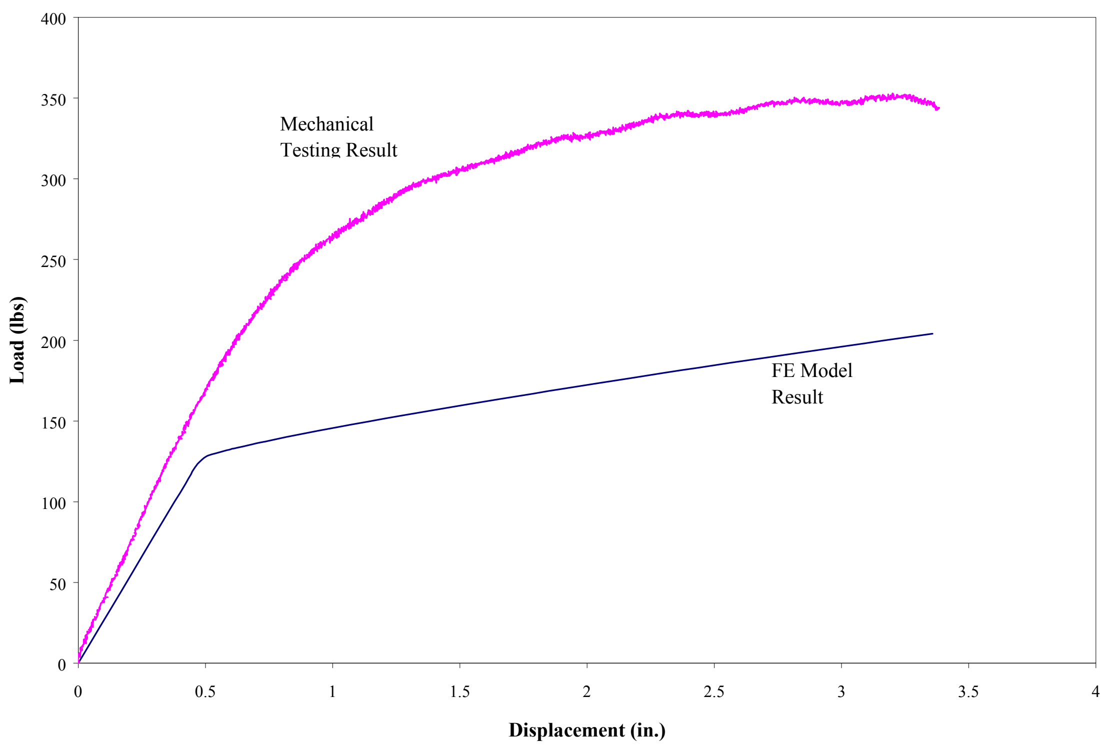

Fig. 4-18 Load-Displacement Curves for a 8 Foot Long Beam with 6 Foot Span and Load Applied at 1/3 of the Span (Bilinear Model)

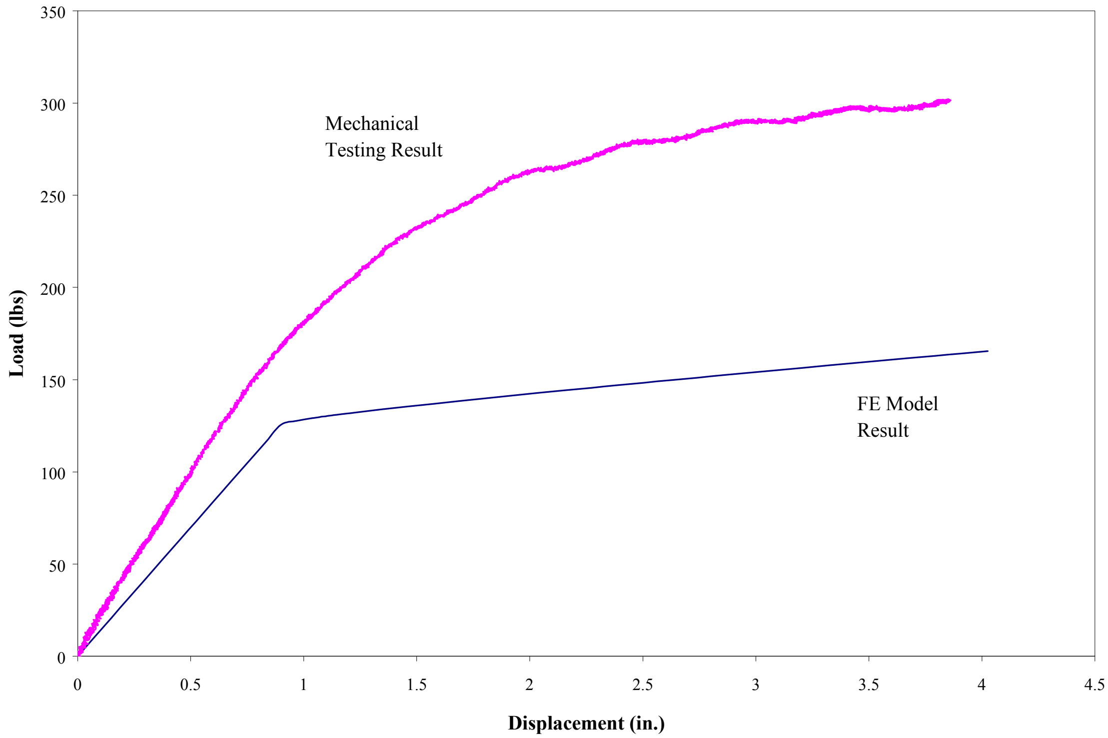

Fig. 4-19 Load-Displacement Curves for an 8 Foot Long Beam with 8 Foot Span and Load Applied at 1/3 of the Span (Bilinear Model)

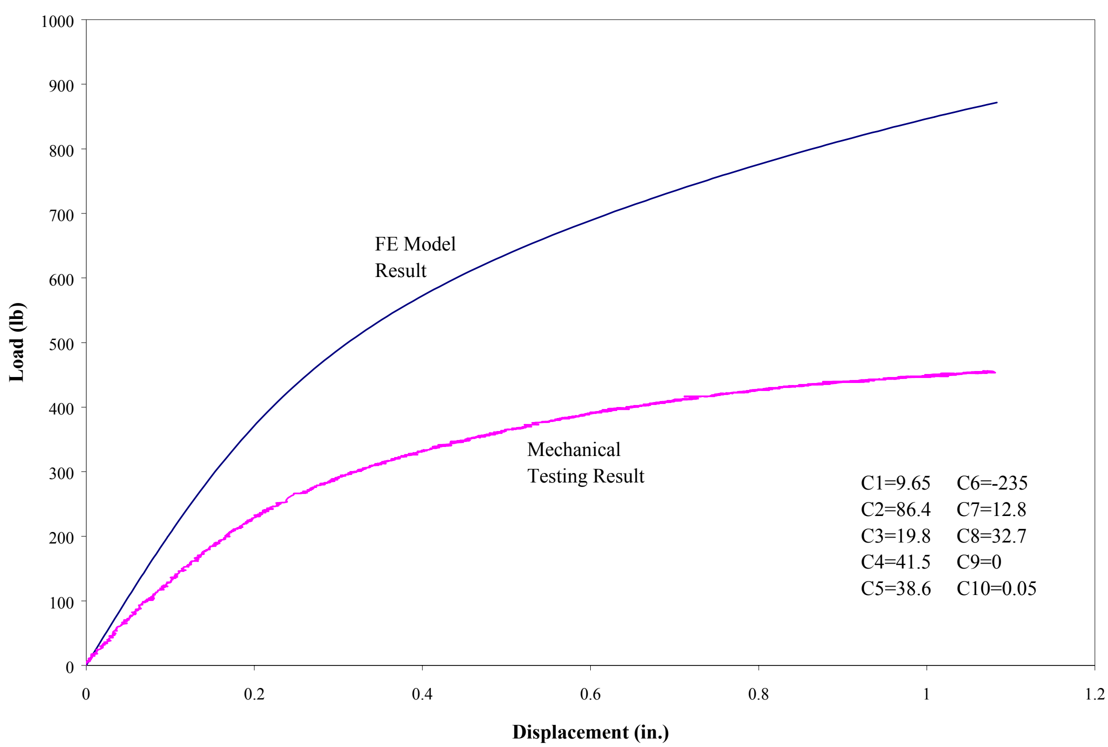

Fig. 4-20 Load-Displacement Curves for a 3 Foot Long Beam 2 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 1)

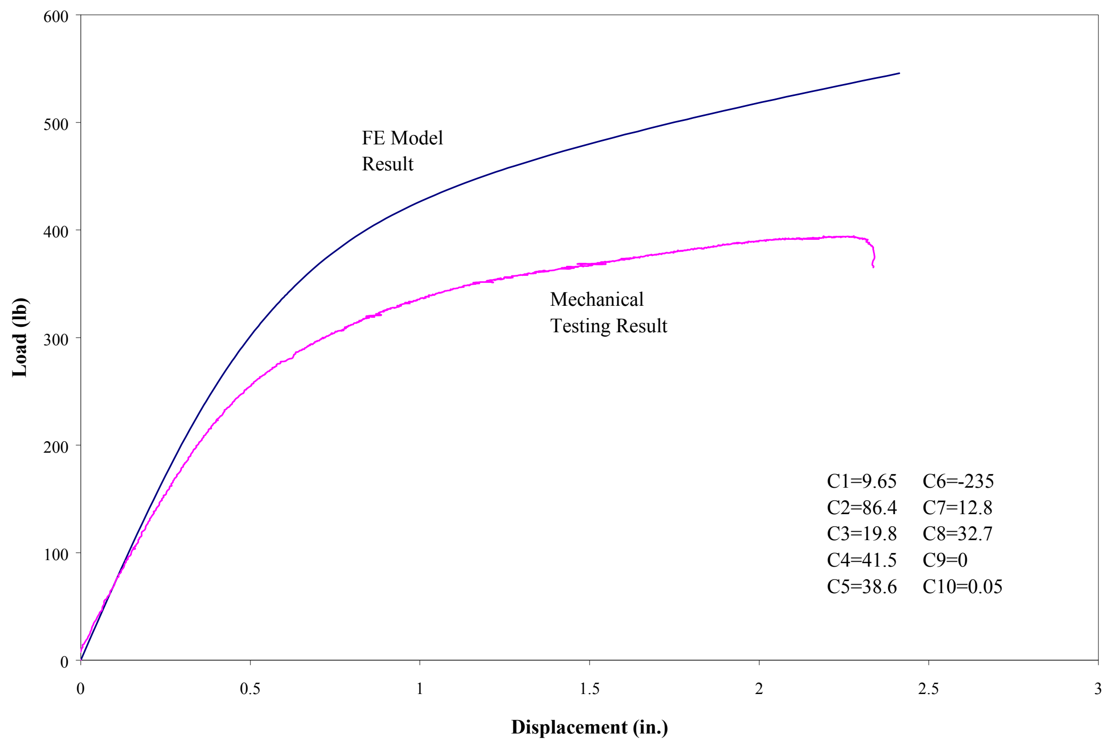

Fig. 4-21 Load-Displacement Curves for a 5 Feet Long Beam with 4 Feet Span and Load Applied at 1/3 of the Span (User-Supplied Material 1)

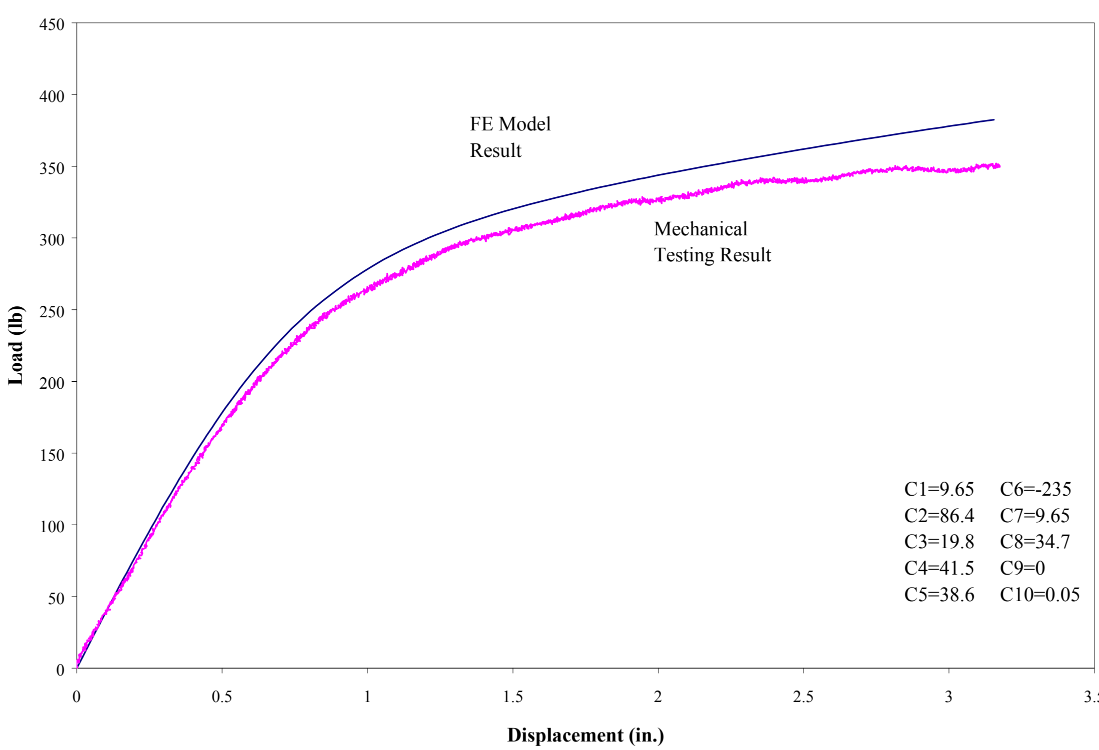

Fig. 4-22 Load-Displacement Curves of a 7 Foot Long Beam with 6 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 1)

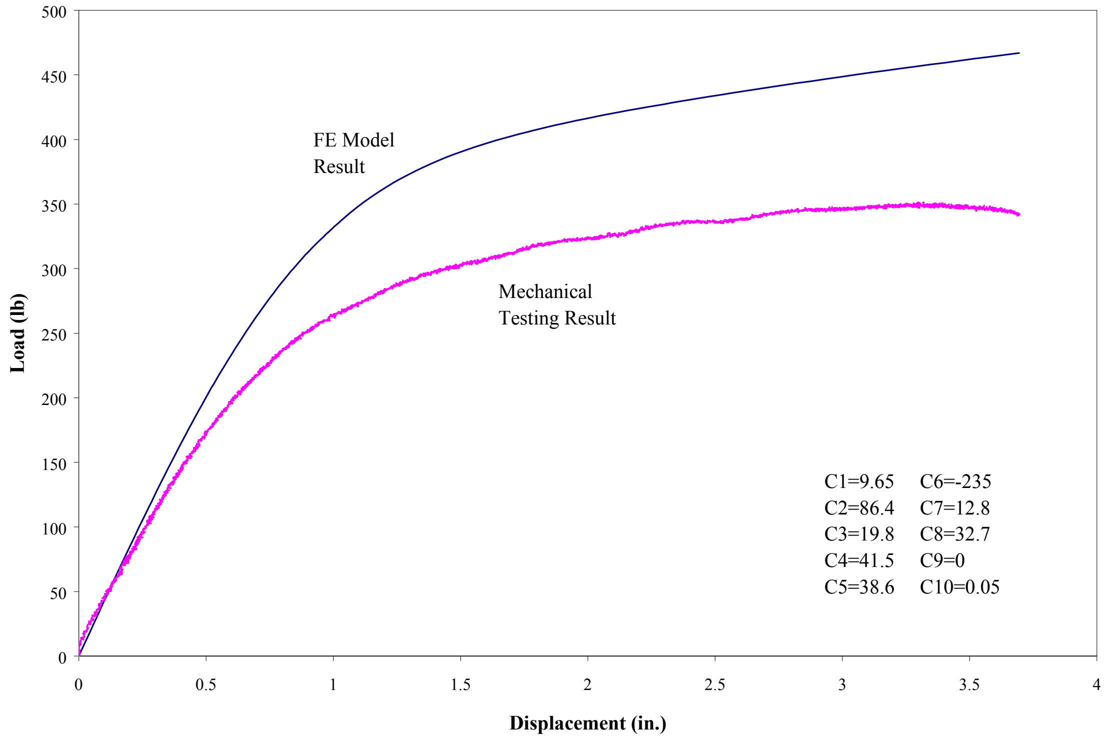

Fig. 4-23 Load-Displacement Curves of an 8 Foot Long Beam with 6 Foot Span and Load Applied at 1/3 of the Span (User-Supplied material 1)

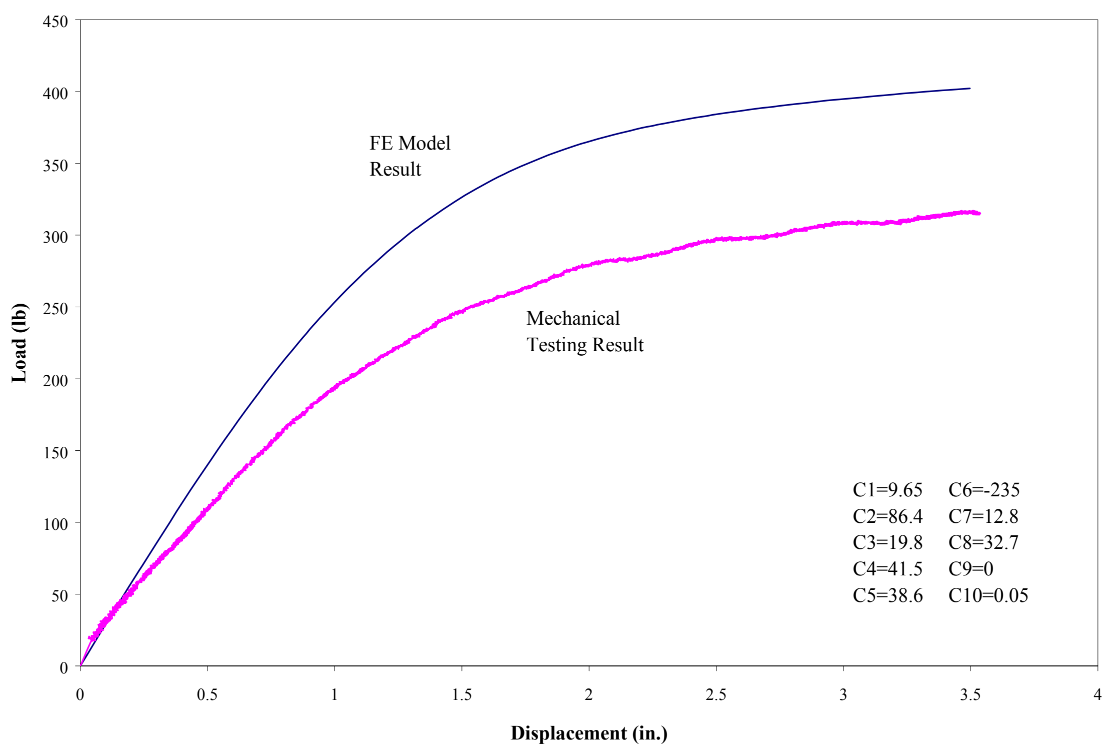

Fig. 4-24 Load-Displacement Curves of an 8 Foot Long Beam with 8 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 1)

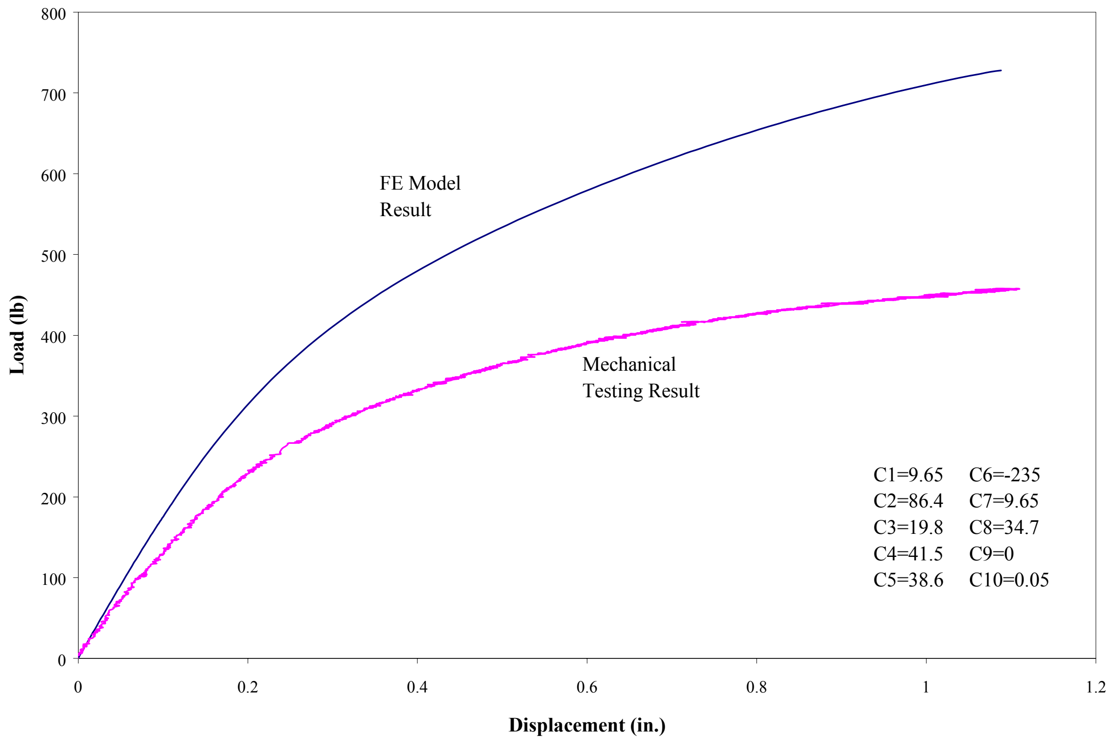

Fig. 4-25 Load-Displacement Curves of a 3 Foot Long Beam with 2 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 2)

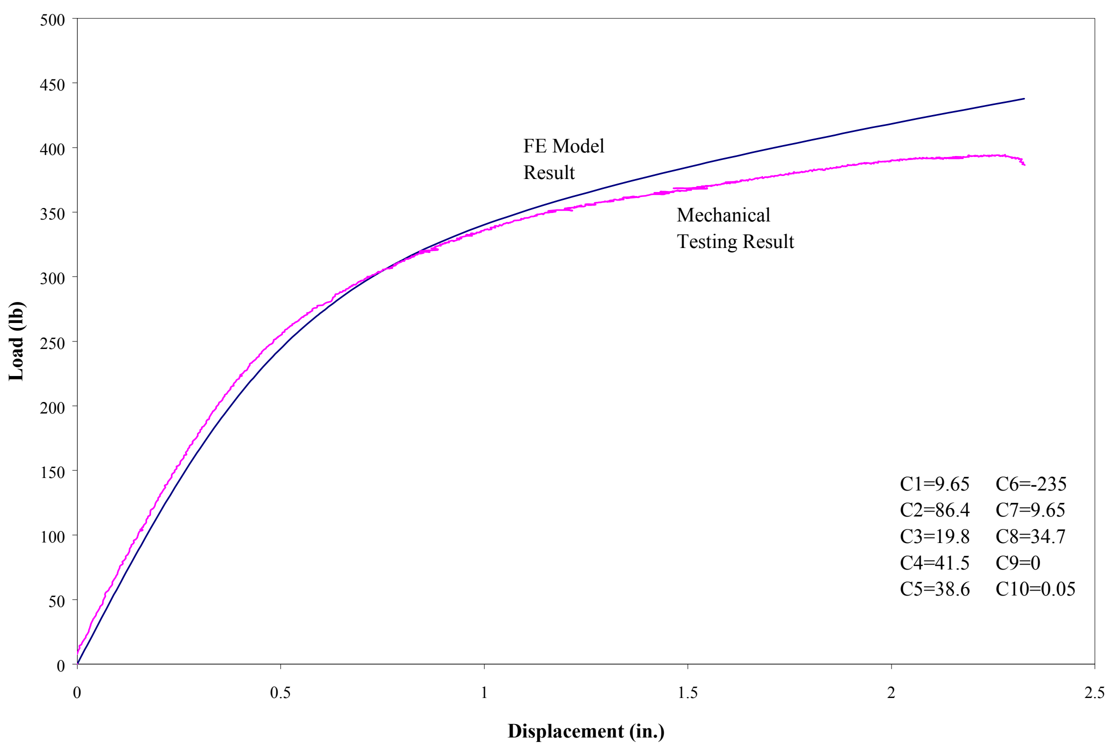

Fig. 4-26 Load-Displacement Curves of a 5 Foot Long Beam with 4 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 2)

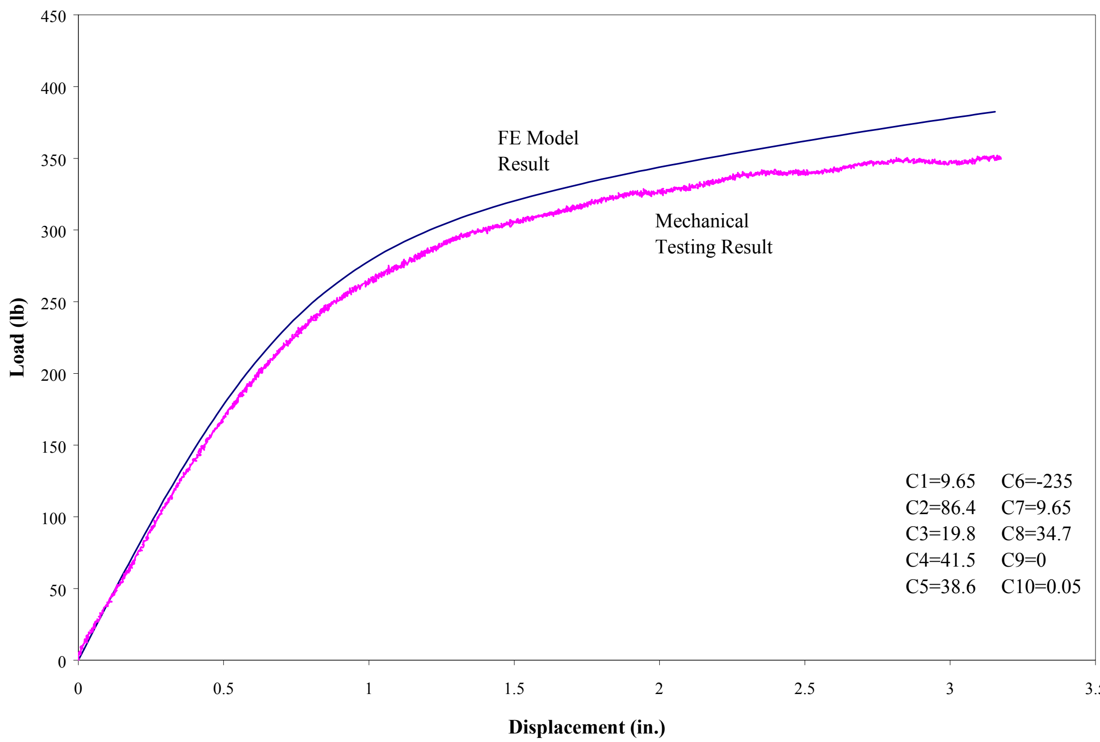

Fig. 4-27 Load-Displacement Curves of a 7 Foot Long Beam with 6 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 2)

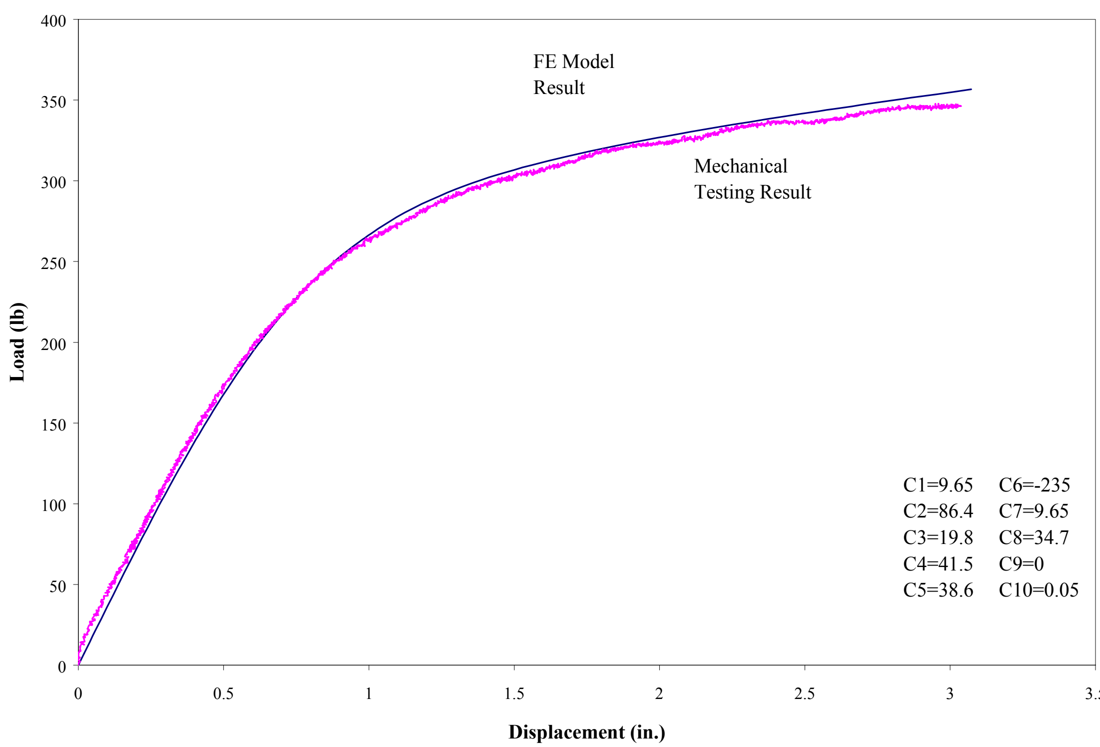

Fig. 3-28 Load-Displacement Curves of an 8 Foot Long Span with 6 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 2)

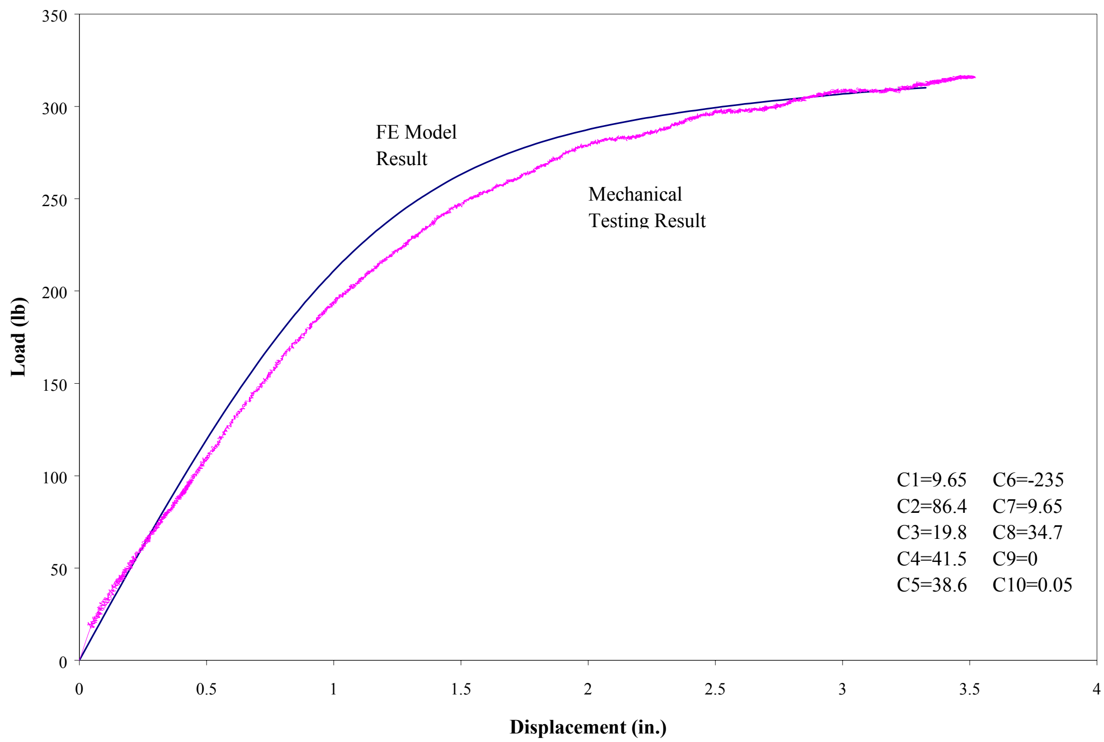

Fig. 4-29 Load-Displacement Curves of an 8 Foot Long Beam with 8 Foot Span and Load Applied at 1/3 of the Span (User-Supplied Material 2)

Chapter 5
DISCUSSION AND CONCLUSIONS

DISCUSSION

Discussion consists of three categories: material strength properties, material stress-strain curves, and design points in APA design specification. First, calculated values for material strength properties were compared to the published ones. Then, from analyzing EPS stress-strain curves in compression, tension, and shear, a better understanding of SIPs mechanical behaviors could be developed. Last, design point provided by APA was sketched in the load-displacement curves for its corresponding beam under four-points loading.

MATERIAL PROPERTIES

EPS mechanical properties were published in ASTM C578. Huntsman Corporation modified the typical physical properties for EPS with density at 1 lb/ft^3. The comparison of calculated and published values of EPS strength properties is shown in Table 5-1 and Table 5-2.

Strength Properties (psi)	Published Values		Calculated Values
	Density, minimum (pcf)		Density (pcf)
	0.90	1.15	0.95
Compressive 10% Deformation	10	13	11.6

Table 5-1 Published by ASTM and Calculated Values of EPS Compression Properties

Strength Properties (psi)	Published Values	Calculated Values
	Density, minimum (pcf)	Density (pcf)
	1.0	0.95
Tensile	28	28.5
Shear	16	13.4
Shear Modulus	440	419/335

Table 5-2 Published by Huntsman Corporation and Calculated Values of EPS Strength Properties

It is easy to see that calculated values of EPS properties agree with the published values well except the ones for shear strength and shear modulus. Huntsman Corporation obtained the shear strength by punch tool test (ASTM D 732-93). The ones calculated in this research make more sense since they were obtained from beam bending tests and would be used to model beam bending behavior. An EPS shear modulus of 335 psi was obtained from the SIP beam bending tests and 419 psi was from double foam shear tests. Finite element models show that it is more accurate to use shear property inputs calculated from SIP beam bending tests rather than the ones from shear test.

STRESS-STRAIN CURVES

Plotting EPS compression, tension, and shear strain-stress curves in one diagram (Figure 5-1), it shows the relationship between these three properties: EPS modulus of elasticity in tension is greater than its modulus of elasticity in compression, which is greater than its shear modulus; maximum tension stress is greater than maximum shear stress, which is greater than compression stress.

Recall when c_1 and c_2 determined from the double foam shear tests were used as shear inputs for modeling SIP beams, finite element model results (load-displacement curves) are somewhat off the actual mechanical testing data. The Mohr's circle for pure shear is shown in Figure 5-2. Any loading direction change will result a non-pure-shear situation, which can be the combination of shear, compression, and tension. The load-displacement recorded in the double foam shear tests could be the result of stress combination rather than pure shear.

Also Figure 5-1 shows that maximum shear stress and compression stress are very close. In user-supplied material 2 of the finite element model, c_1 for shear was equal to c_1 for compression. This proved to be a good assumption when comparing load-displacement curves from finite element models and mechanical tests and obtaining the agreement between these two.

Another finding from Figure 5-3 is that when EPS is loading, shear and compression may govern test result while tension may play unimportant role in material's behavior. Figure 5-3 shows two load-displacement curves of an 8 foot long beam with 8 foot span under four-points loading generated by finite element models. The values obtained from EPS tension testing ($c_1 = 41.5$, $c_2 = 38.6$, and $c_3 = -235$) were used as tension properties in curve 1. And the arbitrarily chosen values ($c_1 = 30$, $c_2 = 80$, and $c_3 = 60$) were used as tension properties in curve 2. Those two curves are nearly identical.

DESIGN POINT

"Acceptance Criteria for Sandwich Panels", issued by ICBO ES, states that the highest load reached for each test should be assumed to be the ultimate if tests are not conducted to failure. Safety factor three is applicable to the ultimate load. For the beams with different spans subjected to double loading at 1/3 of span, the maximum deflections due to their ultimate loads and the maximum loads correspond to their limit deflections (L/180) are shown in Table 5-3.

Figures 5-4 to 5-7 show the correspond load with deflection at L/180 and correspond deflection at 1/3 of ultimate load of beams with different spans in the load-displacement curves. As shown in Table 5-3, APA design equations under-predict the load at a given displacement between 9% to 16% and over-predict the displacement at a given load by 10% to 24%.

Though these are not dramatic differences, APA design equations only predict the SIP beam's behavior within linear range. Table 5-4 shows the comparison of flexural load, shear, and deflection predicted by APA design equations and the maximum load of beams with different span. APA equations can not accurately predicate the maximum load and its correspond load at all. For beams with different span, APA equations predicate that all beams will fail when load reaches at 119 lb. But actually, beams failed ranged from 302 lb to 471 lb.

CONCLUSIONS

A method to analyze SIPs under static, transverse loading has been developed using finite element analysis. In this method, the EPS core is modeled as a user-supplied material and the OSB sheathing is modeled as an elastic-orthotropic material. Modeling EPS as a bilinear material, which was suggested by Esvelt, does not predict SIPs behavior well. Also, modeling EPS as a hyperfoam material is not recommended because they can't predict SIP beam behavior under transverse loading.

The models for SIP beams with different spans show that they are not sensitive to the tensile properties of the EPS core. For EPS, compression and shear properties govern SIP beam behavior.

A hyperbolic and linear equation $p = c_1 Tanh(c_2 x) + c_3 x$ can be used to describe stress-strain curves of EPS in compression, tension, and shear and SIP beams in bending. This equation describes the load-displacement curves with four parameters, c_1 to c_4. The initial slope of the load-displacement curve equals $c_1 \times c_2 + c_3$, c_3 is the secondary slope of the curve, and c_4 stands for the intercept on the x-axis.

To assume EPS with same maximum compression and shear stress proved to be a good assumption. In real life, it is hard to conduct pure shear tests for EPS and obtain its shear properties. With this assumption, shear input for finite element modeling could be described with c_1 equaling to c_1 in compression and c_2 equaling to shear modulus obtained from SIP beam bending tests divided by c_1.

Beams	Given Load at 1/3 of Max. (lb)	Predicted Δ (in.)	Δ Obtained from Testing (in.)	Error	Given Δ at L/180 (in.)	Predicted Load (lb.)	Load Obtained from Testing (lb.)	Error
3’ Beam with 2’ Span	157	0.137	0.124	10%	0.133	152	167	-9%
5’ Beam with 4’ Span	130	0.252	0.203	24%	0.267	138	164	-16%
7’ Beam with 6’ Span	116	0.395	0.343	15%	0.400	118	132	-11%
8’Beam with 8’ Span	100	0.545	0.497	10%	0.533	97.8	108	-9%

Table 5-3 Comparison of APA Predicted Values and Testing Results at 1/3 of Max Load and Deflection at L/180 Situations

Beams	Max Load (lb)	APA Equations Predicted			Actual Deflection (in.)
		Flexural Load Check (lb)	Shear Check (lb)	Deflection (in.)	
3’ Beam with 2’ Span	471	955	119	0.411	1.63
5’ Beam with 4’ Span	394	477	119	0.765	2.28
7’ Beam with 6’ Span	341	318	119	1.16	3.39
8’ Beam with 8’ Span	302	239	119	1.65	3.84

Table 5-4 Comparison of APA Predicted Values and Testing Results at Max Load Situation

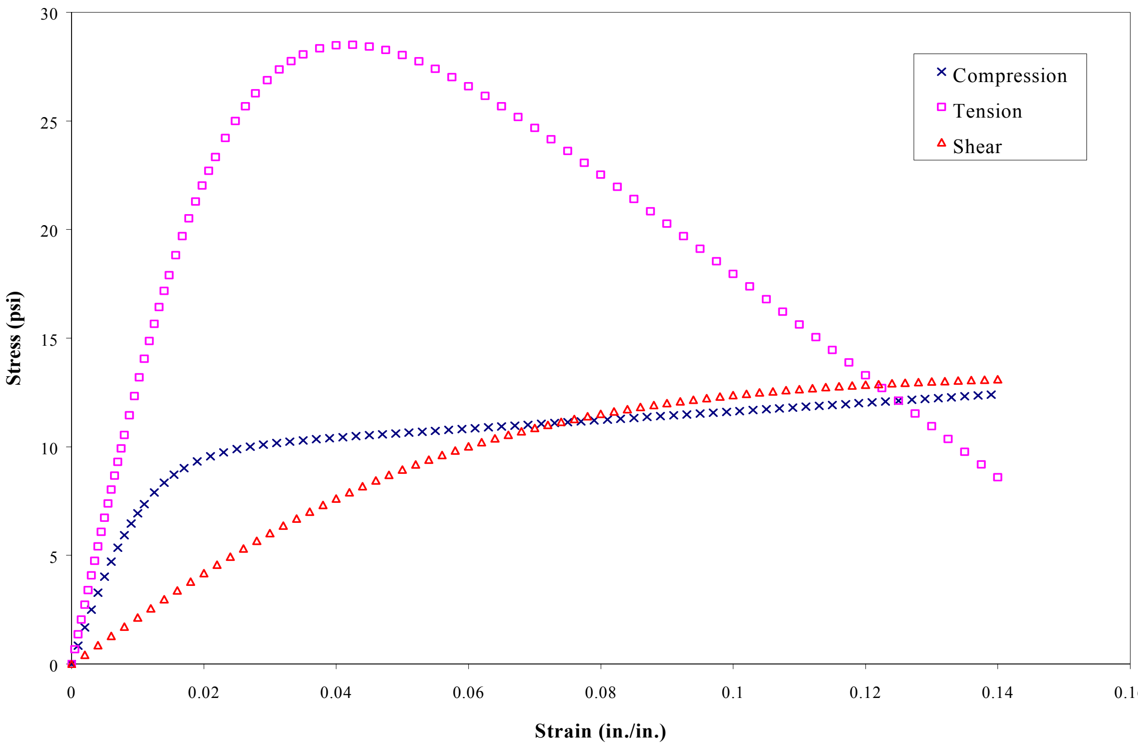

Fig. 5-1 EPS Compression, Tension, and Shear Stress-Strain Curves

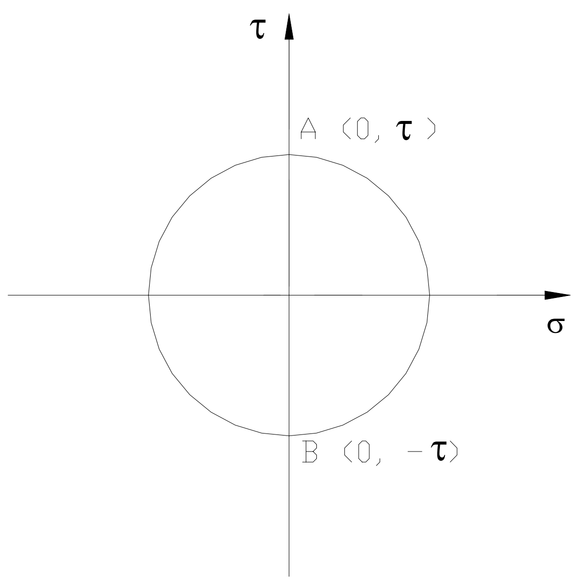

Fig. 5-2 Mohr's Circle for Pure Shear Condition

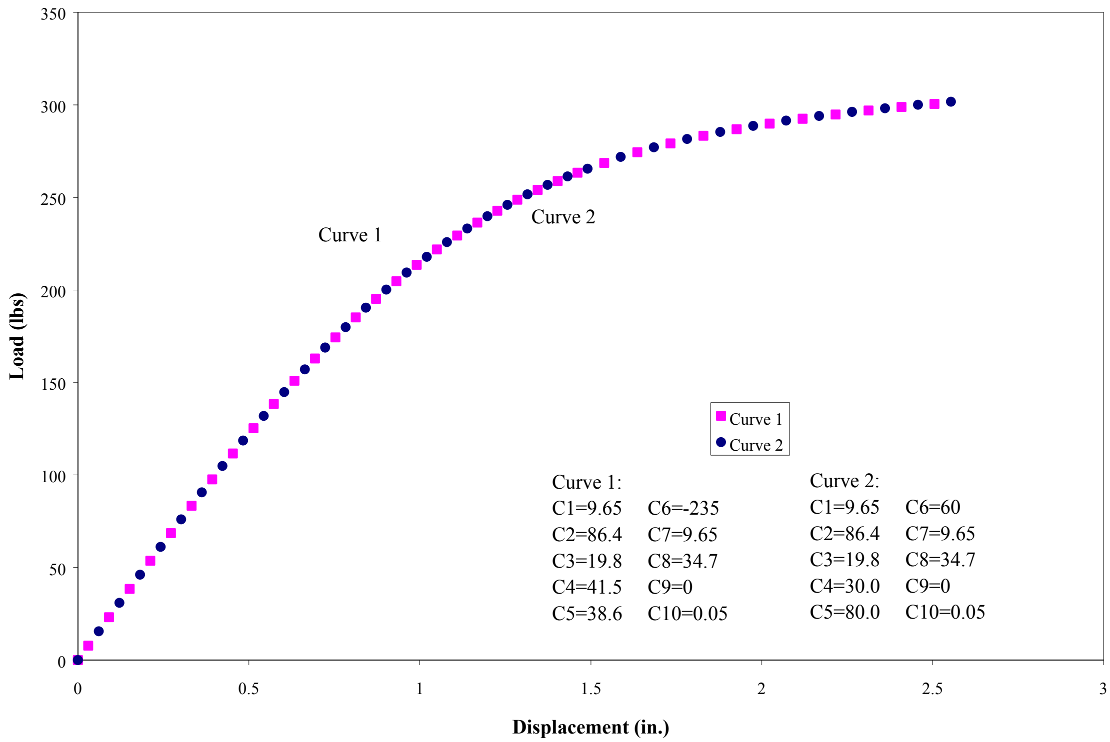

Fig. 5-3 Comparison of Load-Displacement Curves with Different Input for Tension Properties

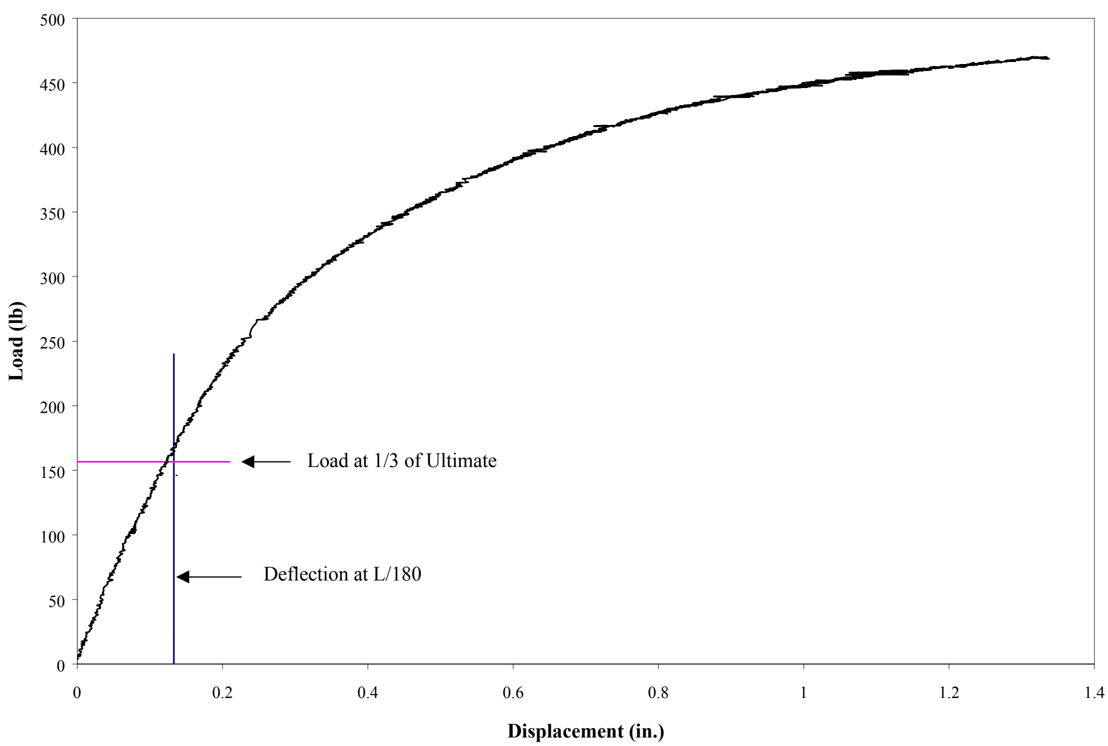

Fig. 5-4 Design Point for a 3 Foot Long Beam Provided by APA

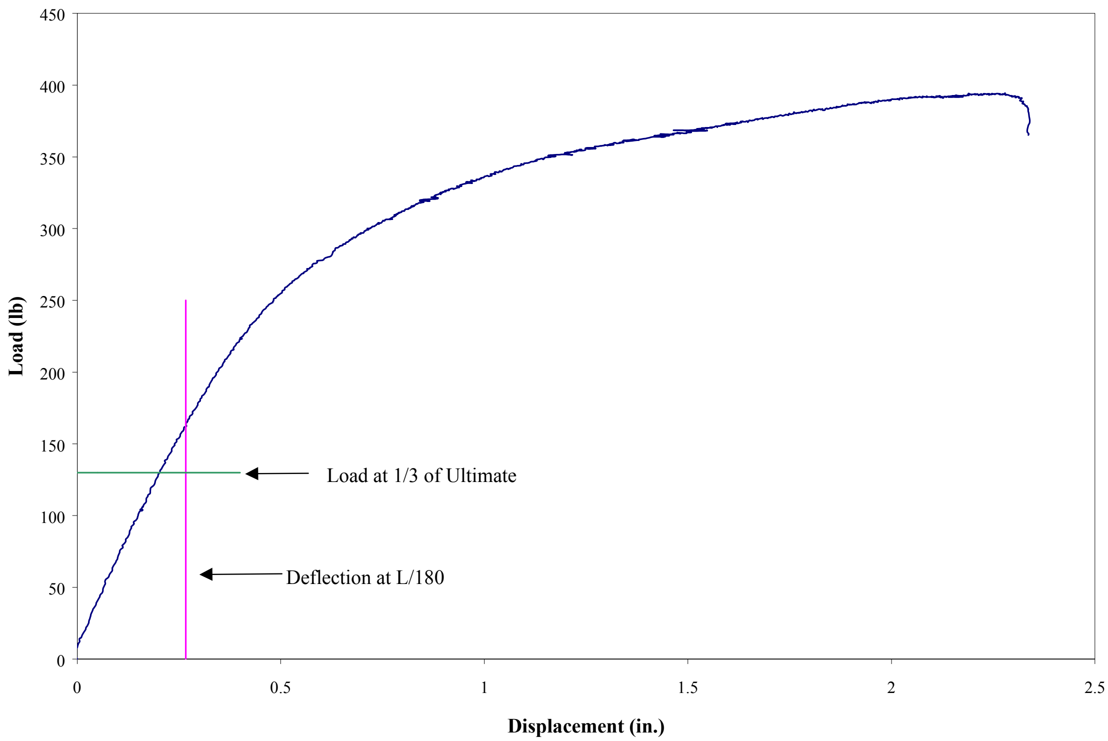

Fig. 5-5 Design Point for a 5 Foot Long Beam Provided by APA

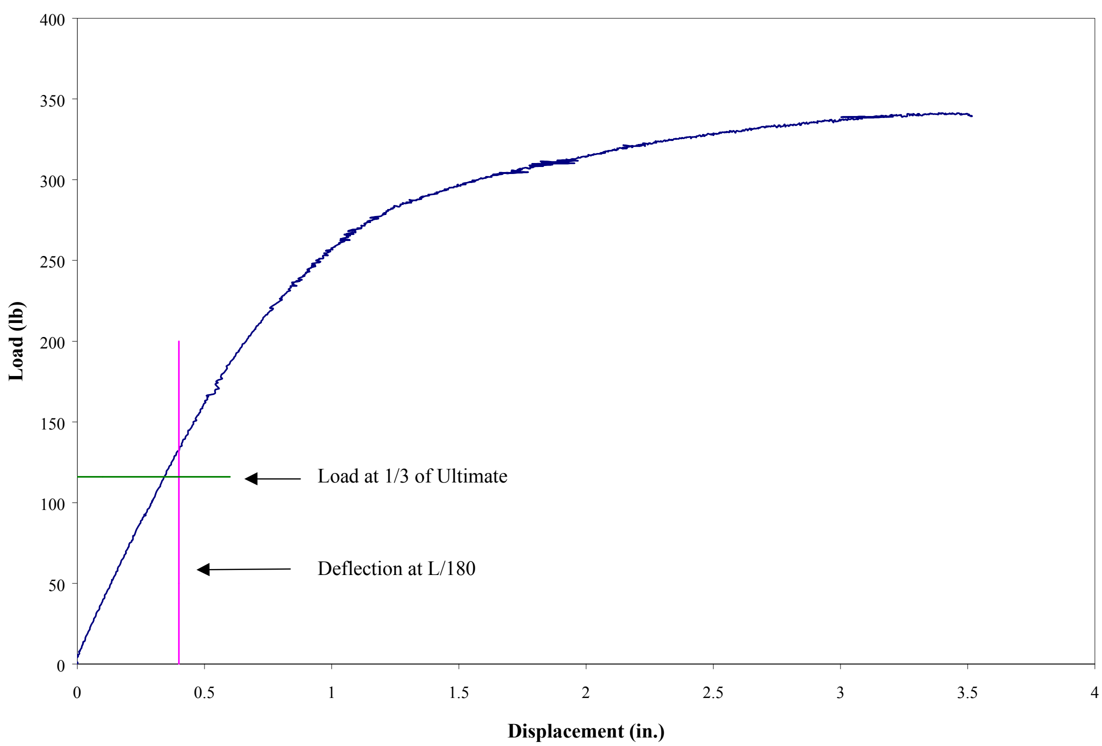

Fig. 5-6 Design Point for a 7 Foot Long Beam Provided by APA

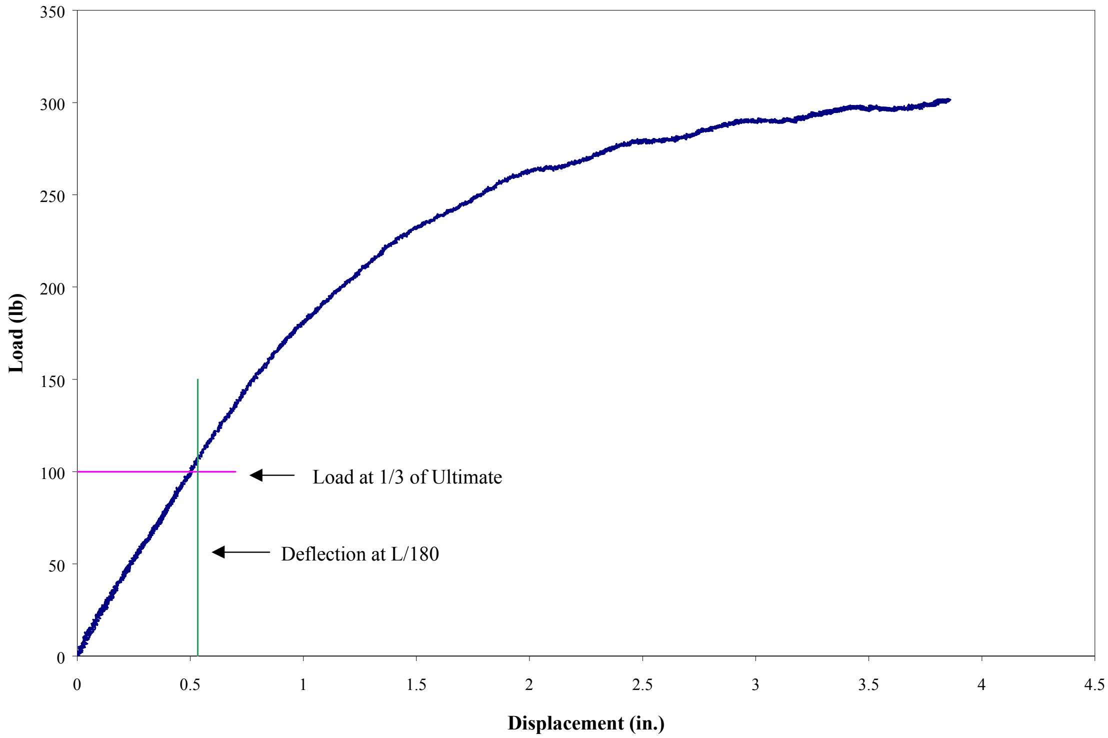

Fig. 5-7 Design Point for an 8 Foot Long Beam Provided by APA

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Appendix A

The Uniaxial Compression Mode of the Hyperfoam Model

For uniaxial compression mode:

```
<< Statistics`NonlinearFit`
<< Graphics`

λ = {λ1, λ2, λ3};
α = {α1, α2, α3};
μ = {μ1, μ2, μ3};

n = 3;
J = 1;
λ1 = 1 + ϵu;
```

$$\lambda 2 = \mathrm{Sqrt}\left[\frac{J}{1+\epsilon u}\right];$$

```
λ3 = λ2;
```

$$Tc = \sum_{j=1}^{3} \frac{2}{\lambda[[j]]} \sum_{i=1}^{3} \frac{\mu[[i]]}{\alpha[[i]]} \left(\lambda[[j]]^{\alpha[[i]]} - J^{-\alpha[[i]]\,\beta[[i]]}\right)$$

```
Part::partd : Part specification β⟦1⟧ is longer than depth of object.

Part::partd : Part specification β⟦2⟧ is longer than depth of object.

Part::partd : Part specification β⟦3⟧ is longer than depth of object.

General::stop : Further output of Part::partd will be suppressed during this calculation.
```

$$\frac{4\left(\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 1/2}\right)\mu 1}{\alpha 1}+\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 2/2}\right)\mu 2}{\alpha 2}+\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 3/2}\right)\mu 3}{\alpha 3}\right)}{\sqrt{\frac{1}{1+\epsilon u}}}+\frac{2\left(\frac{\left(-1+(1+\epsilon u)^{\alpha 1}\right)\mu 1}{\alpha 1}+\frac{\left(-1+(1+\epsilon u)^{\alpha 2}\right)\mu 2}{\alpha 2}+\frac{\left(-1+(1+\epsilon u)^{\alpha 3}\right)\mu 3}{\alpha 3}\right)}{1+\epsilon u}$$

```
data = Import["C:\Heming\Testing Data\Cofer\Compression\Notepad\C-3.txt", "Table"];

equation = Simplify[Tc]
```

$$\frac{4\left(\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 1/2}\right)\mu 1}{\alpha 1}+\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 2/2}\right)\mu 2}{\alpha 2}+\frac{\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{\alpha 3/2}\right)\mu 3}{\alpha 3}\right)}{\sqrt{\frac{1}{1+\epsilon u}}}+\frac{2\left(\frac{\left(-1+(1+\epsilon u)^{\alpha 1}\right)\mu 1}{\alpha 1}+\frac{\left(-1+(1+\epsilon u)^{\alpha 2}\right)\mu 2}{\alpha 2}+\frac{\left(-1+(1+\epsilon u)^{\alpha 3}\right)\mu 3}{\alpha 3}\right)}{1+\epsilon u}$$

```
fiteq = NonlinearFit[data, equation, {ϵu}, {α1, α2, α3, μ1, μ2, μ3}, MaxIterations -> 10000]
```

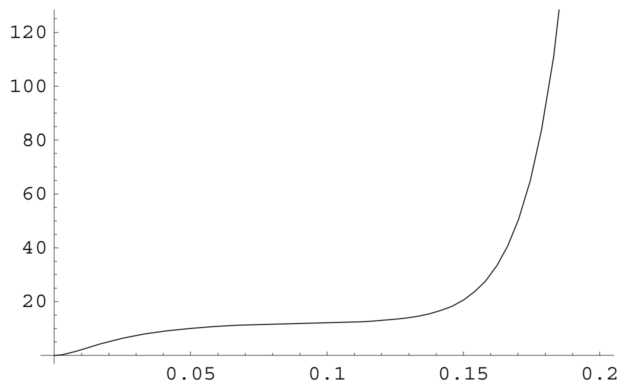
$$\frac{4\left(2.0495\left(-1+\frac{1}{\left(\frac{1}{1+\epsilon u}\right)^{80.6109}}\right)-1.87857\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{41.0553}\right)-2.22041\left(-1+\left(\frac{1}{1+\epsilon u}\right)^{41.0555}\right)\right)}{\sqrt{\frac{1}{1+\epsilon u}}}+\frac{1}{1+\epsilon u}$$

$$\left(2\left(2.0495\left(-1+\frac{1}{(1+\epsilon u)^{161.222}}\right)-1.87857\left(-1+(1+\epsilon u)^{82.1107}\right)-2.22041\left(-1+(1+\epsilon u)^{82.1111}\right)\right)\right)$$

```
chang = BestFitParameters /. NonlinearRegress[data, equation, {εu},
   {α1, α2, α3, μ1, μ2, μ3}, RegressionReport → BestFitParameters, MaxIterations -> 10000]
```

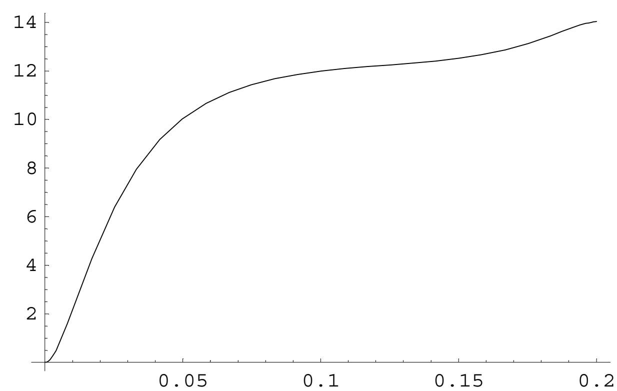
```
{α1 → -161.222, α2 → 82.1111, α3 → 82.1107, μ1 → -330.423, μ2 → -182.32, μ3 → -154.251}
```

```
foamfit = Plot[equation /. {α1 → -161.222, α2 → 82.1111,
    α3 → 82.1107, μ1 → -330.423, μ2 → -182.32, μ3 → -154.251}, {εu, 0, .2}]
```



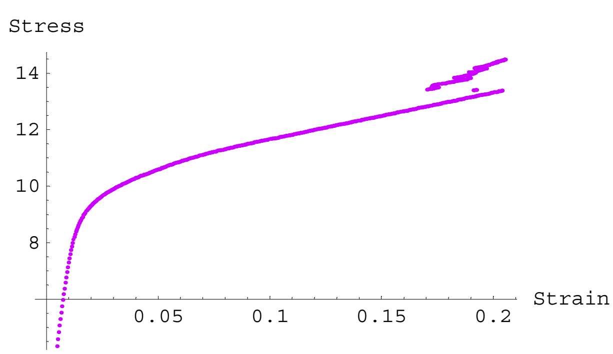
- Graphics -

```
foamfit2 = Plot[fiteq, {εu, 0, 0.2}]
```



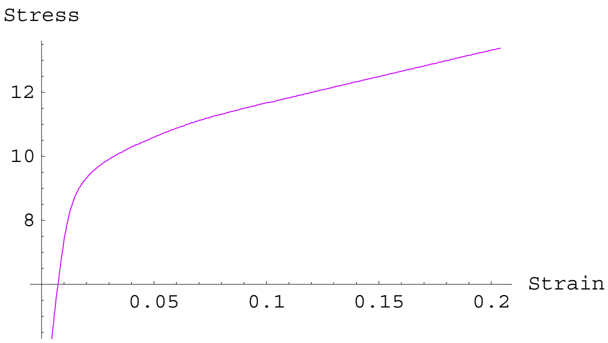
- Graphics -

```
rawdata = ListPlot[data, AxesLabel → {"Strain", "Stress"}, PlotStyle -> Hue[.8]]
```

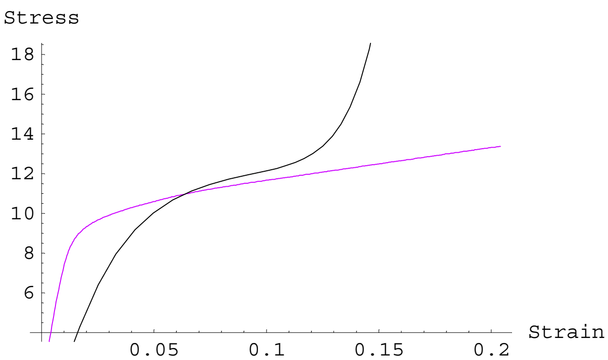


- Graphics -

```
For[i = 1, data[[i, 1]] < 0.2040, i ++]
i;
temp = Take[data, {2, i, 1}];
linshi = Array[aa, {Length[temp], 2}];
linshi[[All, 2]] = temp[[All, 2]];
linshi[[All, 1]] = temp[[All, 1]];
rawdata = ListPlot[linshi,
   AxesLabel → {"Strain", "Stress"}, PlotJoined → True, PlotStyle → Hue[0.8]];
```

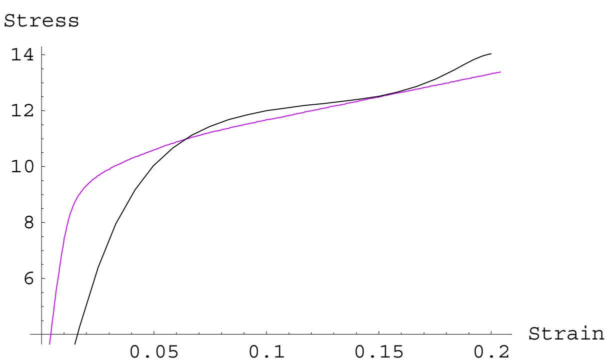



```
Show[rawdata, foamfit]
```



- Graphics -

```
Show[rawdata, foamfit2]
```



- Graphics -

Appendix B

The Simple Shear Mode of the Hyperfoam Model

```
<< Statistics`NonlinearFit`
<< Graphics`
λ = {λ1, λ2, λ3};
α = {α1, α2, α3};
μ = {μ1, μ2, μ3};
```

For shear mode:

```
n = 3;
J = 1;
λ1 = Sqrt[1 + γ^2/2 + γ Sqrt[1 + γ^2/4]];
λ2 = Sqrt[1 + γ^2/2 - γ Sqrt[1 + γ^2/4]];
λ3 = 1;
```

$$ts = \sum_{j=1}^{3}\left(\frac{2\,\gamma}{2\,(\lambda[[j]]^2-1)-\gamma^2}\sum_{i=1}^{n}\frac{\mu[[i]]}{\alpha[[i]]}\,(\lambda[[j]]^{\alpha[[i]]}-1)\right)$$

$$\frac{2\,\gamma\left(\frac{\left(-1+\left(1+\frac{\gamma^2}{2}-\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha1/2}\right)\mu1}{\alpha1}+\frac{\left(-1+\left(1+\frac{\gamma^2}{2}-\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha2/2}\right)\mu2}{\alpha2}+\frac{\left(-1+\left(1+\frac{\gamma^2}{2}-\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha3/2}\right)\mu3}{\alpha3}\right)}{-\gamma^2+2\left(\frac{\gamma^2}{2}-\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)}+$$
$$\frac{2\,\gamma\left(\frac{\left(-1+\left(1+\frac{\gamma^2}{2}+\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha1/2}\right)\mu1}{\alpha1}+\frac{\left(-1+\left(1+\frac{\gamma^2}{2}+\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha2/2}\right)\mu2}{\alpha2}+\frac{\left(-1+\left(1+\frac{\gamma^2}{2}+\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)^{\alpha3/2}\right)\mu3}{\alpha3}\right)}{-\gamma^2+2\left(\frac{\gamma^2}{2}+\gamma\sqrt{1+\frac{\gamma^2}{4}}\right)}$$

```
data = Import["C:\Heming\Testing Data\Testing Data\Shear\Notepad\Shear11.txt", "Table"];
```

```
equation = Simplify[ts]
```

$$\frac{1}{\alpha1\,\alpha2\,\alpha3\sqrt{4+\gamma^2}}\left(2\left(-2^{-\alpha2/2}\,\alpha1\,\alpha3\left(\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{\alpha2/2}-\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{\alpha2/2}\right)\mu2+\right.\right.$$
$$\alpha2\left(-2^{-\alpha1/2}\,\alpha3\left(\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{\alpha1/2}-\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{\alpha1/2}\right)\mu1+\right.$$
$$\left.\left.\left.2^{-\alpha3/2}\,\alpha1\left(-\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{\alpha3/2}+\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{\alpha3/2}\right)\mu3\right)\right)\right)$$

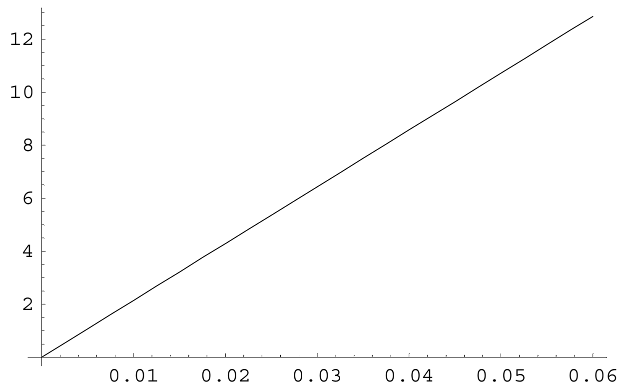
```
fiteq = NonlinearFit[data, equation, {γ}, {α1, α2, α3, μ1, μ2, μ3}, MaxIterations -> 10000]
```

$$\frac{1}{\sqrt{4+\gamma^2}}\left(1.49191\times10^{7}\left(0.0067102\left(\frac{1}{\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{0.000888255}}-\frac{1}{\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{0.000888255}}\right)-\right.\right.$$
$$0.00177651\left(5.44867\left(-\frac{1}{\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{0.000795852}}+\frac{1}{\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{0.000795852}}\right)+\right.$$
$$\left.\left.\left.0.03944\left(\left(2+\gamma^2-\gamma\sqrt{4+\gamma^2}\right)^{0.0237043}-\left(2+\gamma^2+\gamma\sqrt{4+\gamma^2}\right)^{0.0237043}\right)\right)\right)\right)$$

```
chang = BestFitParameters /. NonlinearRegress[data, equation, {γ},
   {α1, α2, α3, μ1, μ2, μ3}, RegressionReport → BestFitParameters, MaxIterations -> 10000]
```

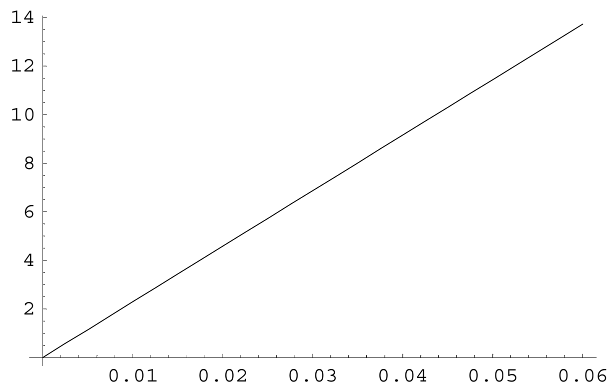
{α1 → 0.0474086, α2 → -0.00177651, α3 → -0.0015917, μ1 → 25.1889, μ2 → 88.8687, μ3 → 114.867}

```
foamfit = Plot[equation /. {μ1 → 23.537, μ2 → 84.1249, μ3 → 106.826,
    α1 → 0.000725729, α2 → 0.00622465, α3 → 5.03091*^-6}, {γ, 0, .06}]
```



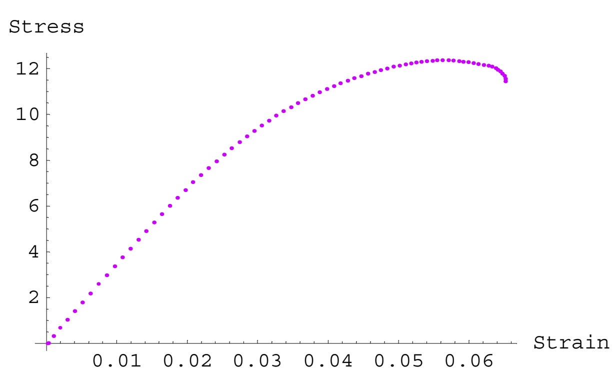
- Graphics -

```
foamfit2 = Plot[fiteq, {γ, 0, 0.06}]
```



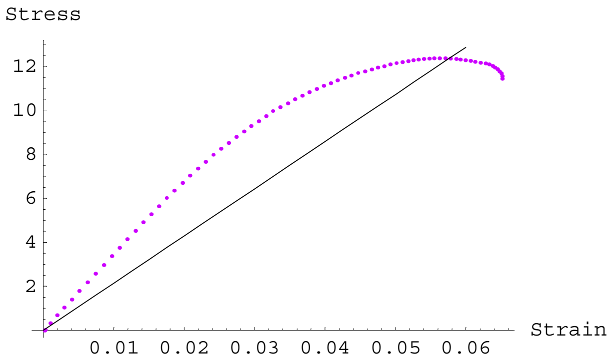
- Graphics -

```
rawdata = ListPlot[data, AxesLabel → {"Strain", "Stress"}, PlotStyle -> Hue[.8]]
```



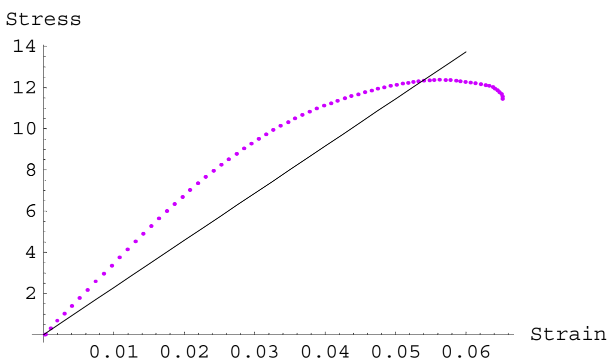
- Graphics -

```
Show[rawdata, foamfit]
```



- Graphics -

```
Show[rawdata, foamfit2]
```



- Graphics -