PDHonline Course S275 (5 PDH)

Design of Folded Plates

Instructor: Ruben A. Gomez, PE

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5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

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Design of Folded Plates

Ruben A. Gomez, P.E.

1.0 Introduction

This course describes how to proceed in the folded plate analysis and design done by hand and by using principles well known to most engineers. Although a folded plate is indeed a three-dimensional problem, we will see how to do it in a two-dimensional manner and without differential equations or calculus, just with the help of simple statics, moment distribution, trigonometry and flexural theory basic principles.

While it is quite true that there are a myriad of computer programs available to solve any folded plate condition and the only requirement for the user is to input a few design descriptive characteristics, it is not less true that engineers in all states have been warned about their ultimate responsibility as generators of construction documents, in terms of not only being responsible for all drawings they sign and seal but also to have a full knowledge of their proposals and being the full fledged authors of the documents they produce. Should something go wrong with the structural design, specifications, construction or general performance, they must prove their knowledge and familiarity with the contents of all engineering documents issued by their offices.

Just as a matter of example, let us take the case of the Florida Board of Professional Engineers which is regulated by Chapter 471 of Florida Statutes (F.S.) and the Florida Administrative Code (FAC). Article 61G15-18.011 reads that "An engineer shall only sign and seal engineering documents when he or she either personally prepared them under the engineer's responsible charge." As also stated in such Rule "an engineer who signs and seals engineering documents in responsible charge must be capable of answering questions relevant to the engineering decisions made during the engineer's work on the project, in sufficient detail as to leave little doubt as to the engineer's proficiency for the work performed". Furthermore, Rule 21H-30.008 titled "Use of Computer Software and Hardware" also reads: "The engineer shall be responsible for the results generated by any computer software and hardware that he or she uses in providing engineering services."

Those rules are very clear in stating that having the convenience of a computer does not relieve the engineer of being thoroughly familiar with the design procedures so contained. Therefore, while whether to use hand computations or the convenience of a computer is a judgment call for the engineer, but not so is the knowledge which he must have to demonstrate his proficiency on the matter at hand or under questioning.

Now that we have been reminded of those rules, let us go into the matter of the design of folded plates. Since it has been said that "a picture is worth a thousand words", we will use graphics extensively and more abundantly than usual to effectively convey the principles of analysis and design.

2.0   Generalities on Folded Plates

Folded plates as a structural form seem to have first appeared accidentally in central Europe in about 1929 and introduced to the United States of America immediately after the Second World War where it rapidly became popular and accepted as a new form of construction because of its feasibility of erection, proven performance and its structural clarity on both analysis and design. The basic structural rational is rather logical enough, straightforward and simple although its required numerical computation procedure is on average somewhat tedious.

Let us start from the beginning and make progress by going from the very simple to the less simple. First we will review the idea where it all began, an elementary 6 in. thick strip of concrete slab laying flat on its side on the ground; while in the horizontal position and perpendicular to the gravitational forces, such slab is adequate for a span say between 12 and 16 feet depending on the design loads, its ultimate capacity may vary accordingly whether it is treated as either a regular reinforced concrete (hard) slab or as a post-tensioned (soft) slab.

Should we decide to lift it up on one of its sides and hold it there at a given height and angle, its structural capacity will increase proportionally to the extent of such height. On Figure 2.1 we have illustrated an example where the concrete panel is tilted to sixty (60) inches and held in position by whatever available mechanical means of support. Although the concrete section has remained the same, its section modulus has increased tenfold. As indicated on the same figure, there is a little price to pay for such gain however, and that is a loss of some 13.3% on its coverage ability.

In the next step we are going to examine the basic and fundamental assumptions that are necessary to lay the foundation on folded plate design, in such a manner that we all are on the same basis of reasoning. Here are those assumptions:

1- The material used (concrete in our case) is homogeneous, monolithic and "elastic".

2- Longitudinal vertices are continuous and held in such a way that there is no relative rotation or sliding along the common boundary between two adjoining plates.

3- Plates have very small resistance against twisting and torsion and therefore those stresses are usually considered small and thus ignored.

4- End diaphragms or bents, if any, are provided to transmit end reactions from the plates to the columns, however, they are generally considered unable to provide restraint against rotation at the end of the plates.

5- For longitudinal spans of 50 feet and over, bridging is recommended at midpoint. For spans from 70 to 100 feet, bridging should be provided at the third points of the span. Bridging should be of not less than 4 inches in thickness and have adequate openings at the bottom of the valley to facilitate drainage.

6- The principle of *superposition* holds true and valid; that means, the structure is susceptible to being analyzed separately for its redundant forces and different load conditions, then at the end the results are algebraically added to get to the final answer.

HELPFUL ANALOGY

$$S = \frac{bd^2}{6} = \frac{Ad}{6}$$
$$M = f \cdot S = f \cdot \frac{Ad}{6}$$
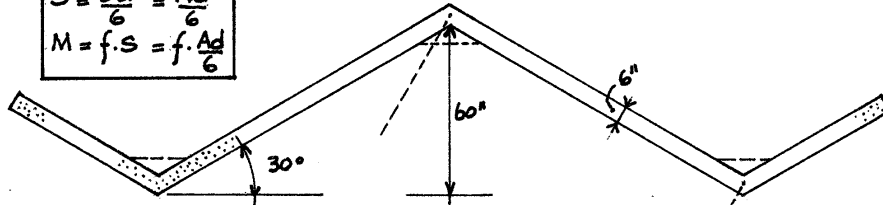
SLAB FLAT ON THE GROUND: $S = \frac{1}{6}(120 \times 6^2) = 720 \text{ in}^3$

SLAB TILTED IN PLACE: $S = \frac{1}{6}(12 \times 60^2) = 7200 \text{ in}^3$

ANALOGOUS SECTION (720 in²)
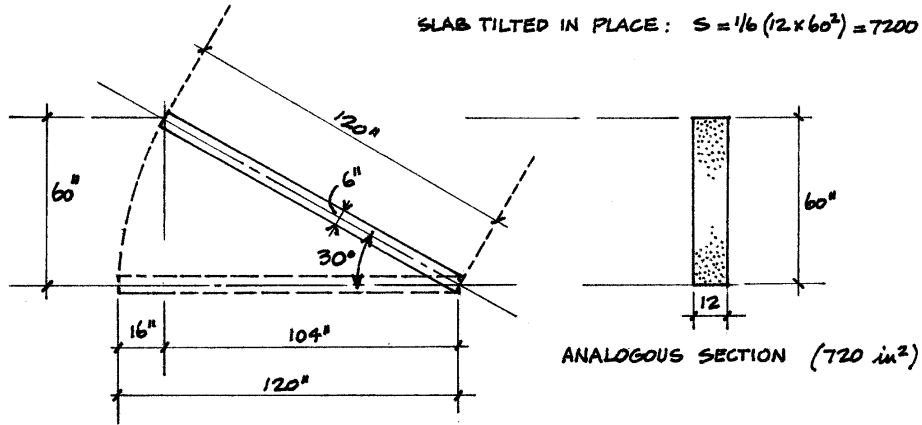
LOSS IN SLAB COVERAGE : $16 \div 120 = 0.133$

IN EXCHANGE FOR 13.3 % IN COVERING ABILITY LOSS, THE SLAB GAINED 10 TIMES ITS ORIGINAL STRENGTH.

FIGURE 2.1

3.0   Structural Idealization

In this section we will review the components of a folded plate in preparation for the analysis of a numerical example which will appear at the end of this lesson. Complete understanding of two of those components are paramount in this process, they are the *transverse slab* and the *longitudinal plate* running perpendicular to the first. Transverse moments in the slab and longitudinal flexural stresses in the plates will be combined with a merging of the shearing forces between the plates and their transmission to their end supports.

When two adjacent plates are continuous and monolithic with each other at a common edge, whether that common joint takes place at a ridge or a valley, the loads applied at the common edges, which are in fact the reactions from the transverse continuous slab separated into two components, each acting in the plane of their corresponding plates. Those components (or plate loads) in turn cause flexural stresses in the plates parallel to said common edge. Unless that common edge is contained in a vertical plane of symmetry, geometrically and loading wise, if those stresses were calculated from those loads at any given point along the common edge, they would not have the same values for the two adjacent plates. However, since the plates are monolithically connected to each other, said stresses must be the same. As result of all that, a set of horizontal shearing forces are generated and acting along the common joint between adjoining plates. Those shearing forces must be of such magnitude and direction that the longitudinal stresses along the edge are also the same when computed for each of the plates separately.

We will start with the simplest conceived form of a folded plate as shown on Figure 3.1(a) and consisting of two inclined rectangular plates built continuous along a common edge (ridge) and connected at the ends by means of either two triangular end plates or a tension rod to take care of the resulting thrust at the supports. A column is provided at each of the four corners to transmit the total forces to the foundation system.

On Figure 3.1(b) we have loaded the structure with a vertical concentrated load on the ridge at mid distance, as a result of that load P the structure will develop a deflection (Δ) on the ridge at the middle of the span which can be determined by using a Williot diagram (for more details on that procedure please see Appendix IV). Since the structure is formed by two identical plates ABEF and BCDE and due to their flexural deformations, they will also deflect in two equal amounts along their own planes.

As the concentrated load P is applied on the center of the ridge, it splits into two equal components P_1 and P_2 in the inclined direction of the two adjacent slabs as it is shown on Figure 3.1(c).

Figure 3.1(d) is a lateral view of the so conceived structure showing the concentrated load on the ridge and the resulting total deflection (or sag).

Figure 3.2(b) depicts the two plates taken apart and under the action of their share of the concentrated load applied at midspan on the ridge and with the resulting deflection δ.

Joint rotations and translations can be plotted accordingly with the help of the Williot Diagram as shown on Figure 3.2(a).
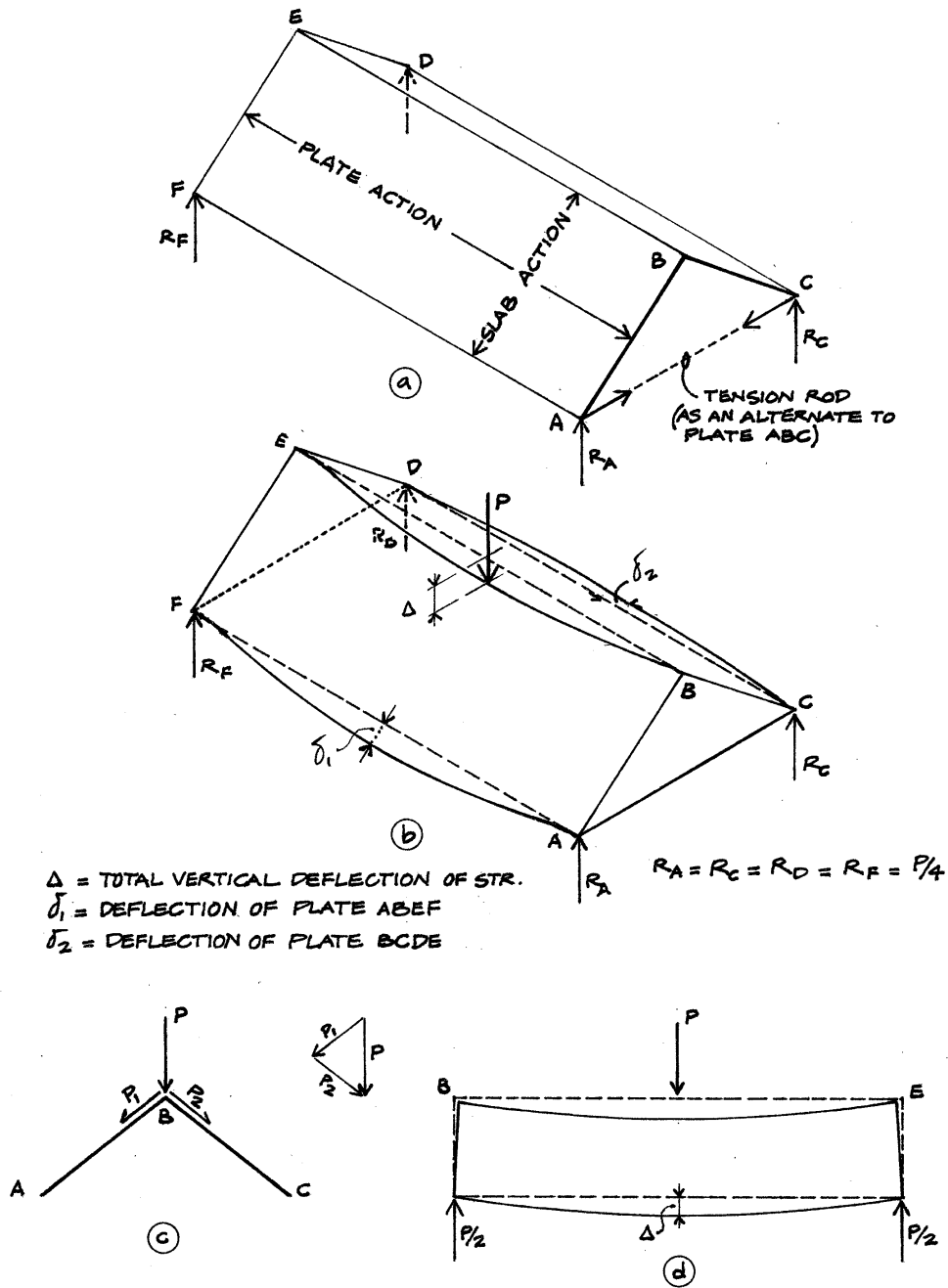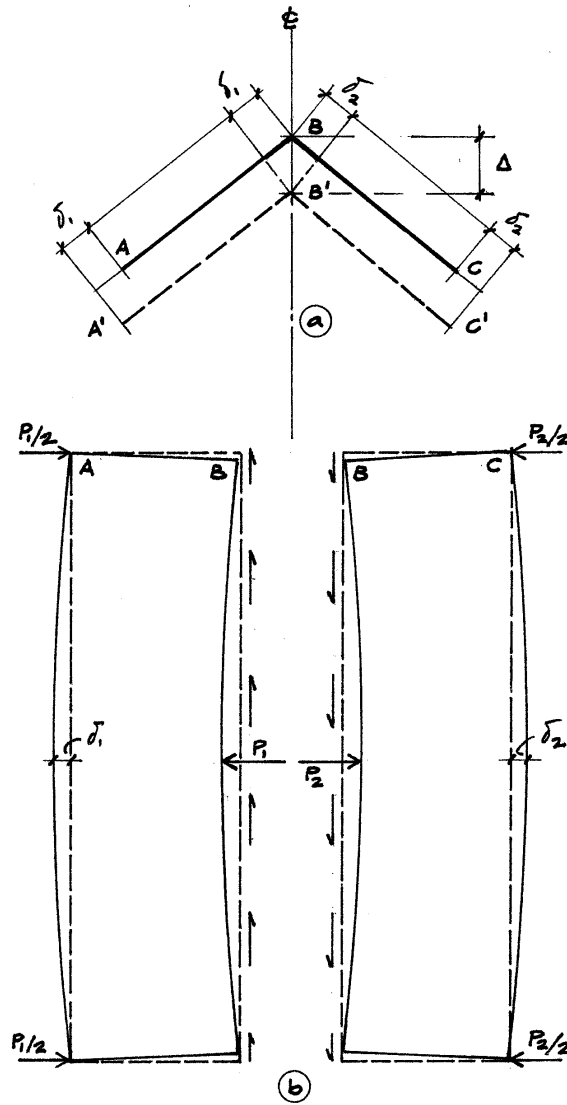

FIGURE 3.1

DEFLECTIONS EXAGGERATED FOR CLARITY.

FIGURE 3.2

4.0   Procedural Steps of Analysis

The following are the ten (10) recommended steps that comprise the entire procedure for

the analysis and design of folded plates. As you read them also examine the details on Figure 4.1 (a through e).

Step #1 (Figure 4.1a)
Consider an imaginary one-foot strip cut along the center line of the proposed folded plate. Calculate one-way slab moments (in foot-pounds) as it were fully supported at every ridge and valley. Balance all unequal fixed-end-moments (FEM's) by using the moment distribution method.

Step #2 (Figure 4.1b)
Determine plate loads in the plane of the plates (in pounds per linear foot) as resulting from slab loads.

Step #3 (Figure 4.1c)
Consider plates disconnected at ridges and valleys, find extreme fiber stresses (in pounds per square inch) at the center line of plate span.

Step #4
Connect plates together again. Balance unequal stresses (PSI) at joints by using the stress distribution method (similar to moment distribution).

Step #5 (Figure 4.1d)
Consider plates disconnected once more, calculate deflections based on the distributed extreme fiber stresses. Those deflections must be on the plane of the plate.

Step #6 (Figure 4.1e)
Plot the deflected positions of the plate vertices (joints) at the centerline of the plate span.

Step #7
Scale deflection magnitudes from the Williot Diagram. Deflections are to be translated perpendicular to the plate.

Step #8
As in steps #5, 6 and 7, the slab transverse bending strength was neglected. Deduct the FEM's from the deflections perpendicular to the plates in step #7. Those moments will tend to be unequal and large.

Step #9
Apply the above corrected FEM's to the transverse section as it was done with the gravity loads in step #1. Repeat steps 1 through 8 and continue the cycles until the corrections converge.

Step #10
Determine and summarize all moments and shears. Calculate all required reinforcing steel accordingly.

FIGURE 4.1
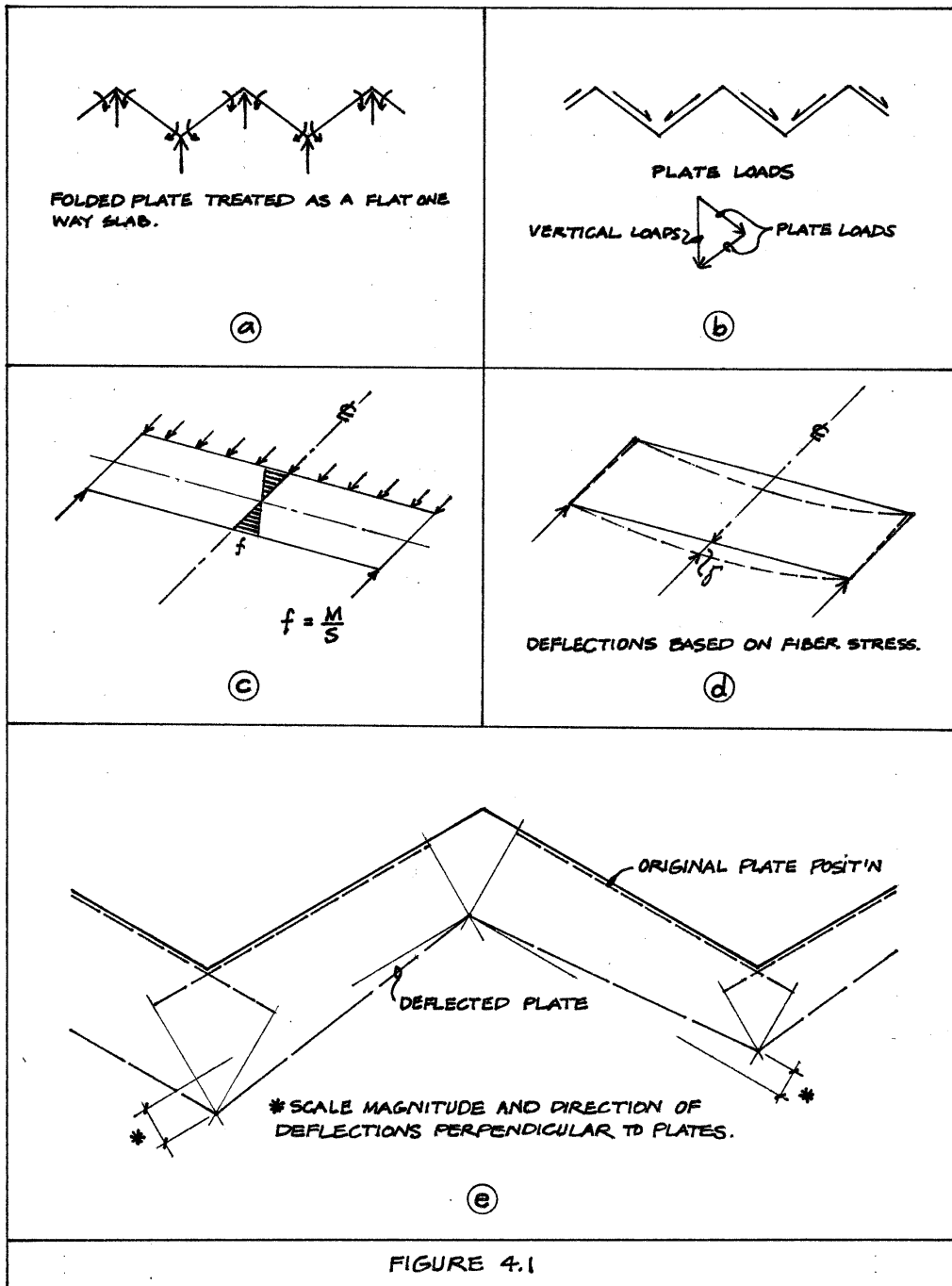
5.0   Formulary Derivation & References

In this section you will find all the formulae needed in the design of folded plates, the reason of their applicability, how they were derived as presented and/or whatever source they came from.

First we will examine the concept of stress distribution as a parallel to Hardy Cross' Moment Distribution Method, for that purpose please follow the sequence illustrated on Figure 5.1.

Shear stresses are an important part of folded plate design, especially edge shears which are the result of unequal free edge stresses commonly present at the joint of two adjacent plates. Those shears are illustrated on Figure 5.2. It is interesting to notice on that same figure that all shears, whether transverse, longitudinal or edge shears decrease to a zero value at the centerline and increase to their maximum values towards the far ends of the plate.

On Figure 5.3 we have derived plate deflection equations by taking the common beam deflections under uniform load from the tables of the Concrete Reinforcing Steel Institute handbook and merging them with the beam's typical extreme fiber stresses, as well as from the Portland Cement Association's values for the ½ sine wave loads and operating in the same explained manner.

Figure 5.4 covers the effect of loads caused by plate deflections in general, such as:

a. plate deflections on their own plane,
b. deflections perpendicular to the plate's plane,
c. corrective fixed-end-moments (FEM's), and
d. plate loads due to corrective FEM's.

All of them have something in common; they follow the configuration of the deflection curve, attain their maximum value at the center line of the plate span and diminish to a zero value at the supports. Therefore, it would be safe to affirm that the corrective plate loads have a distribution pattern following the deflection curve rather than a uniform distribution, as in the case of the gravity loads. In the same manner, the deflection curve can be closely approximated to the configuration of the ½ sine wave curve as suggested on the figures.

Figure 5.4 also depicts a method to obtain the loads generated by plate deflections by establishing a comparison and relationship of moments induced by uniform loads and ½ sine wave loads. In the same manner, Figure 5.5 shows the way to obtain the modified plate load corrections by the same relationship between the end shears of the uniform loads and ½ sine wave loads. Finally, on Figure 5.6 it is shown the fixed-end-moments generated by plate deflections. Emphasis must be placed on two important points:

#1- Since in practice real deflections are too small to be measurable, therefore their scale

need to be exaggerated as much as possible to get the proper visual configurations.

#2- All deflections must be drawn and measured taken perpendicular to the original (before deflection) plate position.

FIGURE 5.1

VARIABLE SHEAR
WITH ZERO VALUE AT
℄ OF PLATE SPAN.
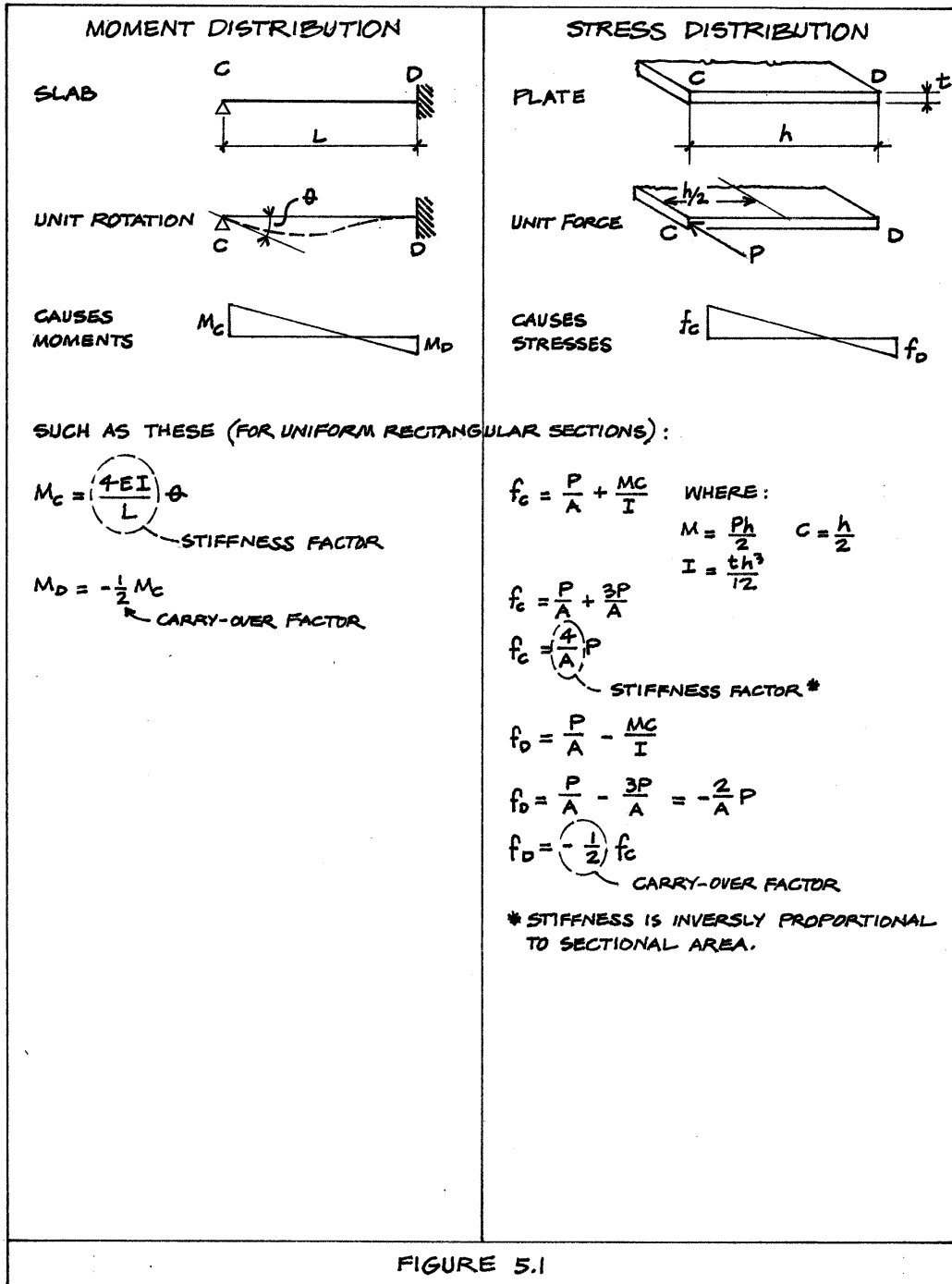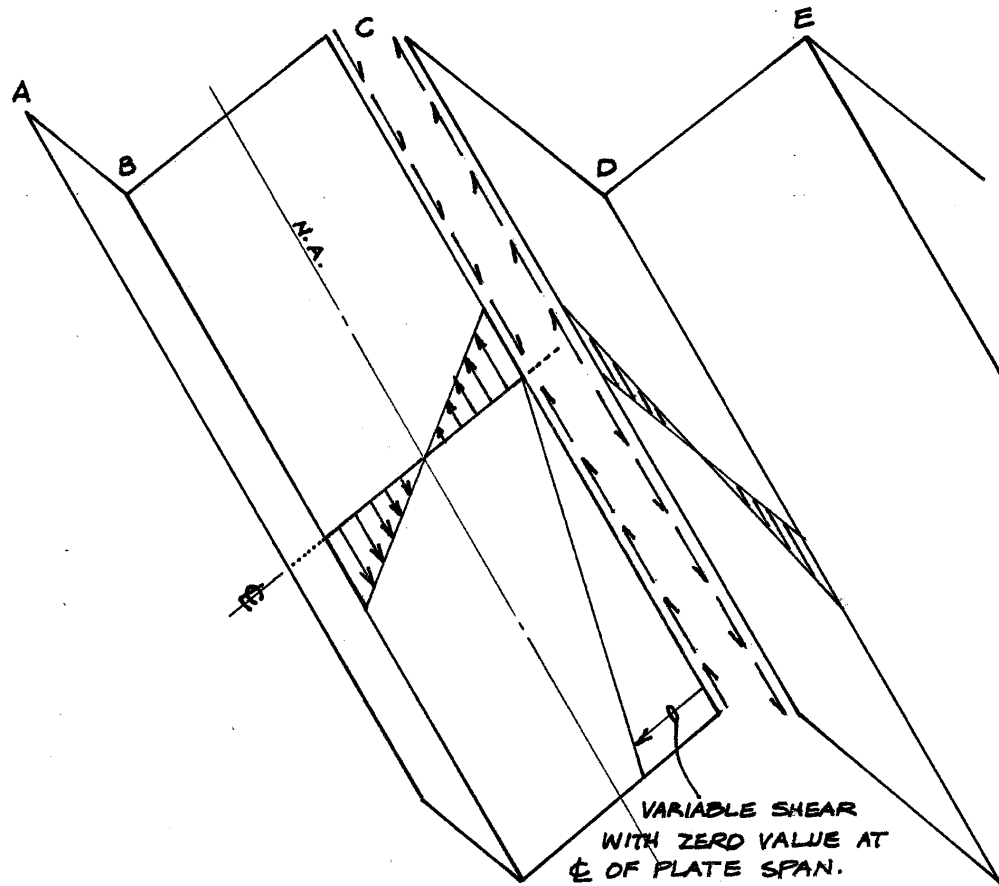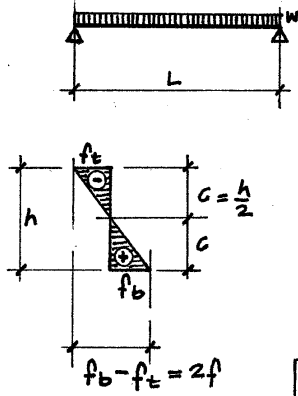
STRESS DISTRIBUTION (CONTINUED)
SHEAR STRESSES ON FOLDED PLATES
FIGURE 5.2

REF.: CRSI HANDBOOK
BEAM DIAGRAMS & FORMULAS

$$\begin{cases} M_{\mathfrak{C}} = \frac{1}{8} w \cdot L^2 \\ \Delta_{\mathfrak{C}} = \frac{5}{384} \left(\frac{wL^4}{EI} \right) \end{cases}$$

$$f = \frac{Mc}{I}$$

$$2f = 2\frac{Mc}{I} = \frac{2}{I} \cdot \left(\frac{wL^2}{8} \right) \left(\frac{h}{2} \right) = \left(f_b - f_t \right) = \frac{wL^2 h}{8I}$$
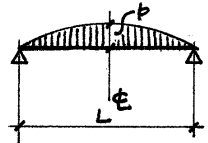
$$\Delta_{\mathfrak{C}} = \frac{5}{384} \left(\frac{wL^4}{EI} \right) = \frac{5}{384} \left(\frac{wL^2 h}{8I} \right) \left(\frac{L^2}{Eh} \right) 8 =$$

$$= \frac{5}{384} \left(f_b - f_t \right) \left(\frac{8L^2}{Eh} \right)$$

$$f_b - f_t = 2f$$

$$\boxed{\Delta_{\mathfrak{C}} = \frac{5}{48} \left(\frac{L^2}{Eh} \right) \left(f_b - f_t \right)} \longleftarrow \text{FOR UNIFORM LOAD}$$

REF.: PCA ST77 P.6
½ SINE WAVE LOADING

$$\begin{cases} M_{\mathfrak{C}} = \frac{pL^2}{\pi^2} \\ \Delta_{\mathfrak{C}} = \frac{pL^4}{EI\pi^4} \end{cases}$$

$$f = \frac{Mc}{I}$$

IN THE SAME MANNER AS ABOVE :

$$2f = \left(f_b - f_t \right) = 2\frac{Mc}{I} =$$

$$= \frac{2}{I} \left(\frac{pL^2}{\pi^2} \right) \left(\frac{h}{2} \right) = \frac{pL^2 h}{I\pi^2}$$

$$\Delta_{\mathfrak{C}} = \frac{pL^4}{EI\pi^4} = \left(\frac{pL^2 h}{I\pi^2} \right) \left(\frac{L^2}{E\pi^2 h} \right) \quad \text{THEREFORE :}$$

$$\boxed{\Delta_{\mathfrak{C}} = \frac{L^2}{E\pi^2 h} \left(f_b - f_t \right)} \longleftarrow \text{FOR ½ SINE WAVE LOAD}$$

SIMPLE SPAN PLATE DEFLECTION EQUATIONS

FIGURE 5.3

UNIFORM LOAD

$M_{UNIF} = 1/8 \, w L^2$

WHEN $w = p$

$9.87 \, M_{SIN} = 8 \, M_{UNIF}$

THUS: $M_{SIN} = \dfrac{8}{9.87} M_{UNIF}$

1/2 SINE WAVE LOAD

P IN PLF @ ¢ OF SPAN.

$M_{SIN} = \dfrac{pL^2}{\pi^2} \cdot \dfrac{\sin \pi x}{L}$ AT $x = 1/2 L$:

$M_{SIN} = \dfrac{pL^2}{\pi^2} \cdot \sin\left(\dfrac{\pi}{L} \cdot \dfrac{L}{2}\right)$

$\sin\left(\dfrac{\pi}{L} \cdot \dfrac{L}{2}\right) = \sin \dfrac{\pi}{2} = \sin 90° = 1$

$M_{SIN} = \dfrac{pL^2}{\pi^2} = \dfrac{pL^2}{9.87}$

LOADS CAUSED BY PLATE DEFLECTIONS
FIGURE 5.4

END SHEAR: $V = \dfrac{wL}{2}$

PCA ST77 p.6

END SHEAR: $V = \dfrac{pL}{\pi}$

WHEN: $w = p$ THEN,

$V_{SIN} = \dfrac{1/\pi}{1/2} V_{UNIF} = 0.637 \, V_{UNIF}$

MODIFIED PLATE LOAD CORRECTIONS
FIGURE 5.5

END PLATE:

CRSI HANDBOOK

$FEM = \dfrac{3EI\Delta}{L^2}$

$FEM = \dfrac{3EI\Delta}{h^2}$

* Δ MEASURED NORMAL TO ORIGINAL PLATE POS.

INTERIOR PLATES:

$FEM = \dfrac{6EI\Delta}{L^2}$

$FEM = \dfrac{6EI\Delta}{h^2}$

FEM's GENERATED BY DEFLECTIONS NORMAL TO PLATES
FIGURE 5.6

6.0   Preparatory Work Previous to Analysis

Figure 6.1 illustrates the necessary preparation work that is necessary previous to start the numerical work on a folded plate design. The upper figure shows a partial view of a cross section of the typical structure. Given consideration to the longitudinal conception please notice that folded plate BCD is equivalent to a rectangular concrete section with a base equal to **2a** and a depth **H**. On the same sheet it is shown a calculation for the applicable gravity loads which have been conveniently adjusted for the rising angle.

At the bottom of the page we have shown by using the moment differential on every slab, how to obtain the generated shears which need to be algebraically added to the gravity load shears to obtain the resulting value.

There are other considerations that the design engineer must bear in mind as part of his **notion of the result** anticipation to his particular case, they may or may not be applicable but are worth mentioning:

#1- In our included numerical example we will be assuming that the folded plate structure is being supported by columns, however, if in your practice it happens to lean on bearing walls as in the case of the typical *saw-tooth* warehouse roof, it will affect the moment distribution and even some positive moments are likely to be found at the first and second joints.

#2- Although the maximum negative moment at support B may be many times larger than the normal and customary $0.083wL^2$, in a larger structure by the time you reach the 4th or 5th bay, it will likely be getting close to that value.

GRAVITY LOADS:
DEAD LOAD : 3" SLAB   37.5 PSF
SUPERIMPOSED LOADS   40.0
　　　　　　　TOTAL   77.5 PSF

$77.5 \div \cos\theta = 77.5 \div 0.80 \cong 97$ PSF

DEAD LOAD : 6" SLAB   75.0 PSF
SUPERIMPOSED LOADS   40.0
　　　　　　　TOTAL :   115.0 PSF

$115 \div 0.80 =$ 144.0 "

$M_B = 1/2 \times 144 \times 2.50^2 = 450$ LB-FT
$M_C = 136$ LB-FT
DIFF. MOMENT $M_{DIFF} = 314$ LB-FT
PRODUCES END REACTIONS:
$314 \div 5.0 = 63$ LBS.
PLUS:
END REACTIONS (SHEARS) DUE TO GRAVITY LOADS:
$97 \times 5/2 = 243$ LBS.

WHICH GIVES US THE TOTAL END SHEARS.

PREPARATION PRIOR TO ANALYSIS
FIGURE 6.1

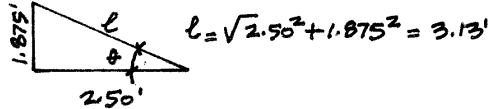FOLDED PLATE NUMERICAL EXAMPLE   CALC. SHEET #1 OF 8

CONSIDERING ONE FOOT STRIP OF SLAB :

$$M_B = \frac{1}{2} wL^2 = \frac{1}{2} \times 144 \times 2.50^2 = 450 \text{ LB-FT}$$

$$M_n = \frac{1}{12} wL^2 = \frac{1}{12} \times 97 \times 6.25^2 = 316 \text{ LB-FT}$$
$$\qquad\qquad 5.00^2 \qquad 202$$

$$\ell = \sqrt{2.50^2 + 1.875^2} = 3.13'$$

SECTIONAL AREAS :

$$AB = 0.50 \times 3.13 = 1.57 \text{ FT}^2$$
$$BC = 0.25 \times 6.25 = 1.56 \text{ "}$$
$$\Sigma = 3.13 \text{ FT}^2$$

DISTRIBUTION FACTORS FOR FREE-EDGE STRESSES :

$1.57 \div 3.13 = 0.50$ FOR BOTH $\underline{BA}$ AND $\underline{BC}$.

SECTION MODULUS :

$$S_{AB} = \frac{1}{6} td^2 = \frac{1}{6} \times 0.50' \times 3.13^2 = 0.82 \text{ FT}^3$$

$$S_{BC} = \frac{1}{6} \times 0.25' \times 6.25^2 = 1.63 \text{ FT}^3$$

$$R = \frac{S}{2} \cdot \frac{1}{\sin\theta} = \frac{S}{2 \times 0.60} = 0.833 S$$
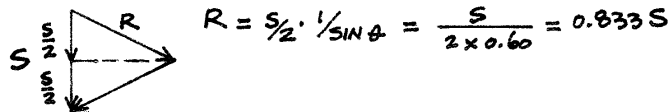
FREE-EDGE STRESSES : $f = \frac{RL^2}{8} \cdot \frac{1}{S}$ (LONG DIRECTION)

PLATE AB : $\frac{R \, 40^2}{8 \, (144)} \times \frac{1}{.82} = 1.695 R$

PLATE BC : $1.389 R \times \frac{1}{1.63} = 0.852 R$

PLATE DEFLECTIONS : $\delta = \frac{5L^2}{48 E.h} (f_b - f_t)$

$$\delta_{AB} = \frac{5 \times 40^2 \times 12}{48 \, E \, 3.13} (f_b - f_t) = 0.0639 \times \frac{10^4}{E} (f_b - f_t)$$

$$\delta_{BC} = \frac{5 \times 40^2 \times 12}{48 \, E \, 6.25} (f_b - f_t) = 0.0320 \times \frac{10^4}{E} (f_b - f_t)$$

FEM's : $I_{SLAB} = \frac{bd^3}{12} = \frac{1}{12} (12 \times 3^3) = 27 \text{ in}^4$

$$BC = \frac{3EI\Delta}{h^2} = \frac{3E \, 27\Delta}{12 \times 144 \times 6.25^2} = \frac{1200 \, E\Delta}{10^6}$$

$$CD = \frac{6EI\Delta}{h^2} = \frac{2400 \, E\Delta}{10^6}$$

SHEET #2 OF 8

144 PSF   97 PSF

3.13'   3"   6.25'

6"   3.75'

2.5'   5.0'   5.0'   5.0'   5.0'

0	1		½	½		½	½		½	½		0	DISTRIB. FACTOR
+450	-202		+202	-202		+202	-202		+202	-202		+202	F.E.M. (LB-FT)
0	-248		0	0		0	0		0	0		0	D
0	0		-124	0		0	0		0	0		0	C.O.
0	0		+62	+62		0	0		0	0		0	D
0	+31		0	0		+31	0		0	0		0	C.O.
0	-31		0	0		-15	-15		0	0		0	D
0	0		-15	-7		0	0		-7	0		0	C.O.
0	0		+11	+11		0	0		+3	+3		0	D
D	+5		0	0		+5	+1		0	0		+1	C.O.
0	-5		0	0		-3	-3		0	0		0	D
0	0		-2	-1		0	0		-1	0		0	C.O.
			+1	+1		0	0						D
+450	-450		+136	-136		+222	-222		+198	-198		+203	DISTRIB'D MOMENTS

DIFF. MOM.
-314   +86   -24   +5

÷
5.0   5.0   5.0   5.0

↓63   63↑   ↑17   17↓   ↓5   5↑   ↑1   1↓

GRAVITY LOADS
360↓   ↓243   243↓   ↓243   243↓   ↓243   243↓   ↓243   243↓

TOTAL REACTIONS
666   406   508   480   488

338   338   400   400
555   555   893   761   423   423   823   807   407   407

R (LBS)
555

MULTIPLIER
1.695   0.852   0.852   0.852   0.852

FREE-EDGE STR. (PSI)
941   761   648   701   688

SHEET #3 OF 8

	A	B		C		D		E		F	
DISTR. FACTOR	0	½	½	½	½	½	½	½	½	0	
F.E.S. (PSI)	+941	+941	−761	−761	+648	+648	−701	−701	+688	+688	
D	0	−90	−90	+56	+56	+26	+26	+13	+13	0	
C.O.	−45	0	+28	−45	+13	+28	+6	+13	0	+6	
D	0	−14	−14	+16	+16	−17	−17	−6	−6	0	
C.O.	−7	0	+8	−7	−8	+8	−3	−8	0	−3	
D	0	−4	−4	+7	+7	−2	−2	+4	+4	0	
C.O.	−2	0	+3	−2	−1	+3	+2	−1	0	+2	
D		−1	−1	+1	+1		−2	−2	0	0	0
									−1		
DISTR'D STRESS	+887	+832	−831	−735	+731	+692	−691	−687	+699	+693	

$(f_b - f_t)$

+1719	−1566	+1423	−1378	+1392	MULTIPLIER
$.0639 \cdot \frac{10^4}{E}$	$.0320 \cdot \frac{10^4}{E}$	$.0320 \cdot \frac{10^4}{E}$	$.0320 \cdot \frac{10^4}{E}$	$.0320 \cdot \frac{10^4}{E}$	

DEFLECTIONS (δ)

$1.098 \frac{10^6}{E}$	$0.501 \frac{10^6}{E}$	$0.455 \frac{10^6}{E}$	$0.441 \frac{10^6}{E}$	$0.445 \frac{10^6}{E}$	
	$0.796 \frac{10^6}{E}$	$0.184 \frac{10^6}{E}$	$0.028 \frac{10^6}{E}$	$0.012 \frac{10^6}{E}$	WILLIOT DIAGRAM
	$1200 \frac{E}{10^6}$	$2400 \frac{E}{10^6}$	$2400 \frac{E}{10^6}$	$2400 \frac{E}{10^6}$	MULTIPLIER
	955	442	67	29	FEM (LB-FT)

	A	B		C		D		E		F	
DISTRIB. FACTOR	0	1	½	½	½	½	½	½	½	0	
FEM		0	+955	+442	+442	+67	+67	+29	+29		
D		0	−698	−698	−254	−254	−48	−48	0		
C.O	−349	0	−127	−349	−24	−127	0	−24			
D	+349	+64	+64	+186	+186	+64	+64	0			
C.O.	+32	+174	+93	+32	+32	+93	0	+32			
D	−32	−134	−134	−32	−32	−46	−46	0			
	·	·	·	·	·	·	·	·	·		
	·	·	·	·	·	·	·	·	·		
	·	·	·	·	·	·	·	·	·		
DISTR. MOM.	0	+453	−453	−219	+219	+32	−32	−12			

+453	−672	+251	−44	RESULTANT
5.0	5.0	5.0	5.0	LEVER ARM

↑91	91↓	↓134	134↑	↑50	50↓	↓9	9↑	REACTION

Σ SHEARS

1.374	0.691	0.691	0.691	0.691	MULTIPLIER
104 C	77 C	23 C	72 C	23 C	STRESS (PSI)

SHEET #4 OF 8

MODIFY FREE-EDGE-STRESS CONSTANTS TO ADJUST FOR ½ SINE WAVE LOAD:
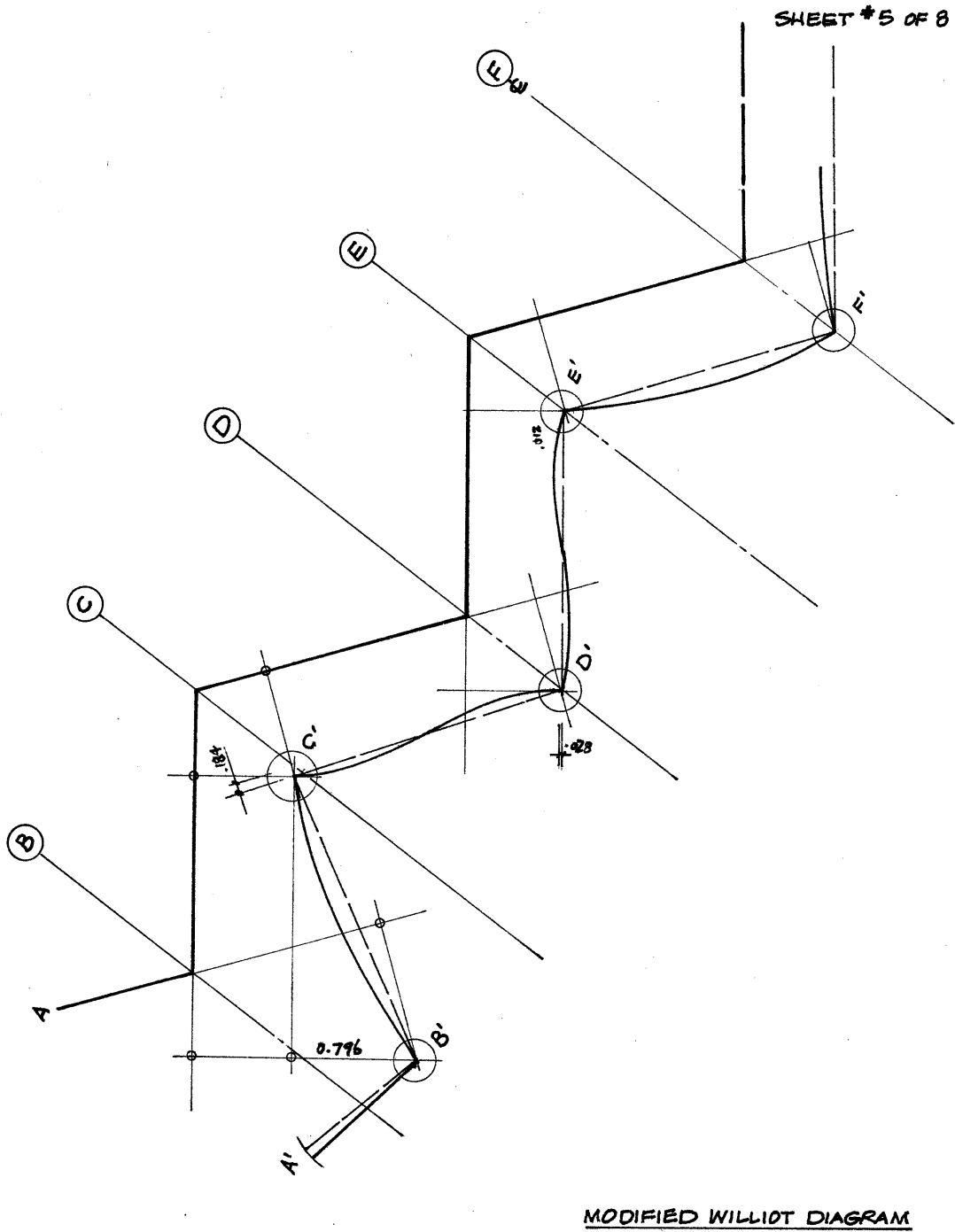
PLATE AB: $1.695 \times \dfrac{8}{9.87} = 1.374$

PLATE BC: $0.852 \times \dfrac{8}{9.87} = 0.691$

BACK TO THE ½ SINE WAVE LOADING: $L = 40'$; $h = \begin{cases} 3.13' \ (AB) \\ 6.25' \ (BC) \end{cases}$

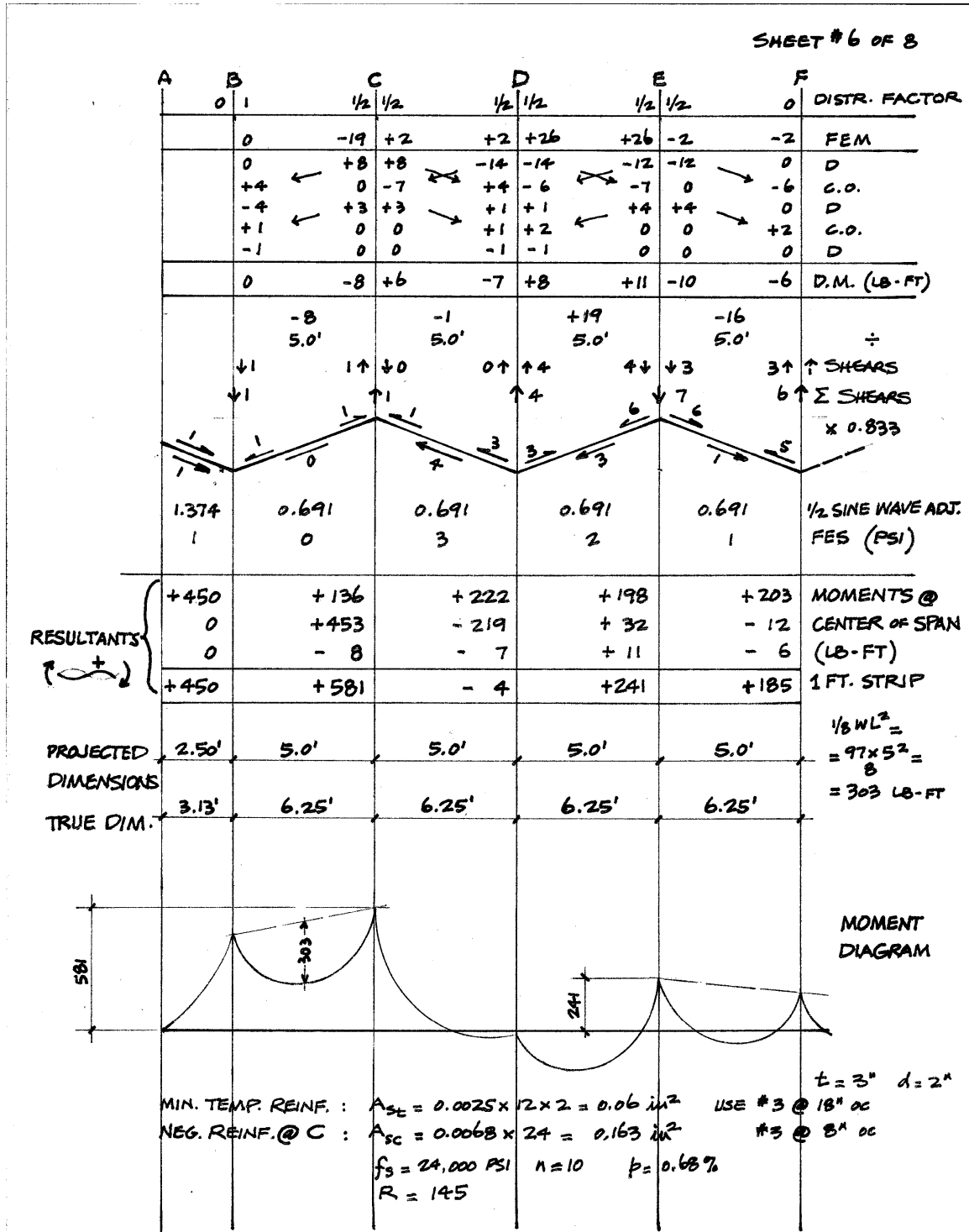$$\delta_{AB} = \frac{L^2}{E\pi^2 h}(f_b - f_t) = \frac{40^2 \times 12}{(9.87 \times 3.13)E}(f_b - f_t) =$$

$$= \boxed{0.0622 \left(\frac{10^4}{E}\right)(f_b - f_t)}$$

$$\delta_{BC} = \frac{19,200}{(9.87 \times 6.25)E}(f_b - f_t) = \boxed{0.0311 \left(\frac{10^4}{E}\right)(f_b - f_t)}$$

	A	B		C		D		E		F	
0		½	½		½	½		½	½		0	DISTR. FACTOR
-104		-104	-77		-77	+23		+23	+72		+72	+23	+23	FES (PSI)
0		+90	+90		+27	+27		-48	-48		-48	-48	0	D
+45		0	+13		+45	-24		+13	-24		-24	0	-24	C.O.
0		-6	-6		-10	-10		+6	+6		+12	+12	0	D
-3		0	-5		-3	+3		-5	+6		+3	0	+6	C.O.
0		+2	+2		0	0		0	0		-1	-1	0	D
+1		0	0		+1	0		0	0		0	0	0	C.O.
-61		-18	+17		-19	+19		-11	+12		-14	-14	+7	(PSI) DIST.STRESS
-79		-2		+8		-2		-7			$(f_b - f_t)$
$0.0622\left(\frac{10^4}{E}\right)$		$0.0311\left(\frac{10^4}{E}\right)$		$0.0311\left(\frac{10^4}{E}\right)$		$0.0311\left(\frac{10^4}{E}\right)$		$0.0311\left(\frac{10^4}{E}\right)$		MULTIPLIER
$.0049\left(\frac{10^6}{E}\right)$		$0.0006\left(\frac{10^6}{E}\right)$		$0.0025\left(\frac{10^6}{E}\right)$		$0.0006\left(\frac{10^6}{E}\right)$		$0.0022\left(\frac{10^6}{E}\right)$		δ
		$0.016\frac{10^6}{E}$		0.0009		$0.011\frac{10^6}{E}$		0.0009		WILLIOT DIAGRAM
		$1200\frac{E}{10^6}$		$2400\frac{E}{10^6}$		$2400\frac{E}{10^6}$		$2400\frac{E}{10^6}$		MULTIPLIER
		19		2		26		2		FEM (LB-FT)

SHEET #5 OF 8

MODIFIED WILLIOT DIAGRAM

SHEET #6 OF 8

	A	B		C		D		E		F	
DISTR. FACTOR	0	1	½	½	½	½	½	½	½	0	
FEM	0	−19	+2	+2	+26	+26	−2	−2			
D	0	+8	+8	−14	−14	−12	−12	0			
C.O.	+4	0	−7	+4	−6	−7	0	−6			
D	−4	+3	+3	+1	+1	+4	+4	0			
C.O.	+1	0	0	+1	+2	0	0	+2			
D	−1	0	0	−1	−1	0	0	0			
D.M. (LB·FT)	0	−8	+6	−7	+8	+11	−10	−6			

÷ −8 / 5.0'   −1 / 5.0'   +19 / 5.0'   −16 / 5.0'

SHEARS: ↓1 1↑ ↓0 0↑ ↑4 4↓ ↓3 3↑ ↑

Σ SHEARS: ↓1 ↑1 ↑4 ↓7 6↑

× 0.833

½ SINE WAVE ADJ.: 1.374   0.691   0.691   0.691   0.691

FES (PSI): 1   0   3   2   1

RESULTANTS { ⌣ + }

MOMENTS @ CENTER OF SPAN (LB·FT):

	+450	+136	+222	+198	+203
	0	+453	−219	+32	−12
	0	−8	−7	+11	−6
1 FT. STRIP	+450	+581	−4	+241	+185

⅛ WL² = = 97×5² / 8 = = 303 LB·FT

PROJECTED DIMENSIONS: 2.50' 5.0' 5.0' 5.0' 5.0'

TRUE DIM.: 3.13' 6.25' 6.25' 6.25' 6.25'

581   303   24H

MOMENT DIAGRAM

MIN. TEMP. REINF. : A_st = 0.0025 × 12 × 2 = 0.06 in²   USE #3 @ 18" OC

NEG. REINF.@ C : A_sc = 0.0068 × 24 = 0.163 in²   #3 @ 8" OC

f_s = 24,000 PSI   n = 10   p = 0.68%

R = 145

t = 3"   d = 2"

NEG. REINF. @ B :   M = 450 LB-FT   R = 112.50
$p = 0.0052$ $n = 10$
$A_s = .0052 \times 24 = 0.125$ #3 @ 11" oc.

NEG. REINF. @ D, E & F :   #3 @ 11" oc.

POS. REINF. :   #3 @ 12" oc   (TYP.)

FREE-EDGE STRESSES :   T = TENSION
                                                        C = COMPR.

A	B	C	D	E	F	
887C	832T	735C	692T	699C	693T	(PSI)
61T	18C	19C	12C	14T	7C	
826C	814T	754C	680T	685C	686T	RESULTANT

PLATE AB
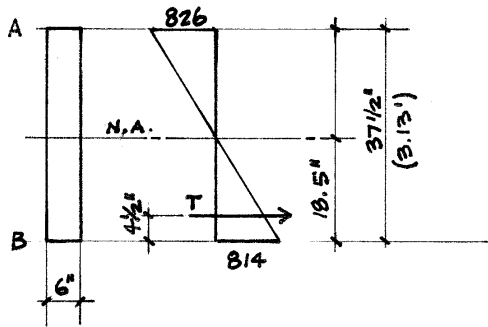
$T = \dfrac{6 \times 18.5 \times 814}{2} = 45{,}177 \text{ LBS}$

$A_s = \dfrac{45{,}177}{24{,}000} = 1.88 \text{ in}^2 \sim (6\#5)$

DIAGONAL TENSION :

$v = \dfrac{45{,}177}{0.50 \times 6 \times 240} = 62.75 \text{ PSI}$ OK

PLATE BC
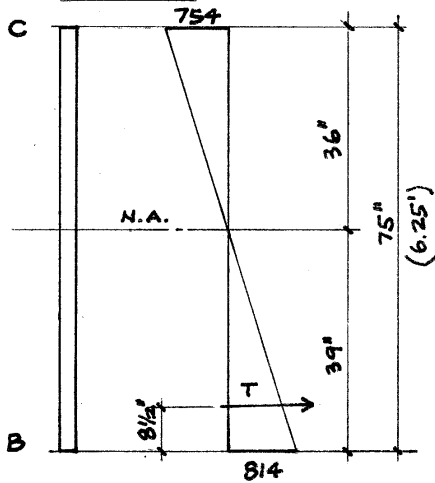
$T = \dfrac{3 \times 39 \times 814}{2} = 47{,}619 \text{ LBS}$

$A_s = \dfrac{47{,}619}{24{,}000} = 1.984 \text{ in}^2 \; (7\#5)$

$v = \dfrac{47{,}619}{0.50 \times 3 \times 240} = 132 \text{ PSI} > 90$

                                USE 4#5 DIAG. BARS

ALSO, ADD VERT. REINF. @ 12" oc. (#3).

PLATE LOADS:

A	B		C	D		E	F	
↘555	↙ 893		↘ 761	↙ 823		↘ 807		PLF
↖ 76	↙ 111		↘ 34	↗ 104		↘ 34		
↘ 1	0		↖ 3	↙ 2		↘ 1		
↘480	↙1004		↘ 792	↙ 721		↘ 842		
384	803		634	577		674		× cos θ
→	←		→	←		→	←	
	419			57			0	RESULTANT
	←			→				
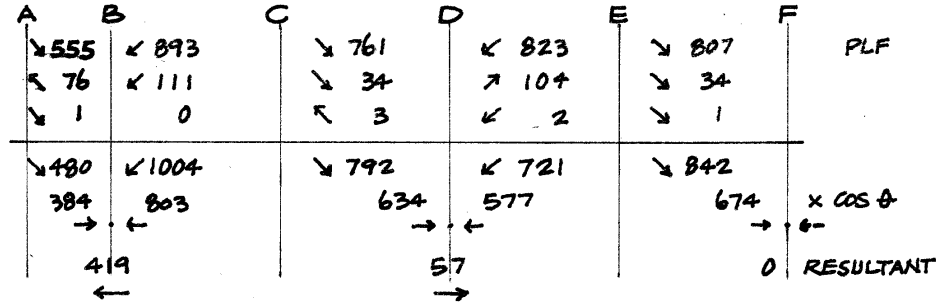
TOTAL THRUST:

MAXIMUM @ COLUMN B: $419 \times \dfrac{40}{2} = 8{,}380$ LBS.   USE 3/4" ROD

TOTAL VERTICAL LOAD:

MAXIMUM @ COLUMN F:
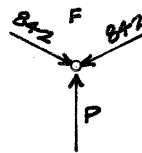
$P = (2 \times 842 \times 20) \sin\theta = 20{,}208$ LBS.

END CONDITION @ EDGE OF PLATE BC (SEE COURSE'S TEXT).

7.0   Numerical Example

We have chosen as a matter of design example, a case at the top of the list on the table of Appendix I. It consists of a four-bay folded plate structure carried by ten (10) concrete columns originally intended to cover a parking space for eight standard size vehicles with access from both ends.

The center-to-center longitudinal span is forty (40) feet and in the transverse direction the columns are spaced at ten (10) feet on centers. The rise of the folded plate is three feet nine inches (3'- 9") at the peak. The slab has been assumed as three (3) inches thick, except the end wings which are six (6) inches in thickness.

A set of typical three-cycle calculations have been included as a matter of an example which may guide the designer through the maze and help to acquire a good and logical understanding of the procedure, even if his final decision is to ultimately use a computer program as a way to accomplish the task.

On sheet #1 we show the necessary parameters such as, sectional areas, distribution factors, section modulus, deflections and fixed-end-moments where the applicable values have been substituted in the terms of the already familiar formulae to the reader.

Sheet #2 takes us through the first cycle of the transverse moment distribution and the determination of the resulting free-edge stresses. In the same manner, sheets #3 and #4 show the subsequent cycles as the process of numerical convergence takes place.

The Williot Diagram shown on sheet #5 was originally prepared at a larger scale for the practical purpose of scaling the deflections and was later reduced to meet publication constraints.

Sheet #6 shows the end of the cycles and the beginning of the necessary recapitulations, as well as the resulting moment diagram and the transverse slab reinforcement which continues on sheet #7 and end up with the sizing of the longitudinal reinforcement for the critical plates AB and BC. At the bottom of the sheet we added a notation warning about large shear forces and the corresponding diagonal tension, for which plate BC will have to be protected against by introducing bent-up diagonal bars and some additional vertical bars as indicated. Figure 7.1 shows the best way to do so without incurring in larger than necessary expenses.

Sheet #8 has a summary of the plate loads which are helpful in determining column loadings and thrusts. In this case we found that the central columns at grid F where the axis of symmetry is drawn, both columns would carry the largest vertical load of 20,208 pounds. In the same manner, the columns on grid B will sustain a thrust of the order of 8,380 pounds requiring the addition of a ¾ in. tension rod to overcome such a force.

The analysis also brought up the fact that the compression forces on the edge of plate BC are beyond the load-carrying capacity of a three-inch slab, for which it would be

necessary to thicken the first and last strips of such slab to double their size, as it is suggested on the Alternate I as part of the enclosed Figure 7.2.
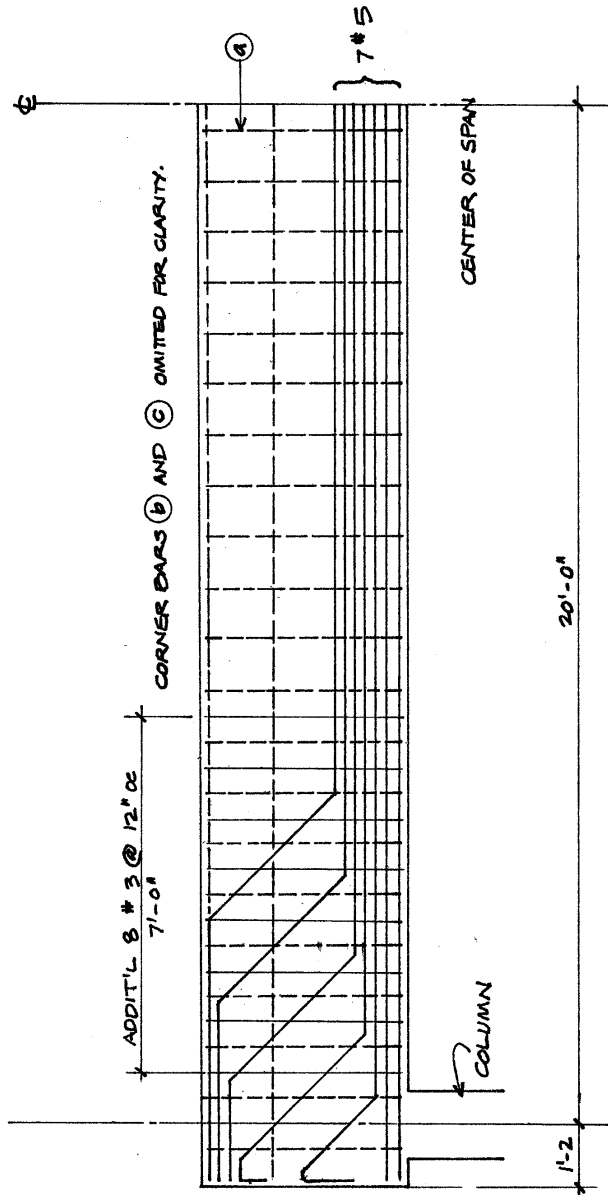

LATERAL VIEW OF PLATE BC

FIGURE 7.1

ISOMETRIC VIEW

EDGE CONDITION DETAIL

ALTERNATE I
FIGURE 7.2
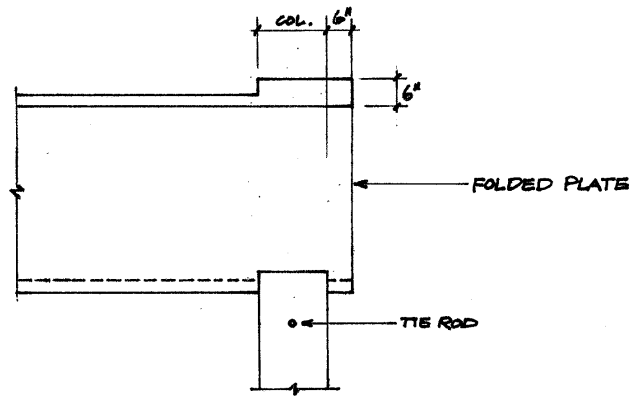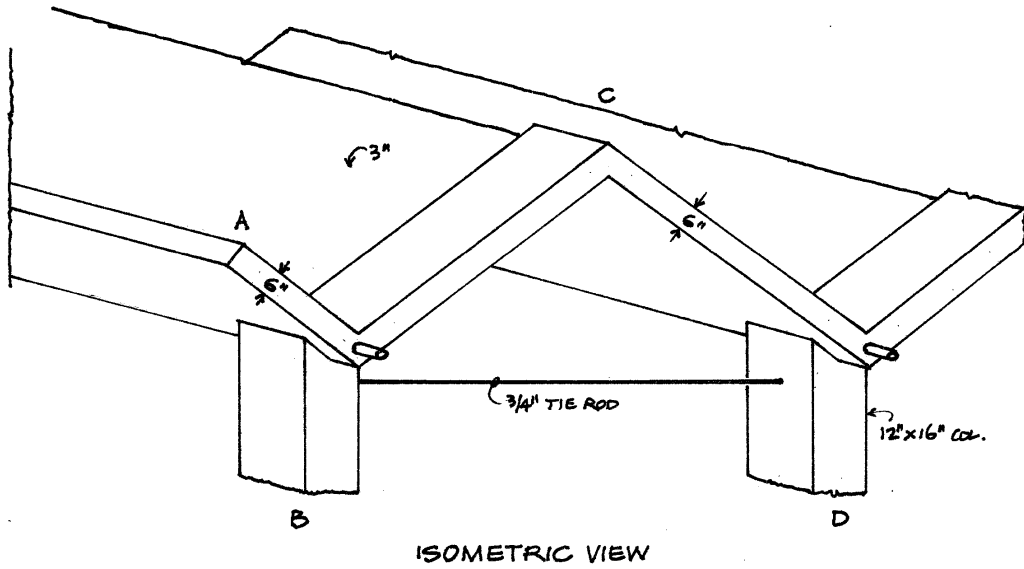
As an alternative to the features above described, the edge condition could be better

served by adding a triangular end gable to solve both problems at the same time, the tension at the top of the column and the compression component coming down the slope. A graphical depiction of such a solution is part of Alternate II in Figure 7.3.

ISOMETRIC VIEW

EDGE CONDITION DETAIL

ALTERNATE II
FIGURE 7.3
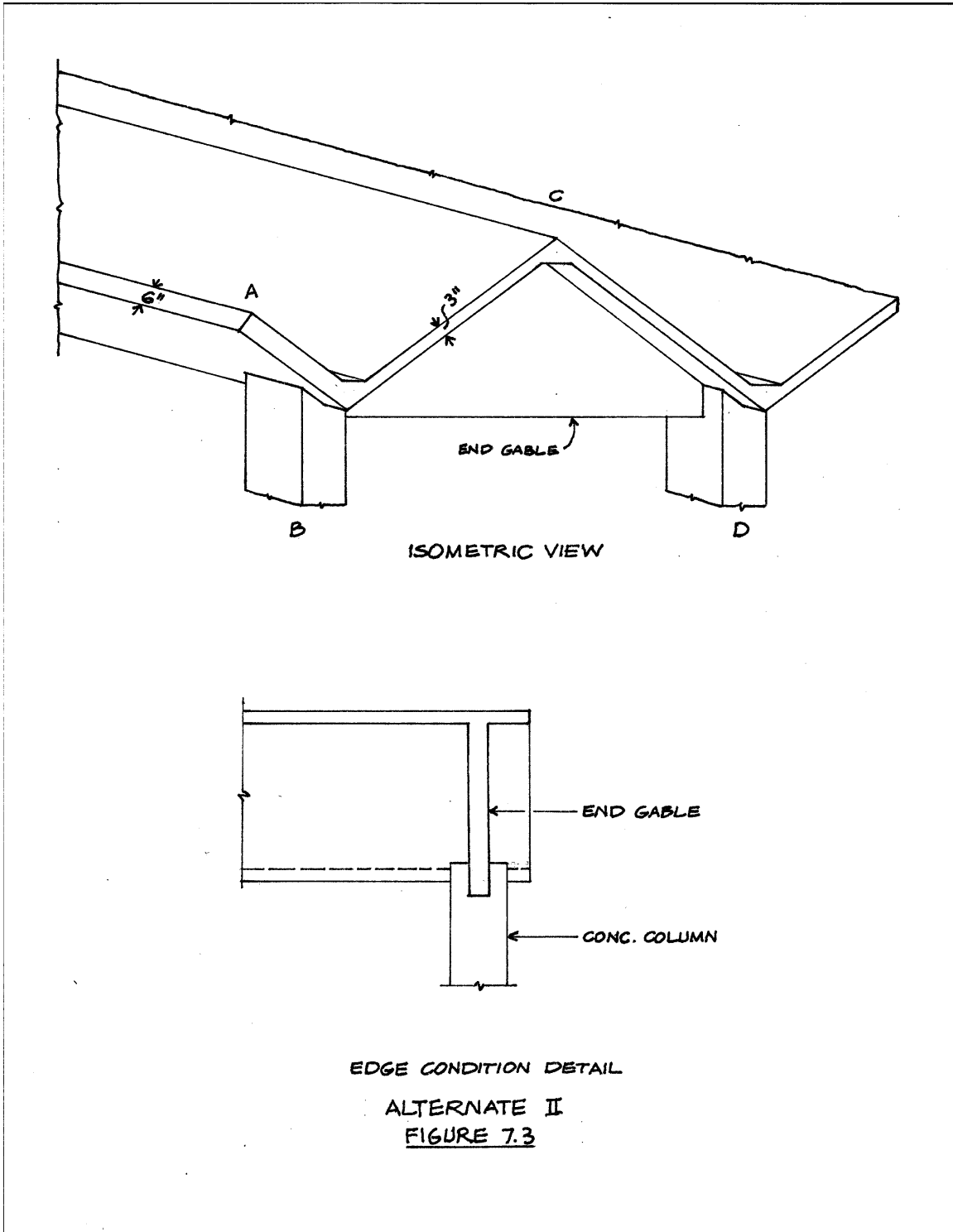
It should also be stated that the alternative of the four (4) end gables is quite desirable so to add lateral stability against hurricane's wind forces.

Figure 7.4 shows a partial cross-section of the folded plates AB-BC-CD-DE which will help to complete understanding of the overall intended design concept. Please notice how the sharp vertices have been avoided by introducing fillers which are identified with a boxed letter "X" on the same figure. Those fillers have many desirable and functional advantages, such as:

1. they help to increase joint stiffness and maintain angular integrity,
2. make form stripping easier by preventing the sharp corners,
3. increase room for the otherwise congested area by the longitudinal reinforcement,
4. by enlarging the concrete section available the effects of crushing and punching shear become more manageable,
5. substantially reduce the concentration of secondary stresses, and
6. help improve rain-water drainage by allowing slope towards the end of the valleys.

The reinforcing bars marked with the circled lower case letters on Figure 7.4 are described as part of the list below:

a. #3 @ 12" on centers,
b. #3 @ 11" on centers,
c. #3 @ 8" on centers, and
d. #3 @ 7" on centers.

Larger bars in vertices B and D are the needed longitudinal reinforcement to resist the bending stresses as result of the plates' flexure in that direction. Vertex B requires a total of 10 #5 bars. Please notice that in order to be consistent with the moment's lever arm assumed on Sheet #7 of the calculations, bar M needs to be moved down as shown on the detail at the lower left corner. In the same manner, Vertex D requires a total of 7 # 5 longitudinal bars as indicated on the same figure.

Since all described reinforcing steel needs to be placed within the confines and limitations of a 3 inch concrete slab, a very meticulous planning must be prepared ahead of time to succeed in an installation where the tolerance is a mere one-eighth of an inch.

FIGURE 7.4

TEMP. REINF.: #3 @ 18" oc
FOR ADDIT'L DESCRIPTION OF REINF. SEE TEXT.
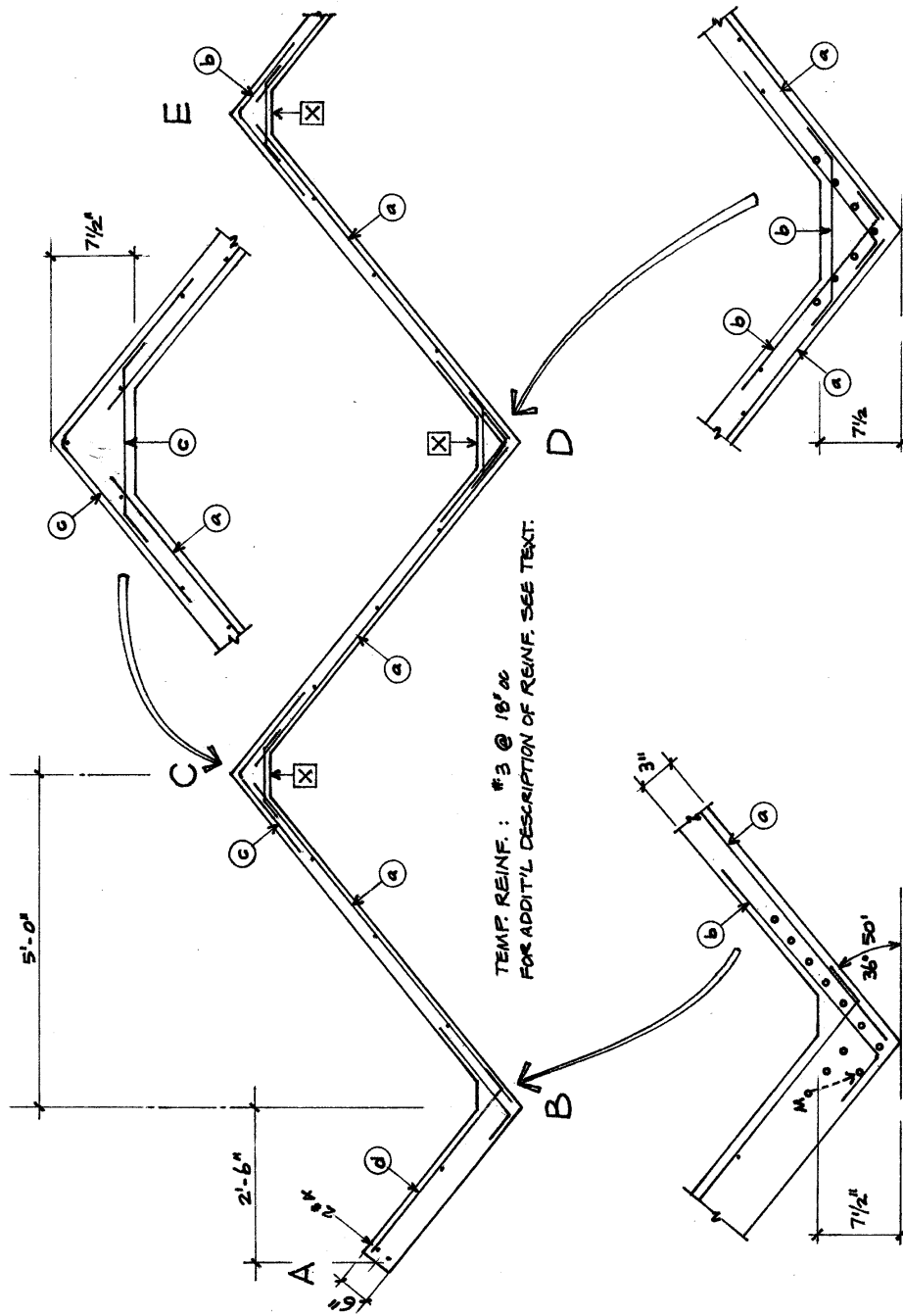
8.0   Rules of Thumb

It does not matter how much of a skillful mathematician or structural analyst and theoretician you may be cut out to be, if you do not possess the practical view and

anticipation to the problems ahead, the former is not going to help you that much. The late Dr. Jacob Feld, PhD, P.E. used to say "while a poor design seldom can be made good by the best construction methods; a good design can often become a failure in the hands of a shoddy builder."

Considering all of the above, it would be safe to conclude that the best recipe for a successful construction project needs to have at least three important ingredients: a dedicated, anticipating and experienced engineer, a good and conscientious builder, and a responsible supervision by the former. Do not allow your well thought out mid-night oil designs to be field-decided by poorly trained builders and improvised county inspectors, make sure that when the "tough gets going" you will have a say to help save your design and your reputation.

Here are some of the concepts that you should always keep in mind when designing and preparing permit drawings for the construction of folded plates:

#1. If you had the chance of close following the enclosed numerical example, you realized that structural sags and deflections have their way of generating their own secondary moments and shears which would burden your structure and trigger more sagging which in turn would generate more stresses and so on all the way on down to failure in the worst of the scenarios. In shorter words, folded plates are not very tolerant to undue deformations without consequences; therefore, it is our recommendation that folded plates be built on a sturdy and substantial set of columns. In the case at hand, the column design load is a mere 20 kips which could theoretically be handled by a small sized column. But please don't, be conservative and have at least some 12 x 16" columns, even though on paper they may look like overkill, folded plates need sturdy column support and good foundations on well drained and compacted soil to guarantee good control of differential settlements. In addition, because of their shape and geometry, folded plates give the impression of being bulky, therefore you should have the columns proportioned accordingly, otherwise your design will look like a "chicken" with a bulky body and skinny legs.

#2. As already covered in two of our previous courses, namely: "Unconventional Construction Methods" and "Dome Design", it is a well known fact that concrete has the propensity to crack, which in most cases can be tracked down to either shrinkage or temperature changes (not to mention under-design); fortunately, it is also known (or should be known) that they both can be remediated and/or prevented on permanent basis by using post-tensioning as suggested on Figure 8.1. However, if you decide to use that solution, the plate (slab) thickness should be increased to a minimum of four (4) inches to accommodate the regular post-tensioning hardware and further, to eliminate the possibility of warping and buckling.

By comparing Figures 7.1 and 8.1 and by using simple visual inspection, it can be deducted that the dollar savings (if any) is almost negligible by switching to post
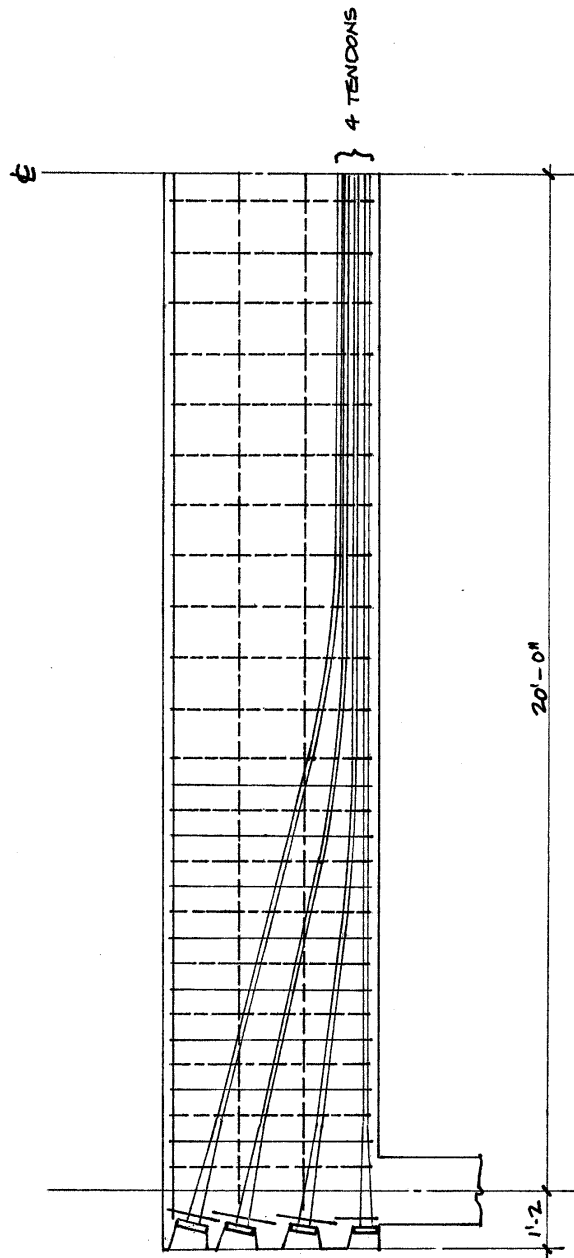
LATERAL VIEW OF PLATE BC - POST-TENSIONING VERSION -

FIGURE 8.1

tensioning, however, there are some other advantageous considerations that should not be overlooked. The first one is the fact that the "soft slab" afforded by post-tensioning is more forgiving when it comes to sensitivity to differential settlements. In the second place, the application of such system will avoid most undesirable cracks and the inconvenience they represent.

Cracks are unsightly and may cause leaks, water stains and efflorescence on the concrete surface, therefore, you must do your best to prevent them and include solutions and recommendations in your design drawings to that end. Keep in mind that there are many good and long lasting roof coatings in the market that would give you an adequate job performance even in the worst of conditions.

#3. Steel work must be fabricated and installed by a dependable and experienced operator. Reinforcement needs to be placed to accurate perfection, as if the bars were drawn to full scale on the forms. Bar spacing must be exact, avoid unnecessary splices, no crooked bends, no short stops. Concrete coverage must be attained by using proper spacers and chairs and their adequacy must be effectively verified before pouring.

#4. Concrete pouring should be a well organized and smooth operation with no stops, delays or interruptions. Overheated or stale concrete should not be allowed at the jobsite.

#5. The plates of the structure, especially when they are facing the southern skies, seem to represent an irresistible opportunity for the owners and their representatives to install signs, solar panels, cameras and dishes. If there is such a possibility, their installation should be planned ahead and the proper inserts, bolts and welding plates should be cast in the concrete in such a way that drilling is completely avoided and forbidden, very particularly in those projects where post-tensioning is planned or has been used.

9.0 Final Word

Now that we have gone through all the "motions" of the design process, we can decide which computer program would be best to tackle the calculations of our next folded plate project while having the certainty that we know all that there is to it and can convincingly sign and seal those computer output printouts as our own. A prose from the American poet Robert Frost comes to mind, as it reads in one of his poems and very applicable to the typical engineer's daily work: "before I would build a wall, I would like to know what I was walling in, and even more important, what I was walling out…."

APPENDICES

S (ft)	C (ft)	H (ft)	Tan (θ)	T (in)	T' (in)
40	10	3	0.60	6	3
40	12	3	0.50	6	3
40	12	3.6	0.60	6	3
40	14	3.5	0.50	6	3
40	14	4.2	0.60	6	3
40	16	4	0.50	6	3
40	16	4	0.50	8	4
40	16	4	0.50	6	4
40	16	4.8	0.60	8	4
40	18	4.5	0.50	8	4
50	12	3.6	0.60	6	3
50	14	3.5	0.50	6	3
50	14	4.2	0.60	6	3
50	16	4	0.50	6	3
50	16	4	0.50	8	4
50	16	4	0.50	6	4
50	16	4.8	0.60	8	4
50	18	4.5	0.50	8	4
60	16	4.8	0.60	8	4
60	18	4.5	0.50	8	4
70	18	4.5	0.50	8	4

Values reproduced from the author's paper titled **Notion Of The Result,** which can be used as part of the design engineer's first computation trial.

APPENDIX I
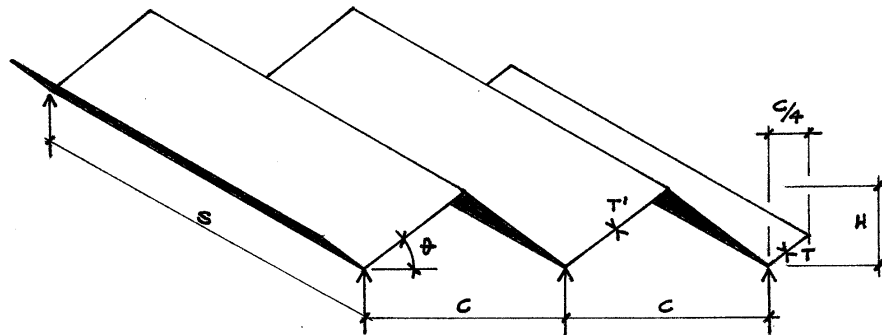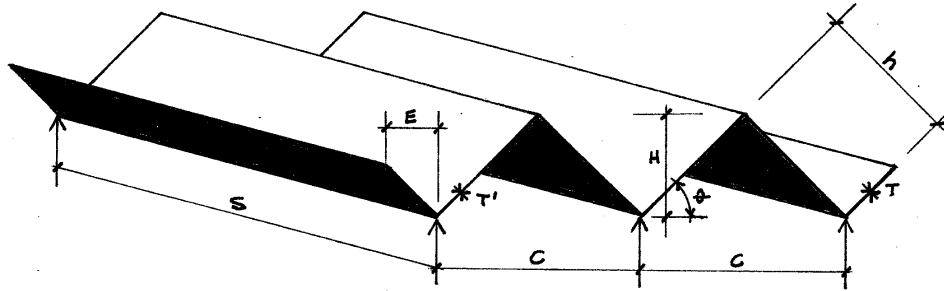
DESIGN GUIDELINES

1- $H/S = 1/18$ MIN. PREFERABLY NEAR $1/10$.

2- θ SHOULD BE BETWEEN 25 AND 40°.

3- $C = 8 - 28$ FT.

4- $E = 0.20 - 0.25C$ (YOU SHOULD ALSO MEET CONDITION #6)

5- $T' = 3 - 5''$

6- T SHOULD BE SELECTED IN SUCH A WAY TO PRODUCE A CROSS SECTIONAL AREA OF THE EDGE PLATE EQUAL TO $1/2$ TO 1 TIMES THE AREA OF THE FIRST INTERIOR PLATE.

7- S SHOULD NOT EXCEED 100 FT.

8- CONVERGENCE OF DESIGN APPROXIMATE METHOD IS LIMITED TO THE FOLLOWING LIMIT:

$$\frac{15}{384}\left(\frac{S^2 T'}{h^3}\right)^2 \frac{1}{\sin^2 2\theta} \leqslant 0.5$$

IF THE 0.5 VALUE IS EXCEEDED, THE CORRECTIONS MAY NOT CONVERGE.

APPENDIX II

APPENDIX III

Notable Contributors to Mathematics and the Structural Theory

We could not pass up the opportunity to render tribute to those men of the past who contributed with principles, methods, theories, procedures, theorems and ideas which provided the foundation and direction to our present structural arsenal of knowledge.

Although we have no names of those contributors at the beginning of history and they are likely hidden in the mists of ancient ages, we do know that those men invented ingenious simple machines such as, the inclined plane, the wedge, the lever, the screw, the wheel and the axle.

The names that were available to us from different sources have been listed below as well as a short description of their main accomplishments. We engineers all should feel grateful for those who came before us and made a contribution for the betterment of our profession.

Our apologies to those contributors whose names may have escaped our research, for those omissions, if any, have not been intentional nor for the lack of effort on our part. With that being said, here is a list of those geniuses who brought us where we are today:

1- Imhotep, one of the only two commoners deified during the long history of Egypt. As the creator and builder of the stepped pyramid a Sakkara in the year 3000 BC, he justly gained the reputation of being the first known structural engineer in the world's history.

2- Pythagoras, Greek born in the year 582 BC. He is known to the modern engineers for his theorem related to the right triangle and having been the first who articulated the word "mathematics" as meaning the "science of learning".

3- Aristotle (384-322 BC), another great Greek mind who mastered wide knowledge in many areas of the arts and sciences who was credited as having written more than twenty-five books in different fields of knowledge. Dean of the Lyceum, a reputable college in the city of Athens, Greece. He was man of unquestionable abilities who certainly gained his place in the history of mechanics and structural analysis.

4- Euclid (315-250 BC), first professor of geometry of the greatest advanced school in the ancient world, the University of Alexandria.

5- Archimedes (287-212 BC), considered by many as a greater mind than Aristotle's. He likely was the greatest physicist of the ancient world and one of the most notable mathematicians of all times. His classic book "On Equilibrium" and his work on the "center of gravity" principles were responsible to establish him as the undisputable father of statics. Further, it is believed that his work on geometry enabled Newton and Leibnitz into the development of calculus. After the fall of Syracuse to the Romans he failed to appear before Marcellus, the Roman conqueror of Syracuse and as punishment for his neglect he was slain. The death of Archimedes marked the end of the great

golden age of the Greek philosophers.

6- *Lucretius Carus* (99-55 BC), a Roman poet and scientist with an innate ability for physics and science, qualities which were unknown amongst the Romans who although of a more practical nature when it came to fighting and building than the Greeks, but generally incapable of "abstract thinking". He is credited by having made the first discovery and derivation of the "law of conservation" of mass eighteen centuries before the French chemist Lavoisier formally published it.

7- *Leonardo da Vinci* (1452-1519 AD), his abilities in music, sculpture, painting, building construction, flying machines and geology has gained him the reputation as the most versatile genius of all times. He is of particular interest to engineers because based on his statement of the "law of the lever" he was able to conceive the concept of "the moment of a force". In his writings he stated two hundred years earlier the principle that has come to be known as "Newton's third law of motion".

8- *Andrea Palladio* (1518-1580 AD), an Italian architect who has been credited by having used the first truss, although his conception was not based on rational analysis.

9- *Simon Stevin* (1548-1620 AD), a Dutch engineer who published a book on statics in 1586. He had a good understanding on the composition and resolution of the "triangle of forces" which he used to solve the equilibrium of truss joints as it is known today.

10- *Galileo Galilei* (1564-1642 AD). His work marked the beginning of the age of reasoning in structural analysis. He dedicated most of his time to investigate the resistance of solids to rupture and likely was the scientist who originated the mechanics of materials. He also investigated the behavior of cantilevering beams and although mostly wrong in his assumptions, his contributions are considered of great impact.

11- *Robert Hooke* (1635-1703 AD). Professor of Geometry and Survey at the Gresham College in London, he dedicated some of his efforts to the study of elasticity which led him to the principle of the Hooke's Law which although he never applied to the solution of engineering problems, but led others to do so.

12- *Sir Issac Newton* (1642-1727 AD). Rare coincidences took place on Newton's birthday, he was born on Christmas day the same year of Galileo's death. He was a notable Professor of Mathematics at the renowned Cambridge University. Author of what is perceived as the greatest scientific book of all times, "Philosophiae Naturalis Principia Mathematica" or as simply called "Principia". In his book he stated the laws of motion, the law of universal gravitation and the infinitesimal calculus. Newton acknowledged his indebtness to the great men of science who preceded him by humbling saying "if I have been able to see a little farther than some others, it was because I stood up on the shoulders of giants."

Sir Isaac Newton was a very modest and humble man, to the point he had to be persistently persuaded by his disciple, Dr. Edmund Halley to publish his "Principia". After much insistence he finally pulled his manuscript from a dusty trunk and had his

book published in 1687. His dedication and genius served as inspiration to those who followed him and in turn "stood on his shoulders", after his death and thanks to his legacy, knowledge accelerated tenfold and cleared the way for greater and better things.

13- James Bernoulli (1654-1705 AD), Swiss mathematician who became interested in Galileo's work and was the first to have assumed that the plane section of a beam remains plane during bending but failed to consider the importance of the neutral axis.

14- Johann Bernoulli (1667-1748 AD), brother of James and equally gifted mathematician, came up with the principle of "virtual velocities" which gave way to methods to determine elastic deflections in structures.

15- Daniel Bernoulli (1700-1782 AD), son of Johann, became interested in determining the elastic curves of bent bars and the vibration of beams. In 1735 he succeeded in the formulation of a differential equation on bar vibrations.

16- Leonhard Euler (1707-1783 AD), collaborator of Daniel who succeeded in figuring the elastic curves of beams and columns. By using the method of least work he prepared valuable discussions on column buckling that we still use today.

*17- Charles Augustin Coulomb (*1736-1806 AD), a French military engineer and physicist. In 1776 Coulomb published the first correct analysis on the fiber stress in a flexed beam with rectangular cross-section. He developed the equilibrium of forces on the cross section with the external forces and correctly evaluated the stresses and placed the neutral axis where it belonged.

18- Louis Henri Navier (1785-1836). Distinguished French engineer, mathematician and professor who in 1826 published the first edition of "Lessons", a great book on engineering mechanics where he presented a study on the strength and deflection of beams of any cross section form. He also included in his book the study of arches, columns with eccentric loadings and suspension bridges.

19- Barre de Saint-Venant (1797-1886). He was known to have a combination of great mathematical abilities and practical view of the problems brought upon him. In 1855 he wrote a book dealing with the topic of torsion and in 1856 another one dealing with flexure, shear stresses, elasticity and plasticity.

20- B. P. Clapeyron (1799-1864 AD). With the collaboration of his associate G. Lame' published in 1833 a paper on elasticity with an exceptional contribution on stresses in hollow cylinders and spheres. He also authored the theorem of equal external and internal work, which became of great importance in the analysis of indeterminate structures. In 1857 he presented his famous "theorem of the three moments" so helpful in the analysis of continuous beams.

21- Squire Whipple (1804-1888 AD) was the first contributor originated in the United States. In 1847 he published his book "Bridge Building" with a detailed procedure for the design of jointed trusses. He must be credited with being the contributor who made

the design of jointed trusses a practical possibility.

22- William John Rankine (1820-1872 AD), English engineer who published his book titled "Manual of Applied Mechanics" where he presented a collection of empirical formulas for the design of columns.

23- James Clerk Maxwell (1830-1879 AD), Professor of Experimental Physics at Cambridge University. In 1864 he published his book titled "On the Equilibrium and Stiffness of Frames" where he presented an analysis of indeterminate frameworks based on the equality of the internal strain energy of a loaded structure and the external work of the applied loads, similar to what Clapeyron had previously proposed. However, unlike Clapeyron, Maxwell expressed the necessary conditions for geometric coherence by a set of equations in which the variables were the redundant stresses.

24- Otto Mohr (1835-1918 AD). His contributions to the structural theory are extremely significant to the engineering community. In 1808 he presented an outstanding study on elastic curves viewed as string polygons which became what we now know as the "method of the elastic weights". Concomitantly he developed a closed allied method for computing deflections as bending moments in a fictitious beam loaded with elastic weights, that is, the "conjugate beam method".

In 1877 a French engineer named Williot disclosed a new method for determining deflections in articulated structures. Since Williot's Diagram had fundamental limitations, in 1887 Mohr devised a series of corrections eliminating its original shortcomings. As a result of those corrections it became the Williot-Mohr diagram, which is an extremely valuable procedure for determining deflections. Finally, in 1892 Mohr came up with the slope-deflection method of analysis as applied to secondary stresses.

25- Alberto Castigliano (1847-1884 AD), an Italian railways engineer presented his "theorem of the least work" as his graduation thesis in 1873. In the following years he extended and perfected his work and in 1876 came back with his work that is now known as "Castigliano's Second Theorem". Again in 1879 Castigliano published his work in a book form and extended it to the area of indeterminate structural analysis, which proved to be more comprehensive than the work done previously by Maxwell and Mohr.

26- Heinrich Muller-Breslau (1851-1925 AD), for many years a distinguished professor of the Technische Hochschule in Berlin, also published in 1886 a basic method for the analysis of indeterminate structures as a variation to the previous work developed by Maxwell and Mohr. He used the principle of superposition of displacements as caused by the individual redundant stresses and reactions.

27- Hardy Cross (1885-1959 AD) began to teach his iterative "moment distribution method" at the University of Illinois in 1924. After the publication of his method in 1930, it became one of the greatest contributions to the analysis of indeterminate continuous frames. The Hardy-Cross Method of Moment Distribution has been known and well

used by every structural engineer who can call himself so. Furthermore, the method can also be successfully used to solve the problem of fluids flow distribution in pipelines and conduit networks.

28- R.C. Southwell (1886-? AD), although he did not know much about Hardy Cross' iteration work in the United States, he was his contemporary and coincidentally dedicated a lot of his time to similar pursuits. For many years a professor at Oxford University where he developed his own ideas in the area of procedural iteration which he called "structural relaxation methods". His first paper on the topic was published in 1935 and was followed by two other books in 1940 and 1946.

In closing, we must say that structural engineering is not clearly defined neither as an art or a science, since an art is the ability of doing things and on the other hand, a science is the organized knowledge of things. Therefore, it is apparent that at the beginning structural engineering existed only as a trial-and-error art well into the times of Galileo when he made a serious attempt to analyze the cantilevering beam. By the mid 1800's scientific analysis of structural problems was an accepted procedure amongst practitioners such as Clapeyron, Whipple and Saint-Venant. Later on, the principles of indeterminate structural theory became matter of extended and elaborate knowledge thanks to the efforts of Maxwell, Mohr, Muller-Breslau, Cross and Southwell.

Some of us find fascination in going back to history and examine how knowledge evolved with time and how a man's legacy was passed from one to the next one in line until it snowballed into a collection of ideas and methods and finally into a science, for understanding is most palpable, clear and perfect when it is based on the knowledge of how things were from the very beginning and how they got to being.

APPENDIX IV

Structural Deflection Methods

Since the determination of deflections is an important part in the design of folded plates, we have included two of the methods available for that purpose. As already described in Appendix III, Item 24, the Williot Diagram as originally proposed by his author in 1877, was an important step forward towards the solution of relative deflections and displacements in articulated structures; however, it had some flaws and limitations that render its use and application of little value. Fortunately, ten years later Professor Otto Mohr of Dresden, Germany, introduced some adjustments and corrections that made the Williot-Mohr Diagram an invaluable tool to determine the absolute magnitude and direction of joint displacements.

THE WILLIOT DIAGRAM

For a better understanding of Williot's method, let us start by considering a small cantilevering truss as shown on Figure IV.1 under the assumption that its members are made of a highly elastic material able to endure large strains.

The task consists in finding the deflections (if any) at all six truss joints. Since the truss is structurally attached at joints **a** and **d**, they are fixed and therefore have no possibility of displacement or deflection.

Every unit of strain is assumed to be equal to one tenth of the member's original length. A positive sign should be considered as meaning a strain caused by tension and therefore resulting in an increase in length, conversely, a negative strain is caused by compression and resulting in shortening of the member in question. As a matter of example, member #1 extending from joint **a** through **b**, has a **positive one unit strain**, therefore, it is under tension and subject to lengthening.

Since the Williot Diagram finds relative displacements, it requires one member to have infinite stiffness (zero strain), the selected member is #5 running from joint **b** to **c**. We have already said above, as a given, that joints **a** and **d** are fixed. Now we will work on joint **e,** since is the joint with the least possibility of displacement for being held by members #2 and #3 with their far ends fixed. Add (positive) one unit to the original length of member #2 and using as center the joint **a** swing an arc in the clockwise direction. In the same manner reduce (negative) length of member #3 in one unit, then draw an arc and where the two arcs intersect is the new position of the displaced joint **e'**.

Next we move to joint **b** and proceed the same way, member #1 is lengthened one unit and member #4 is shortened one unit, swing the arcs and at the intersection is the displaced joint **b'**. Following the same procedure, position of joints **f'** and **c'** can be located. If the deflected truss (b) is superimposed on the original truss (a), then the actual displacement of each joint can be measured directly.

THE MOHR ROTATION DIAGRAM

As noticed on the Williot Diagram we had to assume that member #5 had zero strain and for that purpose we had to conceive it as of infinite stiffness, which is hardly the case in reality.

In 1887 Professor Otto Mohr, being aware of the shortcomings of the Williot Diagram and at the same time realizing of its great potential, he introduced his modification in form of a rotation diagram which has come to be known as the Williot-Mohr Diagram.

In this case, joint displacements are determined on the assumption that joint **c** remains fixed in position and so is member #8. Using a similar procedure as in Williot, we start the construction at joint **c'** and proceed to locate **f'**, **b'**, **e'**, **a'** and **d'.**

If the original truss is then superimposed on the strained truss, the deflection on each joint will become evident but the direction of every displacement would be erroneous, to correct such an error it would be necessary to rotate the entire figure in a clockwise direction until the imaginary line connecting the supports **a'd'** becomes vertical as it should be.

Graphical representations of both diagrams next to each other are included on the next page for the convenience of the reader.

THE MOHR ROTATION DIAGRAM
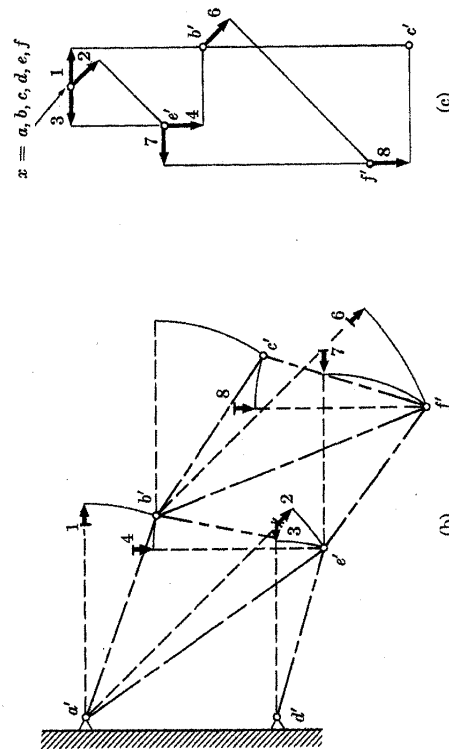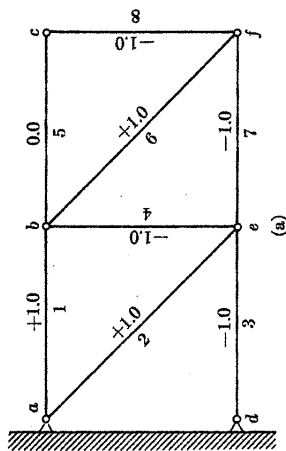
THE WILLIOT DIAGRAM
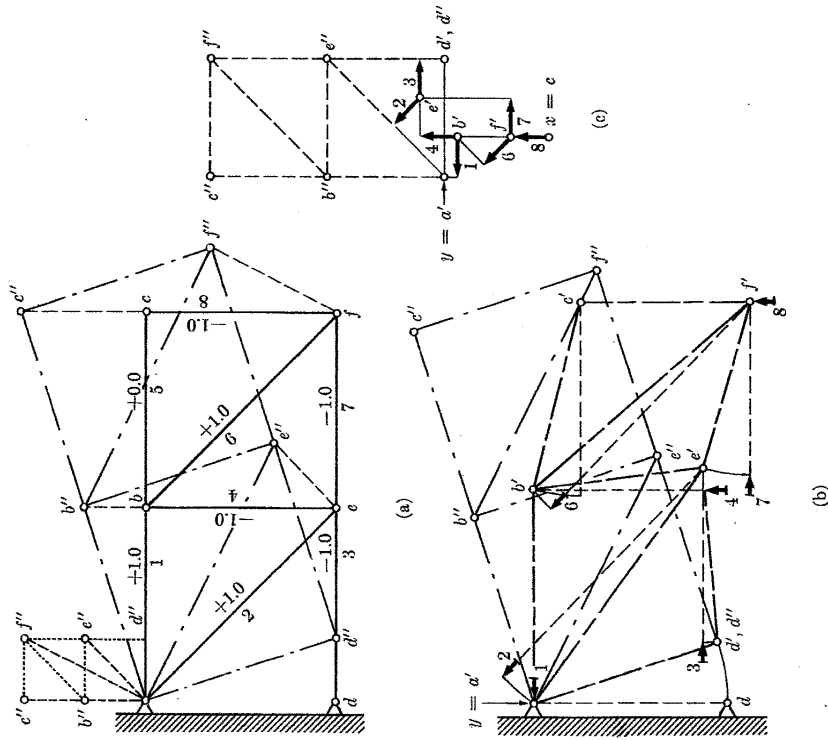
APPENDIX V

An Uncommon Application

We are all familiar with the application of folded plates to serve as roof coverings and subjected to the usual gravity loads encountered in our routine engineering design assignments. However, what it is described below is an unusual application where the 18 in. thick folds were used in an inverted position to carry soil pressure instead. We are referring to the twenty-four story tower completed in 1959 as the City Hall of the City of Havana, Cuba.

According to the reports and historic data available, the structural engineers found themselves with a lot of considerations to ponder on, such as a soil composed of a highly compacted marl with scattered large boulders and a soil report with a recommended maximum design bearing capacity of 6,000 PSF; a heavy reinforced concrete building built such as to resist high wind pressures, as the City of Havana is located within the path of a large number of hurricanes sweeping by every year through the Caribbean Sea.

Due to the difficulties encountered at the site, a pile foundation was eliminated as a possible solution and the design team decided to solve the foundation scheme by using a 90 x 260 ft. raft foundation with a massive concrete depth of 4 ft. After diligent efforts and considerations, they decided to use a variant to the idea by designing a reversed folded plate as the most economical solution available. Previous cost analysis which was later confirmed by practical results showed that a folded plate raft indeed represented savings of some 30% over the cost of a conventional straight leveled raft foundation.

Figure V.1 depicts a partial cross section of the folded plate foundation as it was finally built. It can be observed how the conventional horizontal slab used as the basement floor was also used as an active tension member to overcome the thrust generated by the folded plate action.
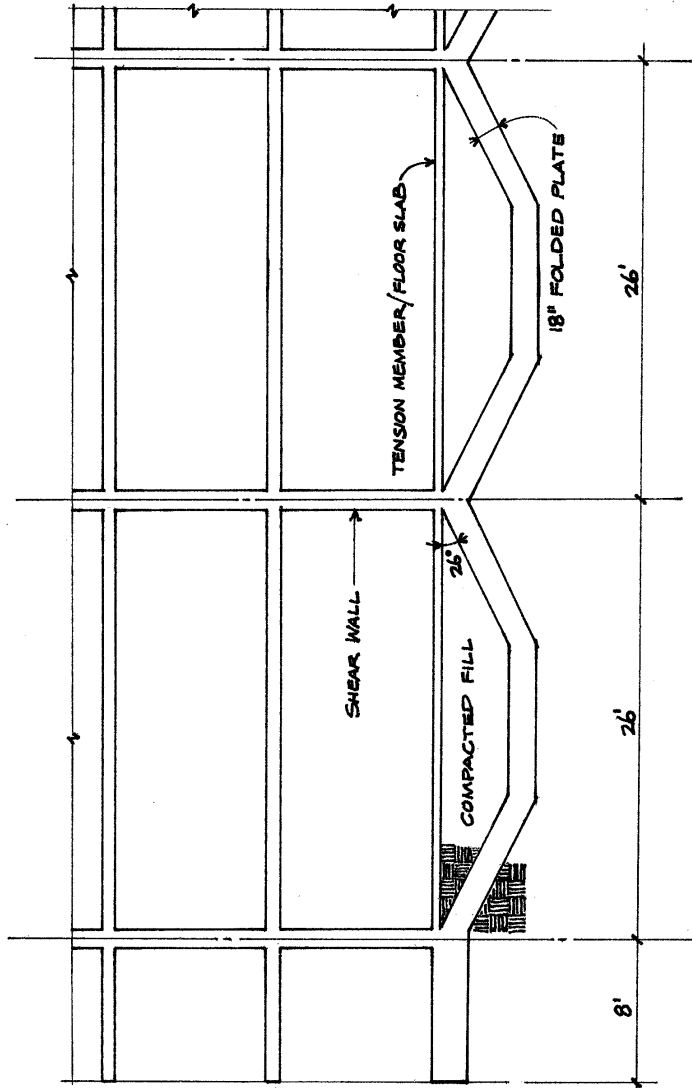
FOLDED PLATE RAFT FOUNDATION AT CITY HALL OF HAVANA.
FIGURE V.1

References used in the preparation of this course:

1. *Direct Solution of Folded Plate Concrete Roofs,* A. L. Parme and John A. Sbarounis. Bulletin #6 issued by the International Association of Shell Structures in 1960. Re-printed with permission by the Portland Cement Association.

2. *Design of Folded Plate Roofs,* Howard Simpson, Associate Professor MIT. Paper #1508, Proceedings of the American Society of Civil Engineers.

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