PDHonline Course K102 (4 PDH)

Over-all Heat Transfer Coefficients in Agitated Vessels

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Course Content

The over-all heat transfer coefficient in a jacketed agitated vessel system can be determined from the energy balance equation and empirical correlations. This course will use one-dimensional flow of heat across the radial direction (Assumption #1). The energy balance equation is composed of a series of resistances which relate the bulk stream temperatures of the agitated liquid batch and the cooling/heating medium through the vessel wall. Figures 1 and 2 are common industrial examples of a jacketed vessel used to heat or cool a liquid batch. (Figures used in this course are located in the following pages.)

Figure 1 represents an arrangement for heating a batch liquid by an isothermal medium which is passed through a jacket that surrounds the section of the vessel containing the liquid. Saturated steam is indicated in this example as the heating medium.

Figure 2 represents an arrangement for cooling a batch liquid by a non-isothermal medium which is passed through a jacket that surrounds the section of the vessel containing the liquid. Chilled water is indicated in this example as the cooling medium.

In both examples, the volume of batch liquid is well mixed by an internally mounted mixing device. Typically this device is a propeller or turbine blade connected to a shaft that is usually driven by an electric motor at a fixed speed. The result of the "well mixed" volume of liquid is that this allows us to assume that the liquid physical properties, i.e., temperature, heat capacity, viscosity etc., of the liquid are uniform throughout the entire liquid batch volume, (Assumption 2). The transport mechanism assumed for heat transfer from the jacket wall to the agitated liquid batch is convective heat transfer (Assumption 3).

Figure 3 is a graphical representation of the temperature trend for the exchange of heat from a hot fluid to a cold fluid through a vessel wall. In addition to the agitated liquid batch, the heating/cooling media is assumed to be in fluid flow. The transport mechanism for heat transfer to or from the heating/cooling medium is convective heat transfer (Assumption 4).

The rate of convection heat transfer, $-Q_A$, from the hot fluid to the vessel wall is defined as equal to the temperature driving force divided by the resistance to exchange heat. The temperature driving force is the difference between the hot fluid bulk temperature, T_A, and the vessel wall temperature on the hot fluid side, T_{wA}. Note that a negative value indicates heat being released. The resistance to heat

Figure 1. Agitated Liquid in a Vessel with a Heating Jacket

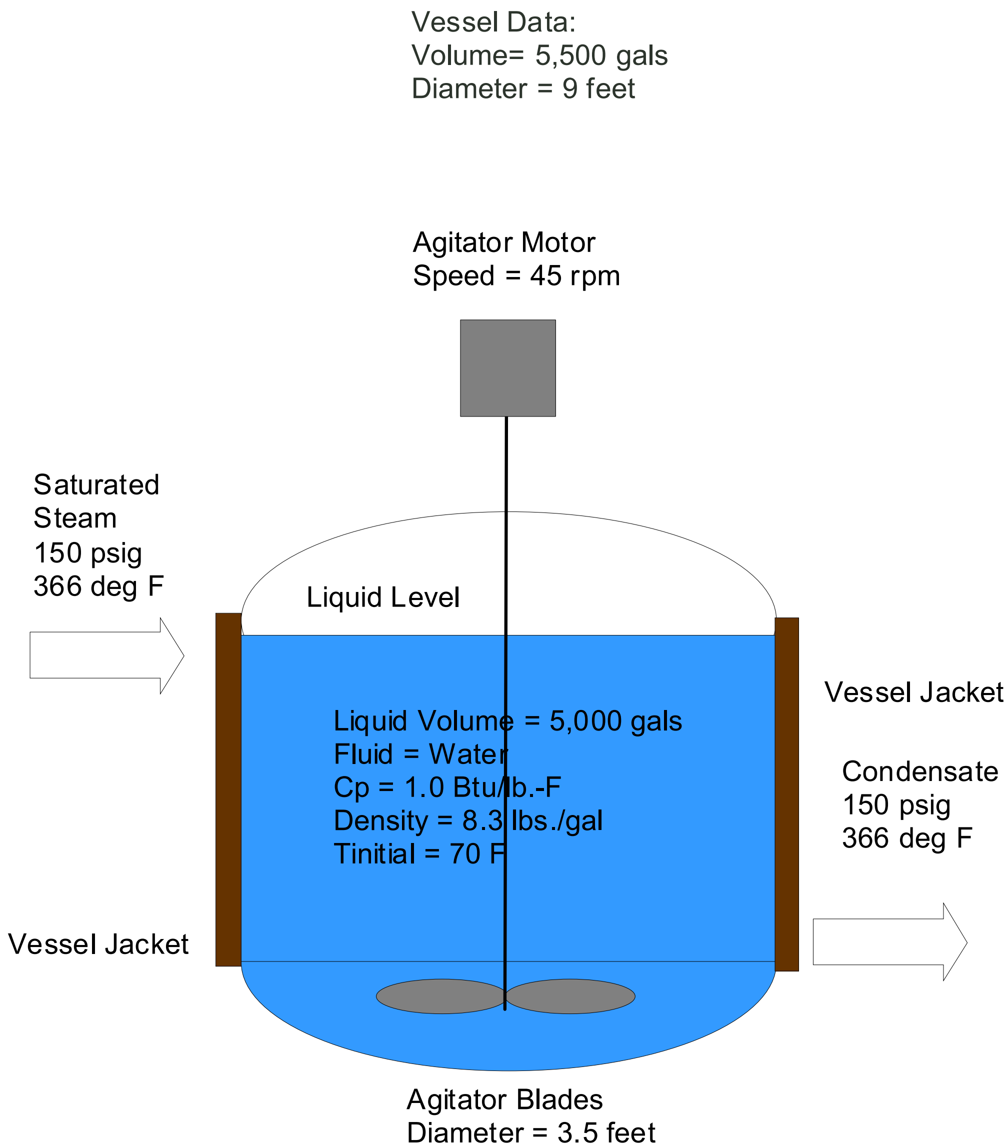

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Figure 2. Agitated Liquid in a Vessel with a Cooling Jacket

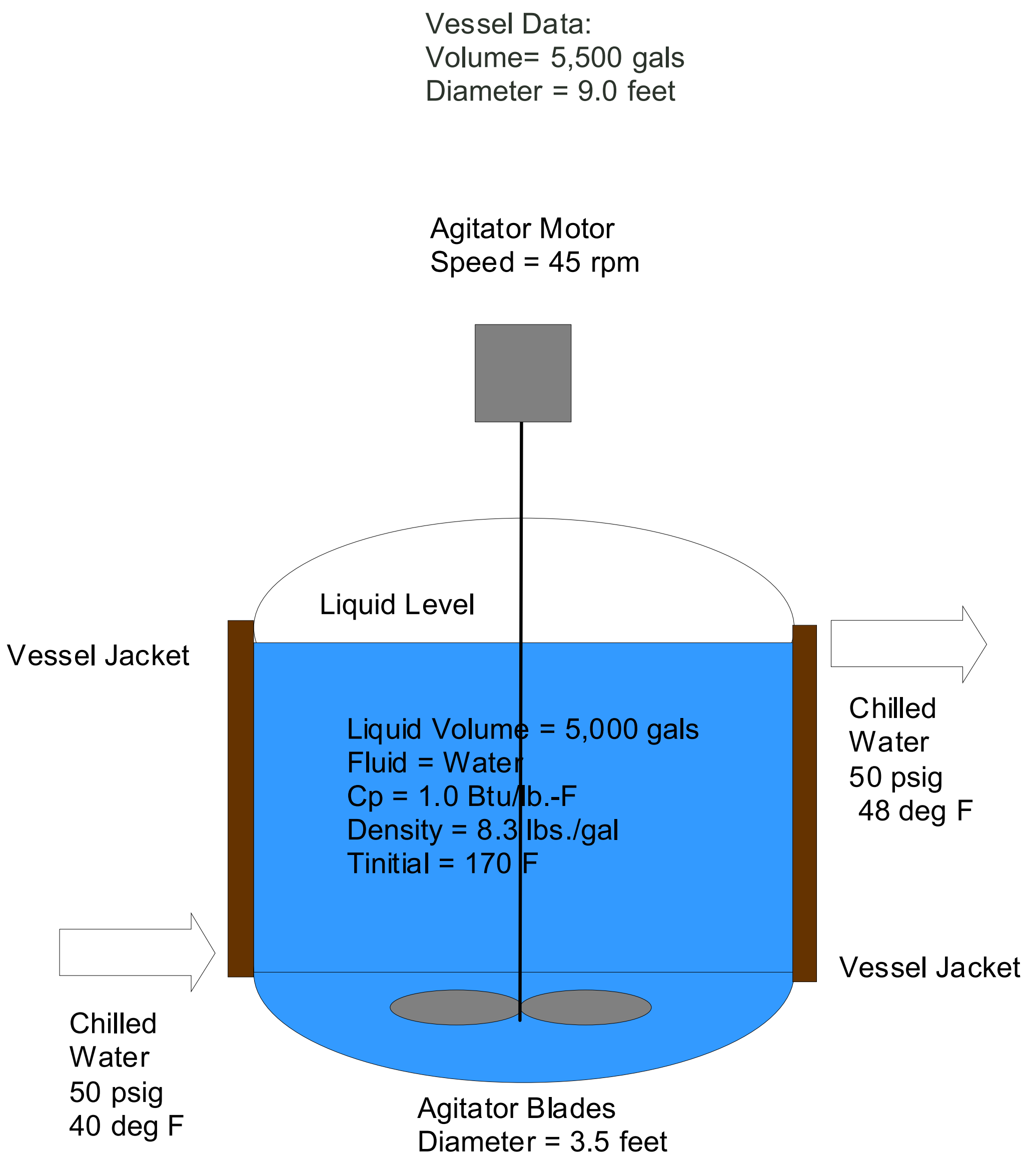

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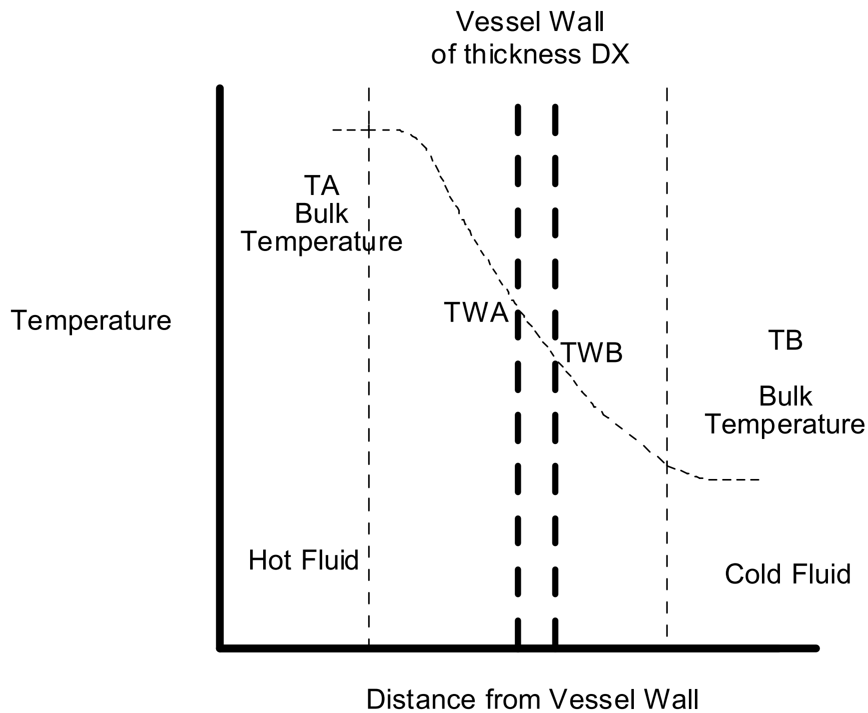
Figure 3. Temperature conditions in the exchange of heat



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exchange, R_A , is the reciprocal of the product terms area, A_A, and heat transfer coefficient, h_A. Expressed mathematically, the temperature driving force is:

$$\Delta T_A = (T_{WA} - T_A) \qquad (1)$$

and the resistance equation across the hot fluid is:

$$R_A = 1 / (h_A * A_A) \qquad (2)$$

The resistance equations for the cold fluid is:

$$R_B = 1 / (h_B * A_B) \qquad (3)$$

The heat transfer through the vessel wall is based on conduction through the vessel material of thickness ΔX, with a material thermal conductivity of k_W (Assumption #5). The conduction transport mechanism for heat transfer is based on the physical contact of one substance to another. The resistance equation for conduction includes a thickness term, ΔX, of the material transferring heat as well as the wall material thermal conductivity term:

$$R_W = \Delta X / (k_W * A_W) \qquad (4)$$

The quantity of heat being transferred by the hot fluid A is obtained by combining equations (1) and (2) yields the convective form of the steady-state heat transfer rate equation:

$$Q_A = - (T_{WA} - T_A) / (1 / (h_A * A_A)) \qquad (5)$$

Likewise for the cold fluid heat gain and the vessel wall heat transfer:

$$Q_B = -\ (\ T_B - T_{WB}\ )\ *\ R_B \qquad (6)$$

$$Q_W = -\ (\ T_{WB} - T_{WA}\ )\ *\ R_W\ ) \qquad (7)$$

Equations (5), (6), and (7) are the Fourier form of the steady-state heat transfer equations which assume a constant temperature driving force across each resistance. If we rearrange equation (5) using its basis equations, (1) and (2), we get:

$$-\ \Delta T_A\ =\ Q_A\ /\ R_A \qquad (8)$$

Likewise for equations (6) and (7) we can derive:

$$-\ \Delta T_B\ =\ Q_B\ /\ R_B \qquad (9)$$

$$-\ \Delta T_W\ =\ Q_W\ /\ R_W \qquad (10)$$

Combining equations (8), (9), and (10) yields:

$$-\ (\ \Delta T_A\ +\Delta T_B\ +\Delta T_W\ )\ =\ (Q_A / R_A) + (Q_B / R_B) + (Q_W / R_W) \qquad (11)$$

At steady state, by definition, the heat transfer through each mechanism is equal and can be represented by $Q_{\text{Over-all}}$, the over-all heat transfer rate (Assumption #6):

$$Q_A = Q_B = Q_W = Q_{\text{Over-all}} \ (12)$$

Substituting equation (1) back into the left-hand side of equation (11) for ΔT_A , along with the respective equivalents for ΔT_B and ΔT_W results in the wall temperature terms cancelling out. Using equation (12) in equation (11) results in:

$$Q_{\text{Over-all}} = (T_A - T_B) / (1/R_A + 1/R_B + 1/R_W) \qquad (13)$$

which implies that the heat transfer rate is equal to the temperature difference divided by the sum of the individual resistances:

$$Q_{\text{Over-all}} = (T_A - T_B) / \sum R_{\text{Over-all}} \ (14)$$

where each resistance is calculated as the reciprocal of the product term of area and individual heat transfer coefficient (Assumption #7).

$$\sum R_{\text{Over-all}} = \sum 1 / (h_i * A_i) \qquad (15)$$

Equation (13) can also be restated as relating the over-all heat transfer to the over-all temperature difference divided by the over-all system resistance.

$$Q_{\text{Over-all}} = \Delta T / \sum R_{\text{Over-all}} \ (16)$$

The heating and cooling of agitated batches can be operated process-wise to maintain the constant temperature driving force. An example of constant temperature driving force is represented by the following.

A jacketed vessel is being fed a minimal amount of condensing steam to heat a reacting liquid while it is being agitated. The endothermic reaction with the liquid batch consumes all of the heat released by the steam and the temperature of the batch liquid remains constant. The condensing steam can be assumed to be at its saturation temperature, which for a constant pressure supply, is a constant temperature. In this example the temperature driving force between both bulk liquids is constant during the liquid reaction period.

In most cases agitated batches, however, are not constant temperature driving force processes, and heat transfer must be calculated using a transient, non-steady state application equation. An example of this situation follows.

A jacketed vessel is supplied with condensing steam to heat an agitated liquid batch from 70 to 150 deg F. While the steam can be assumed to be at its constant saturation temperature, the liquid batch temperature is changing throughout the heating time. The driving force during this time is not constant, but variable.

The important concept from the steady-state rate equation derivation is the definition of the over-all resistances as the sum of the individual resistances. This course will use Assumption #7 in developing the application equation for over-all heat transfer coefficients in agitated vessels.

Normally the bulk fluid streams temperatures, T_A and T_B, are easiest to measure. The temperature difference between the two bulk streams, the agitated liquid batch and the heating/cooling medium, will represent the temperature driving force across all the resistances encountered between the two fluids. This ΔT is referred to as the over-all or total driving force. Accordingly, an over-all coefficient, $U_{\text{Over-all}}$ may be defined as a function of the total resistance, $\sum R_{\text{Over-all}}$, and transfer area, A, consistent with the definition of $U_{\text{Over-all}}$ as:

$$1 / (U_{\text{Over-all}} * A) = \sum R_{\text{Over-all}} \qquad (17)$$

From Figure 3, Equation (17) becomes for the agitated liquid batch and heating/cooling medium convective resistances, along with the jacket wall conductive resistance,:

$$\sum R_{\text{Over-all}} = 1/(U_{\text{Over-all}} *A) = 1/(h_A*A_A) + \Delta X/(K_W*A_W) + 1/(h_B A_B) \qquad (18)$$

where:

$U_{\text{Over-all}}$ = over-all heat transfer coefficient, Btu/hr-sq. ft-deg F. This course will assume that it is constant (Assumption #6). It is understood, however, that the coefficient does in fact vary with the temperature of the fluids. It is assumed that its change with temperature is gradual, so that when the temperature ranges are moderate the assumption of constancy is not in error significantly. Based on this, all individual coefficients will be calculated by determining the fluid properties at an average temperature (Reference 1).

A_A or A_B or A_W = transfer area of the hot, cold fluids and the jacket wall, sq. ft.

h_A = heat transfer coefficient for the hot fluid, Btu/hr-sq. ft of hot phase transfer area-deg F.

h_B = heat transfer coefficient for the cold fluid, Btu/hr-sq. ft of cold phase transfer area-deg F.

k_W = thermal conductivity of the vessel wall material, Btu/hr-sq. ft-(deg F/ft).

ΔX = thickness of the vessel wall, ft.

Two additional resistance terms can be added to equation (18) to include the effects of equipment fouling by the process fluid on both sides of the vessel wall. The terms are strictly empirical. Table 1 contains selected values used in industry.

Equation (19) then defines the general over-all heat transfer coefficient for agitated liquid batches:

$$\sum R_{\text{Over-all}} = 1/(U_{\text{Over-all}} {*} A) = 1/(h_A {*} A_A) + 1/(h_{DH}A_{DH}) + \Delta X/(K_W {*} A_W) + 1/(h_B A_B) + 1/(h_{DC}A_{DC}) \qquad (19)$$

where:

h_D = heat transfer coefficient for fouling deposits, Btu/hr-sq. ft of deposit transfer area-deg F. Fouling coefficients for the hot fluid deposits are noted as h_{DH} , for cold fluid deposits are h_{DC} .

A_{DC} or A_{DH} = transfer area of the hot and cold fouling deposits, sq. ft.

Simplification of Equation (19)

For the case where the vessel wall is composed of a material such as iron, steel, copper or aluminium, and the operating pressures are not high, the vessel wall thickness will be relatively small and the thermal conductivity will be relatively high. This ratio of thickness to thermal conductivity will be almost a magnitude smaller than the other resistances and can be assumed negligible (Assumption #8).

$$\Delta X/(K_W{*}A_W) = 0 \qquad (20)$$

Another simplifying assumption can be made relative to the heat transfer areas. For vessels operating at 150 psig jacket pressures with a 3-foot diameter or larger, the ratio of outside-to-inside heat transfer surface areas for 3/8" thick vessel wall is less than 2 percent difference. For a 9-foot vessel, the difference drops to less than 0.7%. This analogy will also be extended to the fouling deposit areas.

Therefore it will be assumed that the relative areas are equal (Assumption #9):

$$A_A = A_B = A_W = A_{DC} = A_{DH} = A \qquad (21)$$

and the simplified over-all heat transfer coefficient equation becomes:

$$\sum R_{\text{Over-all}} = 1/U_{\text{Over-all}} = 1/h_A + 1/h_{DH} + 1/h_B + 1/h_{DC} \qquad (22)$$

Agitated vessels are typically designed and operated so that the internal wetted vessel surface is essentially "clean" at the start of each operation. The vessel side fouling coefficient will be assumed to be zero and therefore the only fouling consideration will be the heating or cooling media. Fouling deposit coefficient subscript for the media will be represented by $1/h_{DM}$, this further simplifies equation (19) to:

$$\sum R_{\text{Over-all}} = 1/U_{\text{Over-all}} = 1/h_A + 1/h_B + 1/h_{DM} \qquad (23)$$

Application Equations

Over-all heat transfer coefficient for the Agitated Batch Liquid general case:

$$1/(U_{\text{Over-all}} * A) = 1/(h_A * A_A) + 1/(h_{DH} A_{DH}) + \Delta X/(K_W * A_W) + 1/(h_B A_B) + 1/(h_{DC} A_{DC}) \qquad (19)$$

Over-all heat transfer coefficient for the simplified case in agitated vessels:

$$1/U_{\text{Over-all}} = 1/h_A + 1/h_B + 1/h_{DM} \qquad (23)$$

Calculation of Jacket Side Heat Transfer Coefficients

Notation for jacket side heat transfer coefficients, h_J , will utilize the subscript "J".

Condensing Vapor Heat Transfer Coefficients

Heat transfer coefficients for condensing vapors, such as steam, can be calculated from a Nusselt type correlation (Reference 2). The condensing vapors in the jacket side of a jacketed vessel are typically introduced at a nozzle along the top of the jacket. The vapor droplets collect as a film layer as they condense on the vertical jacket-vessel wall and eventually run down to the bottom of the jacket. The condensed phase is then collected and removed through a nozzle located at the bottom of the jacket. A steam trap is typically used to maintain the jacket at saturation conditions and minimize the amount of uncondensed steam from leaving the jacket. An empirical correlation which is based on the condensate film for vertical tubes is given in equation (25). This correlation utilizes the physical properties of the liquid condensate to predict the heat transfer coefficient. Equation (25) is valid for condensate film Reynolds numbers from 2,100 to 100,000. Equation (24) defines the condensate film Reynolds number:

$$N_{Re\ Film} = 4 * \Gamma / \mu_J \qquad (24)$$

where:

$N_{Re\ Film}$ = Reynolds number for the condensate film.

Γ = condensate loading rate, lbs./ft.-hr. This is calculated as the mass rate of condensate per length of vessel jacket perimeter.

μ_J = viscosity of condensate film, lb./ft.-hr.

$$h_J * (\mu_J^2 / (k_J^3 * \rho_J^2 * g))^{1/3} = 0.0076 * N_{Re\ Film}^{\ 0.4} \qquad (25)$$

where:

h_J = jacket heat transfer coefficient of the condensate film, Btu/ hr-sq. ft- deg F.

k_J = thermal conductivity of the condensate film at the average jacket saturation temperature, Btu/hr-sq. ft. ft.-(deg F/ft).

ρ_J = condensate film density at the average jacket saturation temperature, lb./cu. ft.

g = gravitational acceleration constant, 32.2 ft/sec^2 , or 4.17 E+08 ft/hr^2.

Liquid Fluids Heat Transfer Coefficients

Typical jacket side liquids used are: water, brine, oils, and organic solvents. The benchmark for predicting heat transfer coefficients for flow in circular tubes is the Dittus-Boelter correlation (Reference 3). This correlation relates the Nusselt number to the jacket fluid Reynolds and Prandlt numbers and utilizes physical properties at the bulk fluid temperature. For Reynolds numbers greater than 10,000, and Prandlt numbers less than 700:

$$(N_{Nu})= A1 * (N_{Re})^{0.8} * (N_{Pr})^{b} * (\mu/\mu_{Jw})^{0.14} \qquad (26)$$

where:

h_J = heat transfer coefficient of the jacket side fluid, Btu/ hr-sq. ft- deg F.

D_J = Equivalent cross flow diameter of the jacket, ft.

k_J = thermal conductivity of the jacket fluid at the average jacket temperature, Btu/hr-sq. ft. ft.-(deg F/ft).

V = fluid velocity, ft/hr.

N_{Nu} = Nusselt number of jacket fluid = ($h_J * D_J / k_J$).

N_{Re} = Reynolds number for jacket fluid = ($D_J * V * \rho_J / \mu_J$).

N_{Pr} = Prandlt number of jacket fluid = ($c_J * u_J / k_J$).

μ_J = viscosity of the jacket fluid at the average jacket temperature, lb./hr-ft.

μ_{Jw} = viscosity of the jacket fluid at the jacket wall temperature, lb./hr-ft.

c_J = jacket fluid heat capacity at the average jacket temperature, Btu/ lb.-deg F.

ρ_J = jacket fluid density at the average jacket temperature, lb./cu. ft.

A1 is the recommended constant for equation (26); A1 = 0.0243 for heating, A1 = 0.0265 for cooling.

b is the recommended constants for equation (26); b = 0.4 for heating, b = 0.3 for cooling.

Rearranging to solve for the individual jacket fluid heat transfer coefficient yields:

$$h_J = (k_J * A1/D_J) * (D_J*V*\rho_J / \mu_J)^{2/3} * (c_J*u_J /k_J)^{b} * (\mu_J/\mu_{Jw})^{0.14} \qquad (27)$$

Calculation of Agitated Liquid Batch Side Heat Transfer Coefficients

Notation for liquid batch side heat transfer coefficients, h_B , will utilize the subscript "B".

The Chilton, Drew, and Jebens benchmark correlation for determining the heat transfer coefficient of the agitated liquid batch to/from the jacketed vessel wall is equation (28) relating the Nusselt number to the agitated liquid batch Reynolds and Prandlt numbers. A correction for viscosity is also included (Reference 4). The assumptions required for this correlation include no phase changes or loss of material for the liquid being agitated.

$$(N_{Nu})= A2 * (N_{Re})^{2/3} * (N_{Pr})^{1/3} * (\mu_B / \mu_{wB})^M \qquad (28)$$

where:

h_B = heat transfer coefficient of the liquid batch being agitated, Btu/ hr-sq. ft- deg F.

D_V = diameter of the vessel, ft.

k_B = thermal conductivity of the batch liquid at the average batch temperature, Btu/hr-sq. ft. ft.-(deg F/ft).

N_{Nu} = Nusselt number = ($h_B * D_V / k_B$).

N_{Re} = Reynolds number for agitated liquids = ($L^2_A * N * \rho_B / \mu_B$).
N_{Pr} = Prandlt number = ($c_B * u_B / k_B$).

μ_B = viscosity of the batch liquid at the average batch liquid temperature, lb./hr-ft.

μ_{wB} = viscosity of the batch liquid at the jacket wall temperature, lb./hr-ft.

c_B = batch liquid heat capacity at the average batch temperature, Btu/ lb.-deg F.

ρ_B = batch liquid density at the average batch temperature, lb./cu. ft.

L^2_A = agitator cross sectional diameter measured tip to tip, ft.

N = agitator speed, revolutions per hour.

A2, M are recommended constants for equation (28) and are given in Table 2.

Rearranging to solve for the individual agitated liquid batch heat transfer coefficient, h_B ,and yields:

$$h_B = (k_B*A2/D_V) * (L^2_A*N*\rho_B /\mu_B)^{2/3} * (c_B*u_B/k_B)^{1/3} * (\mu_B /\mu_{wB})^M \quad (29)$$

Table 1. Typical Fouling Factors, $R_D = 1/h_D$, hr-sq. ft. deg F/Btu
Reference 5.

	LIQUID VELOCITY LESS THAN 3 FT/SEC TO 240 DEG F	**LIQUID VELOCITY GREATER THAN 3 FT/SEC TO 240 DEG F**
Sea Water	0.0005	0.0005
Well Water	0.001	0.001
River Water Clean	0.002	0.001
River Water Muddy	0.008	0.006
Treated Boiler Blowdown	0.001	0.0005
Brine	0.001	0.001
	Gases and Vapors	
Steam (non oil bearing)	0.0	
Air	0.002	
Organic vapors	0.0005	
Refrigerant vapors	0.002	

Table 2. Values for Constants Used in Equation 28.
Reference 6

AGITATOR BLADE TYPE	A2	M	RANGE OF REYNOLD'S NUMBER
3-Blade marine Propeller	0.54	0.14	2,000 (one data point)
Paddle	0.36	0.21	300-300,000
Disk, Flat-Blade Turbine	0.54	0.14	40-300,000
Pitched-blade turbine	0.53	0.24	80-200
Anchor	0.36	0.18	300-40,000
			See Reference 6 for equipment description of each agitator blade type; pp. 19-4, 19-5.

Physical Property Data, Table 3.

COMPOUND	PHYSICAL PROPERTY	REFERENCE #
Water	**Temperature = 125 deg F**	
	Liquid Density = 8.37 lbs./gal	7
	Liquid Heat Capacity = 0.997 Btu/lb.-F	7
	Liquid Viscosity = 0.536 centipoise	7
	Liquid Thermal Conductivity = 0.368 Btu/hr-ft-deg F	7
Water	**Temperature = 44 deg F**	
	Liquid Density = 8.71 lbs./gal	7
	Liquid Heat Capacity = 1.01 Btu/lb.-F	7
	Liquid Viscosity = 1.45 centipoise	7
	Liquid Thermal Conductivity = 0.335 Btu/hr-ft-deg F	7
Condensate	**Temperature = 366 deg F**	
	Liquid Density = 7.20 lbs./gal	7
	Liquid Heat Capacity = 1.07 Btu/lb.-F	7
	Liquid Viscosity = 0.147 centipoise	7
	Liquid Thermal Conductivity = 0.389 Btu/hr-ft-deg F	7
Steam	Saturation Temperature @ 164.7 psia = 366 deg F	9

Example #1

It is proposed to cool a batch of heated water at 170 deg F down to 80 deg F in an agitated vessel similar to the one shown in Figure 2. Chilled well water at 40 deg F is available from a Plant header system at a rate of 100 gpm. A temperature gauge in the return line for a similar process indicates an outlet temperature of 48 deg F. Calculate the over-all heat transfer coefficient for the process. Equipment details are given below:

Equipment Data:
Vessel Diameter = 9.0 ft.
Agitator Diameter = 3.5 ft.
Agitator type = Flat blade disk turbine
Agitator speed = constant 45 rpm
Vessel jacket height = 8.0 ft.
Vessel jacket flow depth = 1.0 in

Step #1.1: Calculate the average temperature of the batch liquid.

$TBatch_{avg}$ = the average temperature of the batch liquid, expressed as deg F. This will be used as the temperature for determining the physical properties.

$$TBatch_{avg} = (T_{initial} + T_{final})/2$$

$$TBatch_{avg} = (170 + 80)/2 = 125 \text{ deg F}$$

$$TBatch_{avg} = 125 \text{ deg F}$$

Step #1.2: Determine the agitated batch liquid physical properties required for heat transfer calculation:

a. From Table 3. the physical properties of the batch liquid at the average temperature are:

ρ_B = Liquid Density = 8.37 lbs./gal
c_B = Liquid Heat Capacity = 0.997 Btu/lb.-F

μ_B = Liquid Viscosity = 0.536 centipoise
k_B = Liquid Thermal Conductivity = 0.368 Btu/hr-ft-deg F

Converting liquid density to lbs./cu. ft. requires multiplying the density from Table 3 by 7.48 and results in a liquid density of ρ_B = 62.6 lbs./cu. ft.

Converting viscosity to lb./hr-ft units requires multiplying the viscosity from Table 3 by 2.42 and results in a liquid viscosity of μ_B = 1.30 lb./hr-ft.

b. The average wall temperature of the chilled water is calculated in Step #1.8 and is 44 deg F.

μ_{wB} = Wall Viscosity = 1.45 centipoise

Converting viscosity to lb./hr-ft units requires multiplying the viscosity from Table 3 by 2.42 and results in a wall viscosity of μ_{wB} = 3.52 lb./hr-ft.

Step #1.3: Determine the agitator speed in appropriate units:

N = revolutions per hour (rph). The agitation speed is given in the example as 45 revolutions per minute, which when multiplied by 60 minutes/hr yields:

$$N = 45 \text{ rpm} * 60 = 2{,}700 \text{ rph}$$

Step #1.4: Determine the agitated batch liquid Reynolds number:

N_{Re} = Reynolds number for agitated liquids = ($L^2_A * N * \rho_B / \mu_B$).

$$N_{Re} = (3.5^2 * 2{,}700 * 62.6 / 1.30)$$

$$N_{Re} = 1.59 \text{ E+06}$$

Step #1.5: Determine the agitated batch liquid Prandlt number:

$$N_{Pr} = \text{Prandlt number} = (c_B * u_B / k_B).$$
$$N_{Pr} = (0.997 * 1.30 / 0.368)$$

$$N_{Pr} = 3.52$$

Step #1.6: Determine the ratio of bulk to wall viscosity:

$$\text{Viscosity Ratio} = (\mu_B / \mu_{w\,B})$$

$$\text{Viscosity Ratio} = (1.30 / 3.75)$$

$$\text{Viscosity Ratio} = 0.347$$

Step #1.7: Determine the agitated liquid individual heat transfer coefficient:

Equation (29) is used with the values as calculated in the above steps:

$$h_B = (k_B*A2/D_V) * (L^2{}_A*N*\rho_B/\mu_B)^{2/3} * (c_B*u_B/k_B)^{1/3} * (\mu_B/\mu_{wB})^M \qquad (29)$$

based on the physical description of the agitator blade values for the constants are obtained from Table 2 are: A2 = 0.54, M = 0.14.

$$h_B = (0.368*0.54 /9.0) * (1.59\ E{+}06)^{2/3} * (3.52)^{1/3} * (0.347)^{0.14}$$

$$h_B = 0.022 * 14{,}406 * 1.16 * 0.869$$

$$h_B = 321 \text{ Btu/ hr-sq. ft- deg F.}$$

Step #1.8: Calculate the average temperature of the chilled water.

$Tmedia_{avg}$ = the average temperature of the chilled water, expressed as deg F. This will be used as the temperature for determining the physical properties.

T_{avg} = ($T_{initial}$ + T_{final})/2

T_{avg} = (40 + 48)/2 = 44 deg F

T_{avg} = 44 deg F

Step #1.9: Determine the chilled water physical properties required for heat transfer calculation:

a. From Table 3. the physical properties at the average temperature are:

ρ_J = Liquid Density = 8.71 lbs./gal
c_J = Liquid Heat Capacity = 1.01 Btu/lb.-F
μ_J = Liquid Viscosity = 1.45 centipoise
k_J = Liquid Thermal Conductivity = 0.335 Btu/hr-ft-deg F

Converting liquid density to lbs./cu. ft. requires multiplying the density in Table 3 by 7.48 and results in a chilled water density of ρ_J = 65.2 lbs./cu. ft.

Converting viscosity to lb./hr-ft units requires multiplying the viscosity in Table 3 by 2.42 and results in a chilled water viscosity of μ_J = 3.51lb./hr-ft.

b. The average wall temperature and the average bulk fluid temperature are identical. The wall viscosity ratio is the 1.

Viscosity Ratio = (μ / μ_w) = 1

Step # 1.10: Determine the Jacket Cross-flow diameter and cross-flow area.

The flow area of the jacket is not circular, but approximated by a rectangular duct that is 1.0 inch wide by 96 inches long. For duct shapes other than circular, an equivalent diameter, $D_{equivalent}$, can be calculated as:

$$D_J = D_{Equivalent} = 4 * A_{CS} / W_{etted}$$

where:

$D_{Equivalent}$ = Equivalent circular diameter of non circular duct, inches; Reference 8.

A_{CS} = Area of cross-sectional flow, square inches.

W_{etted} = Perimeter of duct that has wetted flow, inches.

$$A_{CS} = \text{Length} * \text{Width}$$

$$A_{CS} = 1.0 * 8 * 12$$

$$A_{CS} = 96 \text{ sq. in. or } 0.67 \text{ sq. ft.}$$

$$W_{etted} = (\text{Length} * 2) + (\text{Width} * 2)$$

$$W_{etted} = (1.0 * 2) + (96 * 2)$$

$$W_{etted} = 194 \text{ in}$$

$$D_J = 4 * 96 / 194$$

$$D_J = 1.98 \text{ in. or } 0.165 \text{ ft.}$$

$A_{Equivalent}$ = Equivalent cross-flow area of non circular duct, square feet calculated using equivalent circular diameter.

$$A_{Equivalent} = (\pi /4) * D_J^{\,2}$$

$$A_{Equivalent} = (\pi /4) * (0.165)^{2}$$

$$A_{Equivalent} = 0.0214 \text{ sq. ft.}$$

Step #1.11: Calculate the jacket water velocity:

Velocity = volumetric flow/ equivalent cross sectional area of flow

Volumetric Flow = 100 gal/min * min/60 sec * cu. ft/ 7.48 gals

Volumetric Flow = 0.22 cu ft/sec

Vel = 0.22 cu ft/sec / 0.0214 sq. ft.

Vel = 10.3 ft/sec or 37,080 ft/hr.

Step #1.12: Determine the chilled water jacket Reynolds number:

N_{Re} = Reynolds number for jacket = ($D_J * V * \rho_J / \mu_J$).

$$N_{Re} = (0.165 * 37{,}080 * 65.2 / 3.51)$$

$$N_{Re} = 114{,}926$$

Step #1.13: Determine the chilled water jacket Prandlt number:

N_{Pr} = Prandlt number = ($c_J * u_J / k_J$).

$$N_{Pr} = (1.01 * 3.51 / 0.335)$$

$$N_{Pr} = 10.6$$

Step #1.14: Determine the jacket side heat transfer coefficient using equation (27): A1=0.0265, b=0.3

$$h_J = (k_J * A1 / D_J) * (D_J * V * \rho_J / \mu_J)^{2/3} * (c_J * u_J / k_J)^b * (\mu_J/\mu_{Jw})^{0.14} \qquad (27)$$

$$h_J = (0.335 * 0.0265 / 0.165) * (114{,}926)^{2/3} * (10.6)^{0.3} * (1)^{0.14}$$

$$h_J = 0.0538 * 2{,}355 * 2.03 * 1$$

h_J = 258 Btu/ hr-sq. ft- deg F.

Step # 1.15: Calculate the over-all heat transfer coefficient:

Using Application equation (23) as the Over-all heat transfer coefficient for the simplified case in agitated vessels:

$$1/U_{Over\text{-}all} = 1/h_J + 1/h_B + 1/h_{DM} \qquad (23)$$

h_{DM} is obtained from Table 1 for well water:

$R_{DM} = 1/h_{DM} = 0.001$ or

h_{DM} = 1,000 Btu/ hr-sq. ft- deg F.

$$1/U_{Over\text{-}all} = 1/258 + 1/321 + 1/1{,}000$$

$$1/U_{Over\text{-}all} = 0.004 + 0.003 + 0.001$$

$$1/U_{Over\text{-}all} = 0.008$$

the over-all heat transfer coefficient is:

$U_{Over\text{-}all}$ = 125 Btu/ hr-sq. ft- deg F.

Example #2

It is proposed to heat a batch of cold water from 80 deg F to 170 deg F in an agitated vessel similar to the one shown in Figure 1. Saturated steam is available from a Plant header system at a pressure of 150 psig. Calculate the over-all heat transfer coefficient for the process if it is assumed that 10,000 lbs. per hour of steam will be used in the jacket. Equipment details are given below:

<u>Equipment Data:</u>
Vessel Diameter = 9.0 ft.
Agitator Diameter = 3.5 ft.
Agitator type = Flat blade disk turbine
Agitator speed = constant 45 rpm
Vessel jacket height = 8.0 ft.
Vessel jacket flow depth = 1.0 in

Step #2.1: Calculate the average temperature of the batch water.

$TBatch_{avg}$ = the average temperature of the batch liquid, expressed as deg F. This will be used as the temperature for determining the physical properties.

$TBatch_{avg} = (T_{initial} + T_{final})/2$

$TBatch_{avg} = (80 + 170)/2 = 125$ deg F

$TBatch_{avg} = 125$ deg F

Step #2.2: Determine the agitated batch liquid physical properties required for heat transfer calculation:

a. From Table 3. the physical properties of the batch liquid at the average temperature are:

ρ_B = Liquid Density = 8.37 lbs./gal
c_B = Liquid Heat Capacity = 0.997 Btu/lb.-F
μ_B = Liquid Viscosity = 0.536 centipoise
k_B = Liquid Thermal Conductivity = 0.368 Btu/hr-ft-deg F

Converting liquid density to lbs./cu. ft. requires multiplying the density in Table 3 by 7.48 and results in a liquid density of ρ_B = 62.6 lbs./cu. ft.

Converting viscosity to lb./hr-ft units requires multiplying the viscosity in Table 3 by 2.42 and results in a liquid viscosity of μ_B = 1.30 lb./hr-ft.

Step #2.3: Determine the steam/condensate average temperature.

The average temperature of the steam/condensate is determined by its saturation temperature. From Table 3, this is 366 deg F.

μ_{wB} = Wall Viscosity = 0.147 centipoise

Converting viscosity to lb./hr-ft units requires multiplying the viscosity in Table 3 by 2.42 and results in a wall viscosity of μ_{wB} = 0.356 lb./hr-ft.

Step #2.4: Determine the agitator speed in appropriate units:

N = revolutions per hour (rph). The agitation speed given in the example was 45 revolutions per minute, which when multiplied by 60 minutes/hr yields:

N = 45 rpm * 60 = 2,700 rph

Step #2.5: Determine the agitated batch liquid Reynolds number:

N_{Re} = Reynolds number for agitated liquids = ($L^2_A * N * \rho_B / \mu_B$).

$$N_{Re} = (3.5^2 * 2{,}700 * 62.6 / 1.30)$$

$$N_{Re} = 1.59 \text{ E+06}$$

Step #2.6: Determine the agitated batch liquid Prandlt number:

$$N_{Pr} = \text{Prandlt number} = (c_B * u_B / k_B).$$

$$N_{Pr} = (0.997 * 1.30 / 0.368)$$

$$N_{Pr} = 3.52$$

Step #2.7: Determine the ratio of bulk to wall viscosity:

$$\text{Viscosity Ratio} = (\mu_B / \mu_{w\,B})$$

$$\text{Viscosity Ratio} = (1.30 / 0.356)$$

$$\text{Viscosity Ratio} = 3.65$$

Step #2.8: Determine the agitated liquid individual heat transfer coefficient:

Equation (29) is used with the values as calculated in the above steps:

$$h_B = (k_B*A2/D_V) * (L^2{}_A*N*\rho_B/\mu_B)^{2/3} * (c_B*u_B/k_B)^{1/3} * (\mu_B/\mu_{wB})^M \quad (29)$$

based on the physical description of the agitator blade values for the constants are obtained from Table 2 are: A2 = 0.54, M = 0.14.

$$h_B = (0.368*0.54/9.0) * (1.59 \text{ E+06})^{2/3} * (3.52)^{1/3} * (3.65)^{0.14}$$

$$h_B = 0.022 * 13{,}600 * 1.52 * 1.20$$

$$h_B = 548 \text{ Btu/ hr-sq. ft- deg F.}$$

Step #2.9: Determine the physical properties required for heat transfer calculation for the condensing steam. As per equation (14) the properties are determined by using the values for the condensate film.

From Table 3. the physical properties of the condensate film at the saturation temperature are:

ρ_J = Liquid Density = 7.20 lbs./gal
c_J = Liquid Heat Capacity = 1.07 Btu/lb.-F
μ_J = Liquid Viscosity = 0.147 centipoise
k_J = Liquid Thermal Conductivity = 0.389 Btu/hr-ft-deg F

Converting liquid density to lbs./cu. ft. requires multiplying the density from Table 3 by 7.48 and results in a condensate film density of ρ_J = 53.9 lbs./cu. ft.

Converting viscosity to lb./hr-ft units requires multiplying the viscosity from Table 3 by 2.42 and results in a condensate film viscosity of μ_J = 0.356 lb./hr-ft.

Step #2.10: Calculate the condensate film Reynolds number. Using equation (13):

$$N_{Re\ Film} = 4 * \Gamma / \mu_J \qquad (13)$$

By definition the condensate mass loading rate is:

Γ = condensate loading rate, lbs./ft.-hr. This is calculated as the mass rate of condensate per length of vessel jacket perimeter.

The perimeter is calculated as:

Perimeter = Vessel diameter * π

Perimeter = 9.0 feet * 3.14

Perimeter = 28.3 feet

The steam mass rate was given in the example statement as 10,000 lbs./hour. The condensate film Reynolds number can then be calculated as:

$$N_{Re\ Film} = 4 * 10{,}000 / (28.3 * 0.356)$$

$$N_{Re\ Film} = 3{,}970$$

Step #2.11: Calculate the heat transfer coefficient of the condensate film.

Using equation (25) is valid since the Reynolds number is greater than 2,100.

$$h_J * (\mu_F^2 / (k_F^3 * \rho_F^2 * g))^{1/3} = 0.0076 * N_{Re\ Film}^{\ 0.4} \qquad (25)$$

Rearranging for h_J yields:

$$h_J = 0.0076 * N_{Re\ Film}^{\ 0.4} / (\mu_F^2 / (k_F^3 * \rho_F^2 * g))^{1/3}$$

Substituting values for the parameters on the right hand side of the equations yields:

$$h_J = 0.0076 * (3{,}970)^{0.4} / (0.356^2 / (0.389^3 * 53.9^2 * 4.17\ E{+}08))^{1/3}$$

$$h_J = 0.0076 * 27.5 / 1.22\ E{-}04$$

$$h_J = 1{,}710 \text{ Btu/ hr-sq. ft- deg F.}$$

Step # 2.12: Calculate the over-all heat transfer coefficient:

Using Application equation (23) as the Over-all heat transfer coefficient for the simplified case in agitated vessels:

$$1/U_{\text{Over-all}} = 1/h_J + 1/h_B + 1/h_{DM} \qquad (23)$$

where:

h_{DM} is obtained from Table 1 for steam:

$$R_{DM} = 1/h_{DM} = 0.0$$

substituting in the values for h_B and h_J :

$$1/U_{\text{Over-all}} = 1/\ 1{,}710 + 1/\ 423 + 0$$

$$1/U_{\text{Over-all}} = 0.00058 + 0.0024$$

$$1/U_{\text{Over-all}} = 0.00295$$

the over-all heat transfer coefficient is:

$$U_{\text{Over-all}} = 339 \text{ Btu/ hr-sq. ft- deg F.}$$