



**PDHonline Course E176 (3 PDH)**

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# **Induction Motor Theory**

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# INDUCTION MOTOR THEORY

*Jerry Bednarczyk, PE*

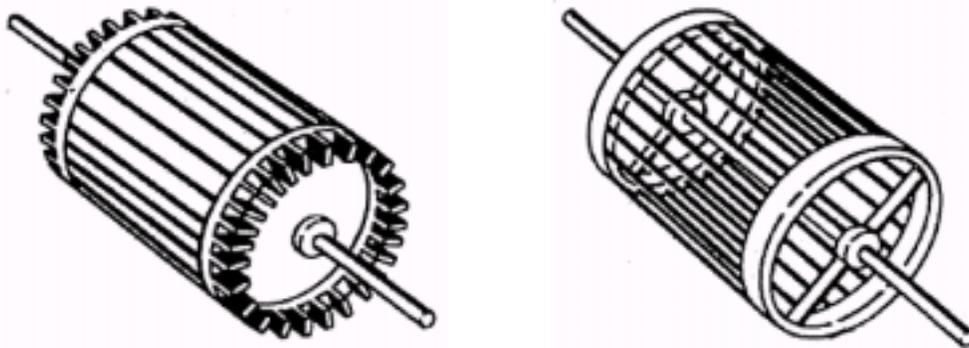
## Course Content

### INTRODUCTION

The AC induction motor is well suited to applications requiring constant speed operation. In general, the induction motor is cheaper and easier to maintain compared to other alternatives.

The induction motor is made up of the stator, or stationary windings, and the rotor. The stator consists of a series of wire windings of very low resistance permanently attached to the motor frame. As a voltage and a current is applied to the stator winding terminals, a magnetic field is developed in the windings. By the way the stator windings are arranged, the magnetic field appears to synchronously rotate electrically around the inside of the motor housing.

The rotor is comprised of a number of thin bars, usually aluminum, mounted in a laminated cylinder. The bars are arranged horizontally and almost parallel to the rotor shaft. At the ends of the rotor, the bars are connected together with a "shorting ring." The rotor and stator are separated by an air gap which allows free rotation of the rotor.



### Typical Induction Motor Rotors

The magnetic field generated in the stator induces an EMF in the rotor bars. In turn, a current is produced in the rotor bars and shorting ring and another magnetic field is induced in the rotor with an opposite polarity of that in the stator. The magnetic field, revolving in the stator, will then produce the torque which will "pull" on the field in the rotor and establish rotor rotation.

In the design of the induction motor, operational characteristics can be determined through a series of calculations. Performing these calculations can help the engineer

provide a motor that is best suited to a particular application. This paper will demonstrate their application.

## SYNCHRONOUS SPEED

The speed with which the stator magnetic field rotates, which will determine the speed of the rotor, is called the Synchronous Speed (SS). The SS is a function of the frequency of the power source and the number of poles (pole pairs) in the motor. The relationship to calculate the SS of an induction motor is:

$$1 \quad SS = (120 \times f) / P$$

Where:

SS = Synchronous Speed (RPM)

f = frequency (cycles / second) = 60

P = number of poles (pole pairs)

Examples:

*For a 2 pole motor:*

$$SS = (120 \times 60) / 2$$

$$SS = 3600 \text{ RPM}$$

*For an 8 pole motor:*

$$SS = (120 \times 60) / 8$$

$$SS = 900 \text{ RPM}$$

## MOTOR SLIP

The rotor in an induction motor can not turn at the synchronous speed. In order to induce an EMF in the rotor, the rotor must move slower than the SS. If the rotor were to somehow turn at SS, the EMF could not be induced in the rotor and therefore the rotor would stop. However, if the rotor stopped or even if it slowed significantly, an EMF would once again be induced in the rotor bars and it would begin rotating at a speed less than the SS.

The relationship between the rotor speed and the SS is called the Slip. Typically, the Slip is expressed as a percentage of the SS. The equation for the motor Slip is:

$$2 \quad \% S = \frac{(SS - RS)}{SS} \times 100$$

Where:

%S = Percent Slip

SS = Synchronous Speed (RPM)

RS = Rotor Speed (RPM)

Example:

SS = 900 RPM  
 RS = 885 RPM

$$\%S = \frac{(900 - 885)}{900} \times 100$$

%S = 1.7% (or S = 0.017)

If the motor slip is known, the rotor speed can be expressed by the equation:

3  $RS = (1 - S) \times SS$

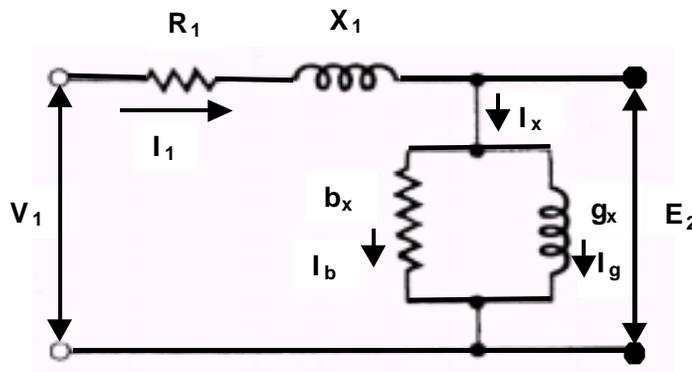
Example (using the information from above):

SS = 900 RPM  
 %S = 1.7% (or S = 0.017)

$RS = (1 - S) \times SS$   
 $RS = (1 - 0.017) \times 900 \text{ RPM}$   
 RS = 885 RPM

**EQUIVALENT CIRCUIT**

To analyze the operating and performance characteristics of an induction motor, an Equivalent Circuit can be drawn. We will consider a 3-phase, Y connected machine, the Equivalent Circuit for the stator is as shown below:



Where:

$V_1$  = Stator Terminal Voltage

$I_1$  = Stator Current

$R_1$  = Stator Effective Resistance

$X_1$  = Stator Leakage Reactance

$Z_1$  = Stator Impedance ( $R_1 + jX_1$ )

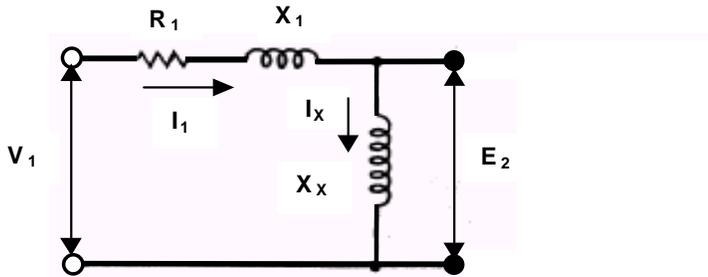
$I_x$  = Exciting Current (this is comprised of the core loss component =  $I_g$ , and a magnetizing current =  $I_b$ )

$E_2$  = Counter EMF (generated by the air gap flux)

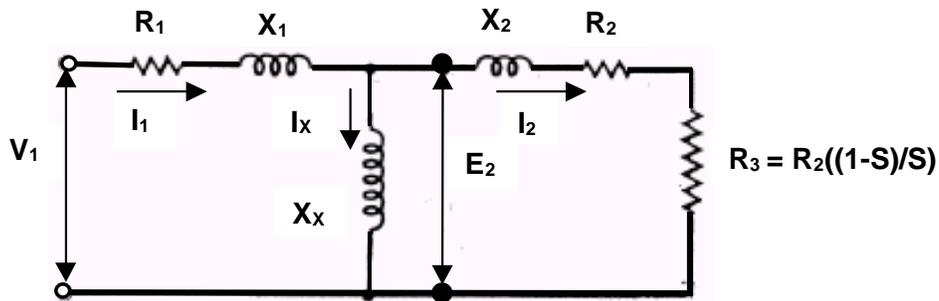
The counter EMF ( $E_2$ ) is equal to the stator terminal voltage less the voltage drop caused by the stator leakage impedance.

4  $E_2 = V_1 - I_1 (Z_1)$   
 $E_2 = V_1 - I_1 (R_1 + j X_1)$

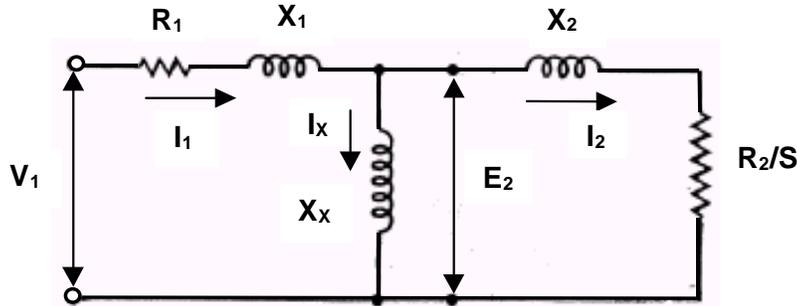
In an analysis of an induction motor, the equivalent circuit can be simplified further by omitting the shunt reaction value,  $g_x$ . The core losses associated with this value can be subtracted from the motor Power and Torque when the friction, windage and stray losses are deducted. The simplified circuit for the stator then becomes:



To complete the circuit, the component for the rotor equivalent must be added.



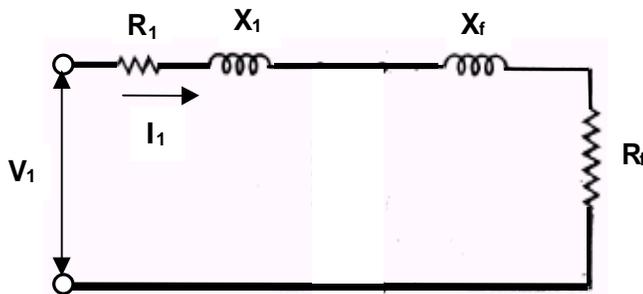
Simplifying the rotor component will yield the following per phase approximate Equivalent Circuit. This circuit will be used in the motor analysis in the sections following.



- $I_2$  = Rotor Current
- $R_2$  = Rotor Resistance
- $X_2$  = Rotor Reactance
- $Z_2$  = Rotor Impedance ( $R_2 / S + jX_2$ )
- $X_x = Z_x$  = Air Gap Impedance

What the stator sees in the air gap is the equivalent of putting an impedance (equal to  $R_2/s + jX_2$ ) across  $E_2$ . In the analysis, all components in the motor equivalent circuit are referred to the stator.

Solving for the Rotor Impedance in parallel with the Air Gap Impedance will yield the following simplified equivalent circuit:



5)  $Z_{RF} = R_f + j X_f = (R_2 / S + jX_2)$  in parallel with  $jX_x$

**EQUIVALENT CIRCUIT ANALYSIS**

The characteristics of an induction motor, the speed, current, starting torque, maximum torque, the motor losses and efficiency can all be determined from an analysis of the equivalent circuit.

### Current and Power Factor

To begin the analysis, the impedance of the motor needs to be calculated. The impedance,  $Z_{RF}$ , is the equivalent per phase impedance seen by the stator across the motor gap.

$$\begin{aligned} \textcircled{6} \quad Z_{RF} &= Z_2 \text{ in parallel with } Z_x \\ &= (R_2/S + j X_2) \text{ in parallel with } jX_x \\ &= (R_f + j X_f) \end{aligned}$$

The total impedance,  $Z$ , is then:

$$\textcircled{7} \quad Z = Z_1 + Z_{RF}$$

The stator current is:

$$\textcircled{8} \quad I_1 = V_1 / Z$$

### Motor Power Calculation:

The total power ( $P_g$ ) that will be transferred across the air gap is given by the equation

$$\begin{aligned} \textcircled{9} \quad P_g &= q \times I_2^2 \times R_2/S \\ &= q \times I_1^2 \times R_f \end{aligned}$$

Where:

$P_g$  = Power transferred across the air gap  
 $q$  = number of stator phases

The rotor copper loss ( $I^2 R$  losses) are given by:

$$\begin{aligned} \textcircled{10} \quad RCL &= \text{rotor copper loss} = S \times (q \times I_2^2 \times R_2/S) \\ &= S \times (q \times I_1^2 \times R_f) \\ &= S \times P_g \end{aligned}$$

The total power ( $P_T$ ) that can then be delivered to the rotor is the power delivered across the air gap ( $P_g$ ) less the rotor copper losses, or:

$$\begin{aligned} \textcircled{11} \quad P_T &= P_g - RCL \\ &= (q \times I_2^2 \times R_2/S) - (q \times I_2^2 \times R_2) \\ &= (q \times I_2^2 \times R_2) \times ((1-S)/S) \\ &= P_g (1-S) \end{aligned}$$

This equation illustrates that as the value of the slip increases in an induction motor, the efficiency decreases.

### Motor Torque:

To determine the internal electromagnetic torque developed as it relates to the internal power ( $P_g$ ), recall that the mechanical power is equal to the torque (T) times the angular velocity, or  $P = W_s T$ .

If  $W_s$  = the synchronous angular velocity of the rotor, then:

$$(12) \quad P_g = (1-s) W_s T$$

Solving for T:

$$(13) \quad T = 1 / W_s (P / (1-s))$$

or

$$T = 1 / W_s (q \times I_2^2 \times R_2 / S)$$

or

$$T = 1 / W_s (q \times I_1^2 \times R_F)$$

Where:

$$W_s = 4\pi f / \# \text{ of poles}$$

$$f = \text{frequency} = 60 \text{ Hz}$$

The calculated rotor torque and power value however, are not the values that are available at the rotor shaft. Losses due to friction, windage and stray load losses need to be subtracted from the calculated Torque & Power values to determine the output at the rotor shaft.

### Motor Efficiency

The last calculation to consider is to determine the motor efficiencies. These can be calculated by comparing the losses to the total power input of the motor.

$$(14) \quad \text{Efficiency} = 1.000 - (\text{Total Losses}) / (\text{Motor Input})$$

**EXAMPLE**

For this example, refer to the motor equivalent circuit and calculations described above. A 3-phase, Y connected, 460 volt (line to line), 25 horsepower, 60 Hz, 8 pole induction motor has the following constants, in ohms per phase referred to the stator:

$$\begin{aligned} R_1 &= 0.342 \\ X_1 &= 0.561 \\ Z_1 &= 0.342 + j0.561 \\ R_2 &= 0.164 \\ X_2 &= 0.265 \\ Z_2 &= (0.164 / 0.02) + j0.265 = 8.20 + j0.265 \\ X_x = Z_x &= 14.75 \\ \text{Friction, windage and stray load losses} &= 265 \text{ Watts} \end{aligned}$$

For a slip of 2.00% (0.02), calculate the speed, stator current, power factor, output torque and power and efficiency when the motor is operated at rated voltage and frequency.

The synchronous speed of the motor is given by equation (1):

$$\begin{aligned} SS &= (120 \times f) / P \\ &= (120 \times 60) / 8 \\ &= 900 \text{ rpm} \end{aligned}$$

Rotor speed (motor slip is known), equation (2):

$$\begin{aligned} RS &= (1 - S) \times SS \\ &= (1 - 0.02) \times 900 \\ &= 882 \text{ rpm} \end{aligned}$$

The impedance,  $Z_{RF}$ , is the equivalent per phase impedance seen by the stator across the motor gap (equation (5)).

$$\begin{aligned} Z_{RF} &= Z_2 \text{ in parallel with } Z_x \\ Z_{RF} &= (R_2/S + j X_2) \text{ in parallel with } X_x \\ Z_{RF} &= (0.164/ 0.02 + j 0.265) \text{ in parallel with } j 14.75 \end{aligned}$$

**CALCULATING THE EQUIVALENT IMPEDANCE:**

In calculating the equivalent impedance, solving for  $Z_{rf}$  becomes very tedious using the standard form. The equation, (6), is:

$$\begin{aligned} Z_{RF} &= [(Z_2) \times (Z_x)] / (Z_2 + Z_x) \\ Z_{RF} &= [(R_2/S + j X_2) \times (j X_x)] / [(R_2/S + j X_2) + (j X_x)] \end{aligned}$$

An alternate method to solve for the equivalent impedance is to convert the impedances to polar form, perform the math and then convert back to the standard form.

### EXAMPLE

Using the data from above, convert each of the components to polar form.

$$Z_2 = (R_2/S + j X_2) = (0.164 / 0.02 + j 0.265) = (8.20 + j 0.265)$$

Convert this to polar form as follows:

$$Z_2 = \sqrt{(8.20^2 + 0.265^2)} \angle \text{TAN}^{-1}(0.265 / 8.20)$$

$$Z_2 = 8.20 \text{ ohm} \angle 1.85 \text{ deg}$$

$$Z_x = X_x = j14.75$$

Convert this to polar:

$$Z_x = \sqrt{(14.75^2)} \angle \text{TAN}^{-1}(14.75 / 0)$$

$$Z_x = 14.75 \text{ ohm} \angle 90 \text{ deg}$$

$$Z_2 + Z_x = (R_2/S + j X_2) + X_x$$

$$= (8.20 + j 0.265) + j14.75$$

$$= (8.20 + j 15.015)$$

Converting to polar:

$$Z_2 + Z_x = \sqrt{(8.2^2 + 15.015^2)} \angle \text{TAN}^{-1}(15.015 / 8.2)$$

$$Z_2 + Z_x = 17.11 \text{ ohm} \angle 61.4 \text{ deg}$$

Therefore,  $Z_{RF}$  becomes:

$$Z_{RF} = [(Z_2) \times (Z_x)] / (Z_2 + Z_x)$$

This equation is solved by multiplying/dividing the real numbers and adding/subtracting the angles (add the angles in the numerator and subtract the angles in the denominator). Therefore:

$$Z_{RF} = [(8.20 \angle 1.85 \text{ deg}) \times (14.75 \angle 90 \text{ deg})] / 17.11 \angle 61.4 \text{ deg}$$

$$= [(8.20) \times (14.75)] / 17.11 \text{ ohms} \angle (1.85 + 90 - 61.4) \text{ deg}$$

$$= 7.07 \text{ ohm} \angle 30.5 \text{ deg}$$

Now then, the result can be converted back to the standard form with the following equation:

$$Z_{RF} = 7.07 (\cosine 30.5 \text{ deg}) + j 7.07 (\sine 30.5 \text{ deg})$$

$$= 6.09 + j3.59 \text{ ohm}$$

$$= R_f + jX_f$$

Finally, the total impedance, Z, is determined by adding the components  $Z_1 + Z_{RF}$  (equation (7))

$$\begin{aligned} Z &= Z_1 + Z_{RF} \\ Z &= (0.342 + j0.561) + (6.09 + j3.59) \\ Z &= 6.432 + j4.151 \end{aligned}$$

$$\begin{aligned} \text{Converting to polar: } Z &= \sqrt{(6.432^2 + 4.151^2)} \text{ TAN}^{-1}(4.151 / 6.432) \\ Z &= 7.66 \text{ ohm } \angle 32.8 \text{ deg} \end{aligned}$$

This result above (Z) will be used to determine the motor stator current ( $I_1$ ) and the power factor.

Continuing with the example:

For a 3 phase motor, the applied voltage to neutral:

$$\begin{aligned} V_1 &= 460V / \sqrt{3} \\ V_1 &= 266V \end{aligned}$$

The stator current is (equation (8)):

$$\begin{aligned} I_1 &= V_1 / Z \\ I_1 &= 266V / 7.66 \text{ ohm} \\ I_1 &= 34.7A \end{aligned}$$

$$\begin{aligned} \text{Power factor} &= \text{Cosine } 32.8 \text{ deg} \\ &= 0.8 \end{aligned}$$

#### Motor Power Calculation:

The total power ( $P_g$ ) that will be transferred across the air gap (equation (9)):

$$\begin{aligned} P_g &= q \times I_2^2 \times R_2 / S \\ &= q \times I_1^2 \times R_f \\ &= 3 \times (34.7)^2 A \times 6.09 \text{ ohms} \\ &= 21,999 \text{ Watts} \end{aligned}$$

The rotor copper loss ( $I^2 R$  losses), equation (10):

$$\begin{aligned} \text{RCL} = \text{rotor copper loss} &= S \times (q \times I_2^2 \times R_2 / S) \\ &= S \times (q \times I_1^2 \times R_f) \\ &= S \times P_g \\ &= 0.02 \times 21,999W \\ &= 440 W \end{aligned}$$

The total power ( $P_T$ ) that can then be delivered to the rotor at full power is the power delivered across the air gap ( $P_g$ ) less the rotor copper losses, or (equation (11)):

$$\begin{aligned}P_T &= P_g - RCL \\ &= P_g - (S \times P_g) \\ &= P_g (1-S) \\ &= 21,999W \times (1.00 - 0.02) \\ &= 21,559W\end{aligned}$$

You will note that this equation illustrates that as the value of the slip increases in an induction motor, the efficiency decreases.

### Motor Torque:

To determine the internal electromagnetic torque developed as it related to the internal power ( $P_g$ ), recall that mechanical power is equal to the torque ( $T$ ) times the angular velocity, or  $P = W_s T$ .

If  $W_s$  = the synchronous angular velocity of the rotor, then (equation (12)):

$$P_g = (1-s) W_s T$$

Where:

$$W_s = 4\pi f / \# \text{ of poles}$$

$$f = \text{frequency} = 60 \text{ Hz}$$

In this example:

$$W_s = 4\pi f / \# \text{ of poles}$$

$$= 4\pi 60 / 8 \text{ poles}$$

$$= 94.2$$

Solving for  $T$  (equation (13)):

$$\begin{aligned}T &= P_g / \{(1-s) \times W_s\} \\ &= 21,999 / \{(1-0.02) \times 94.2\} \\ &= 238.3 \text{ newton-meters (NM)}\end{aligned}$$

### Motor Efficiency

The motor efficiencies can be calculated from the losses (equation (14)):

Total stator copper losses = (# of phases x $I_1^2 \times R_1$ ) = $3 \times (34.7)^2 \times 0.342$	= 1,235 watts
Rotor copper losses = RCL	= 440
Friction, windage and core losses (given)	= <u>265</u>
Total losses	= 1,940 watts
Motor Output ( $P_T = P_g - RCL$ )	= <u>21,550</u>
Motor Input	= 23,490 watts

$$\begin{aligned} \text{(Total losses) / (Motor Input)} &= 1,940 / 23,490 \\ &= 0.083 \end{aligned}$$

$$\begin{aligned} \text{Efficiency} &= 1.000 - 0.083 = 0.917 \\ &= 91.7\% \end{aligned}$$

### **SUMMARY**

These formulas, while presenting idealized conditions in a relatively simplified manner, provide the engineer a basic understanding of the variables to be considered in the design of motors. Factors such as motor starting factors (i.e; high inrush currents), varying speeds and other items all would need to be considered in the total design.