



**PDHonline Course E239 (4 PDH)**

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# **Electrical Fundamentals – Reactance and Impedance**

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# CHAPTER 4

## INDUCTIVE AND CAPACITIVE REACTANCE

### LEARNING OBJECTIVES

Upon completion of this chapter you will be able to:

1. State the effects an inductor has on a change in current and a capacitor has on a change in voltage.
2. State the phase relationships between current and voltage in an inductor and in a capacitor.
3. State the terms for the opposition an inductor and a capacitor offer to ac
4. Write the formulas for inductive and capacitive reactances.
5. State the effects of a change in frequency on  $X_L$  and  $X_C$ .
6. State the effects of a change in inductance on  $X_L$  and a change in capacitance on  $X_C$ .
7. Write the formula for determining total reactance ( $X$ ); compute total reactance ( $X$ ) in a series circuit; and indicate whether the total reactance is capacitive or inductive.
8. State the term given to the total opposition ( $Z$ ) in an ac circuit.
9. Write the formula for impedance, and calculate the impedance in a series circuit when the values of  $X_C$ ,  $X_L$ , and  $R$  are given.
10. Write the Ohm's law formulas used to determine voltage and current in an ac circuit.
11. Define true power, reactive power, and apparent power; state the unit of measurement for and the formula used to calculate each.
12. State the definition of and write the formula for power factor.
13. Given the power factor and values of  $X$  and  $R$  in an ac circuit, compute the value of reactance in the circuit, and state the type of reactance that must be connected in the circuit to correct the power factor to unity (1).
14. State the difference between calculating impedance in a series ac circuit and in a parallel ac circuit.

### INDUCTIVE AND CAPACITIVE REACTANCE

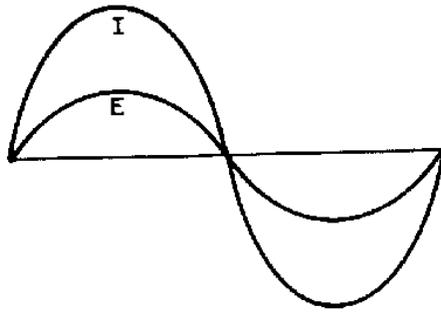
You have already learned how inductance and capacitance individually behave in a direct current circuit. In this chapter you will be shown how inductance, capacitance, and resistance affect alternating current.

## INDUCTANCE AND ALTERNATING CURRENT

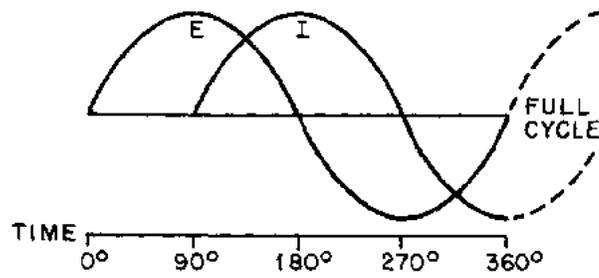
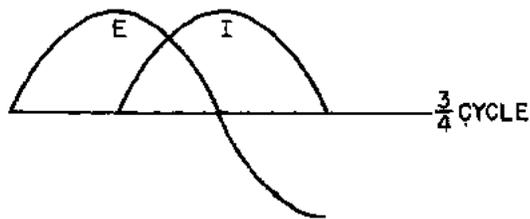
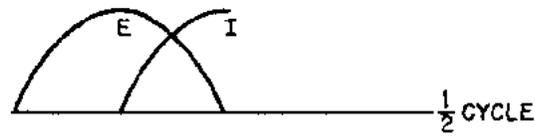
This might be a good place to recall what you learned about phase in chapter 1. When two things are in step, going through a cycle together, falling together and rising together, they are in phase. When they are out of phase, the angle of lead or lag-the number of electrical degrees by which one of the values leads or lags the other-is a measure of the amount they are out of step. The time it takes the current in an inductor to build up to maximum and to fall to zero is important for another reason. It helps illustrate a very useful characteristic of inductive circuits-the current through the inductor always lags the voltage across the inductor.

A circuit having pure resistance (if such a thing were possible) would have the alternating current through it and the voltage across it rising and falling together. This is illustrated in figure 4-1(A), which shows the sine waves for current and voltage in a purely resistive circuit having an ac source. The current and voltage do not have the same amplitude, but they are in phase.

In the case of a circuit having inductance, the opposing force of the counter emf would be enough to keep the current from remaining in phase with the applied voltage. You learned that in a dc circuit containing pure inductance the current took time to rise to maximum even though the full applied voltage was immediately at maximum. Figure 4-1(B) shows the wave forms for a purely inductive ac circuit in steps of quarter-cycles.



(A)



(B)

Figure 4-1.—Voltage and current waveforms in an inductive circuit.

With an ac voltage, in the first quarter-cycle ( $0^\circ$  to  $90^\circ$ ) the applied ac voltage is continually increasing. If there was no inductance in the circuit, the current would also increase during this first quarter-cycle. You know this circuit does have inductance. Since inductance opposes any change in current flow, no current flows during the first quarter-cycle. In the next quarter-cycle ( $90^\circ$  to  $180^\circ$ ) the voltage decreases back to zero; current begins to flow in the circuit and reaches a maximum value at the same instant the voltage reaches zero. The applied voltage now begins to build up to maximum in the other direction, to be followed by the resulting current. When the voltage again reaches its maximum at the end of the third quarter-cycle ( $270^\circ$ ) all values are exactly opposite to what they were during the first half-cycle. The applied voltage leads the resulting current by one quarter-cycle or 90 degrees. To complete the full  $360^\circ$  cycle of the voltage, the voltage again decreases to zero and the current builds to a maximum value.

You must not get the idea that any of these values stops cold at a particular instant. Until the applied voltage is removed, both current and voltage are always changing in amplitude and direction.

As you know the sine wave can be compared to a circle. Just as you mark off a circle into 360 degrees, you can mark off the time of one cycle of a sine wave into 360 electrical degrees. This relationship is shown in figure 4-2. By referring to this figure you can see why the current is said to lag the voltage, in a purely inductive circuit, by 90 degrees. Furthermore, by referring to figures 4-2 and 4-1(A) you can see why the current and voltage are said to be in phase in a purely resistive circuit. In a circuit having both resistance and inductance then, as you would expect, the current lags the voltage by an amount somewhere between 0 and 90 degrees.

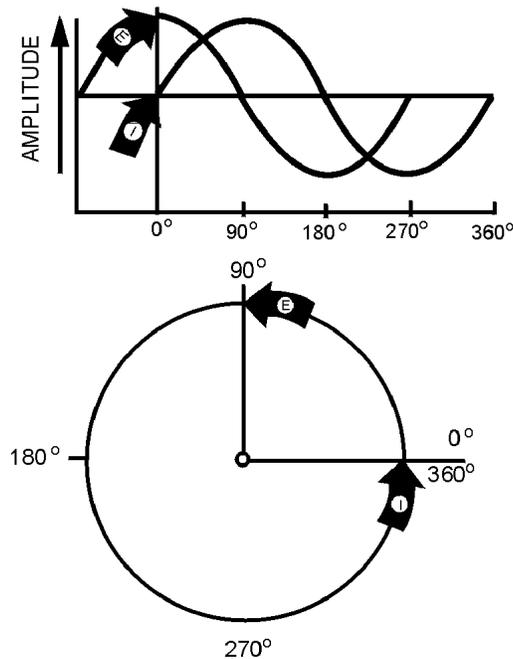


Figure 4-2.—Comparison of sine wave and circle in an inductive circuit.

A simple memory aid to help you remember the relationship of voltage and current in an inductive circuit is the word ELI. Since E is the symbol for voltage, L is the symbol for inductance, and I is the symbol for current; the word ELI demonstrates that current comes after (Lags) voltage in an inductor.

*Q1. What effect does an inductor have on a change in current?*

Q2. *What is the phase relationship between current and voltage in an inductor?*

### INDUCTIVE REACTANCE

When the current flowing through an inductor continuously reverses itself, as in the case of an ac source, the inertia effect of the cemf is greater than with dc. The greater the amount of inductance (L), the greater the opposition from this inertia effect. Also, the faster the reversal of current, the greater this inertial opposition. This opposing force which an inductor presents to the FLOW of alternating current cannot be called resistance, since it is not the result of friction within a conductor. The name given to it is INDUCTIVE REACTANCE because it is the "reaction" of the inductor to the changing value of alternating current. Inductive reactance is measured in ohms and its symbol is  $X_L$ .

As you know, the induced voltage in a conductor is proportional to the rate at which magnetic lines of force cut the conductor. The greater the rate (the higher the frequency), the greater the cemf. Also, the induced voltage increases with an increase in inductance; the more ampere-turns, the greater the cemf. Reactance, then, increases with an increase of frequency and with an increase of inductance. The formula for inductive reactance is as follows:

$$X_L = 2\pi fL$$

Where:

$X_L$  is inductive reactance in ohms.

$2\pi$  is a constant in which the Greek letter  $\pi$ , called "pi" represents 3.1416 and  $2 \times \pi = 6.28$  approximately.

$f$  is frequency of the alternating current in Hz.

$L$  is inductance in henrys.

The following example problem illustrates the computation of  $X_L$ .

Given:  $f = 60 \text{ Hz}$

$L = 20 \text{ H}$

Solution:  $X_L = 2\pi fL$

$X_L = 6.28 \times 60 \text{ Hz} \times 20 \text{ H}$

$X_L = 7,536 \Omega$

Q3. *What is the term for the opposition an inductor presents to ac?*

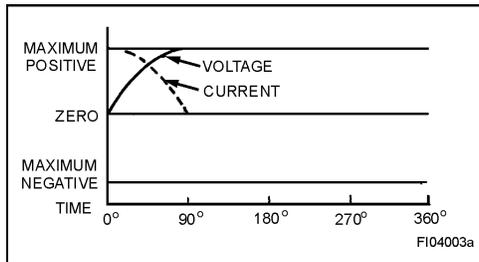
Q4. *What is the formula used to compute the value of this opposition?*

Q5. *What happens to the value of  $X_L$  as frequency increases?*

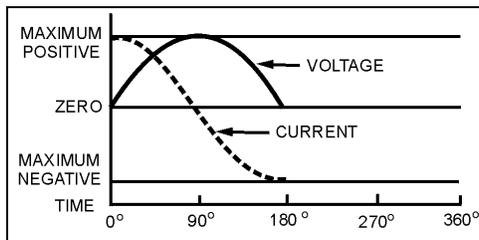
Q6. *What happens to the value of  $X_L$  as inductance decreases?*

## CAPACITORS AND ALTERNATING CURRENT

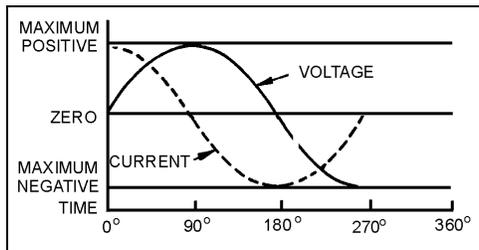
The four parts of figure 4-3 show the variation of the alternating voltage and current in a capacitive circuit, for each quarter of one cycle. The solid line represents the voltage across the capacitor, and the dotted line represents the current. The line running through the center is the zero, or reference point, for both the voltage and the current. The bottom line marks off the time of the cycle in terms of electrical degrees. Assume that the ac voltage has been acting on the capacitor for some time before the time represented by the starting point of the sine wave in the figure.



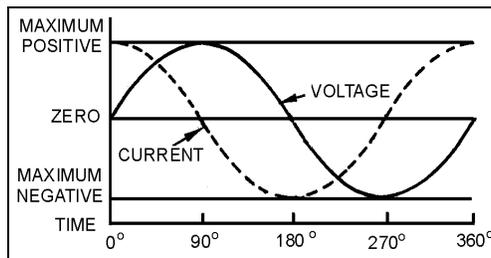
(A)



(B)



(C)



(D)

**Figure 4-3.—Phase relationship of voltage and current in a capacitive circuit.**

At the beginning of the first quarter-cycle ( $0^\circ$  to  $90^\circ$ ) the voltage has just passed through zero and is increasing in the positive direction. Since the zero point is the steepest part of the sine wave, the voltage is changing at its greatest rate. The charge on a capacitor varies directly with the voltage, and therefore the charge on the capacitor is also changing at its greatest rate at the beginning of the first quarter-cycle. In other words, the greatest number of electrons are moving off one plate and onto the other plate. Thus the capacitor current is at its maximum value, as part (A) of the figure shows.

As the voltage proceeds toward maximum at 90 degrees, its rate of change becomes less and less, hence the current must decrease toward zero. At 90 degrees the voltage across the capacitor is maximum, the capacitor is fully charged, and there is no further movement of electrons from plate to plate. That is why the current at 90 degrees is zero.

At the end of this first quarter-cycle the alternating voltage stops increasing in the positive direction and starts to decrease. It is still a positive voltage, but to the capacitor the decrease in voltage means that the plate which has just accumulated an excess of electrons must lose some electrons. The current flow, therefore, must reverse its direction. Part (B) of the figure shows the current curve to be below the zero line (negative current direction) during the second quarter-cycle ( $90^\circ$  to  $180^\circ$ ).

At 180 degrees the voltage has dropped to zero. This means that for a brief instant the electrons are equally distributed between the two plates; the current is maximum because the rate of change of voltage is maximum. Just after 180 degrees the voltage has reversed polarity and starts building up its maximum negative peak which is reached at the end of the third quarter-cycle ( $180^\circ$  to  $270^\circ$ ). During this third quarter-cycle the rate of voltage change gradually decreases as the charge builds to a maximum at 270 degrees. At this point the capacitor is fully charged and it carries the full impressed voltage. Because the capacitor is fully charged there is no further exchange of electrons; therefore, the current flow is zero at this point. The conditions are exactly the same as at the end of the first quarter-cycle ( $90^\circ$ ) but the polarity is reversed.

Just after 270 degrees the impressed voltage once again starts to decrease, and the capacitor must lose electrons from the negative plate. It must discharge, starting at a minimum rate of flow and rising to a maximum. This discharging action continues through the last quarter-cycle ( $270^\circ$  to  $360^\circ$ ) until the impressed-voltage has reached zero. At 360 degrees you are back at the beginning of the entire cycle, and everything starts over again.

If you examine the complete voltage and current curves in part D, you will see that the current always arrives at a certain point in the cycle 90 degrees ahead of the voltage, because of the charging and discharging action. You know that this time and place relationship between the current and voltage is called the phase relationship. The voltage-current phase relationship in a capacitive circuit is exactly opposite to that in an inductive circuit. The current of a capacitor leads the voltage across the capacitor by 90 degrees.

You realize that the current and voltage are both going through their individual cycles at the same time during the period the ac voltage is impressed. The current does not go through part of its cycle (charging or discharging), stop, and wait for the voltage to catch up. The amplitude and polarity of the voltage and the amplitude and direction of the current are continually changing. Their positions with respect to each other and to the zero line at any electrical instant-any degree between zero and 360 degrees-can be seen by reading upwards from the time-degree line. The current swing from the positive peak at zero degrees to the negative peak at 180 degrees is NOT a measure of the number of electrons, or the charge on the plates. It is a picture of the direction and strength of the current in relation to the polarity and strength of the voltage appearing across the plates.

At times it is convenient to use the word "ICE" to recall to mind the phase relationship of the current and voltage in capacitive circuits. I is the symbol for current, and in the word ICE it leads, or comes before, the symbol for voltage, E. C, of course, stands for capacitor. This memory aid is similar to the "ELI" used to remember the current and voltage relationship in an inductor. The phrase "ELI the ICE man" is helpful in remembering the phase relationship in both the inductor and capacitor.

Since the plates of the capacitor are changing polarity at the same rate as the ac voltage, the capacitor seems to pass an alternating current. Actually, the electrons do not pass through the dielectric, but their rushing back and forth from plate to plate causes a current flow in the circuit. It is convenient, however, to say that the alternating current flows "through" the capacitor. You know this is not true, but the expression avoids a lot of trouble when speaking of current flow in a circuit containing a capacitor. By the same short cut, you may say that the capacitor does not pass a direct current (if both plates are connected to a dc source, current will flow only long enough to charge the capacitor). With a capacitor type of hookup in a circuit containing both ac and dc, only the ac will be "passed" on to another circuit.

You have now learned two things to remember about a capacitor: A capacitor will appear to conduct an alternating current and a capacitor will not conduct a direct current.

*Q7. What effect does the capacitor have on a changing voltage?*

*Q8. What is the phase relationship between current and voltage in a capacitor?*

## **CAPACITIVE REACTANCE**

So far you have been dealing with the capacitor as a device which passes ac and in which the only opposition to the alternating current has been the normal circuit resistance present in any conductor. However, capacitors themselves offer a very real opposition to current flow. This opposition arises from the fact that, at a given voltage and frequency, the number of electrons which go back and forth from plate to plate is limited by the storage ability—that is, the capacitance—of the capacitor. As the capacitance is increased, a greater number of electrons change plates every cycle, and (since current is a measure of the number of electrons passing a given point in a given time) the current is increased.

Increasing the frequency will also decrease the opposition offered by a capacitor. This occurs because the number of electrons which the capacitor is capable of handling at a given voltage will change plates more often. As a result, more electrons will pass a given point in a given time (greater current flow). The opposition which a capacitor offers to ac is therefore inversely proportional to frequency and to capacitance. This opposition is called CAPACITIVE REACTANCE. You may say that capacitive reactance decreases with increasing frequency or, for a given frequency, the capacitive reactance decreases with increasing capacitance. The symbol for capacitive reactance is  $X_C$ .

Now you can understand why it is said that the  $X_C$  varies inversely with the product of the frequency and capacitance. The formula is:

$$X_C = \frac{1}{2\pi fC}$$

Where:

$X_C$  is capacitive reactance in ohms

f is frequency in Hertz

C is capacitance in farads

$\pi$  is 6.28 ( $2 \times 3.1416$ )

The following example problem illustrates the computation of  $X_C$ .

Given:  $f = 100 \text{ Hz}$   
 $C = 50 \mu\text{F}$

Solution:  $X_C = \frac{1}{2\pi fC}$   
 $X_C = \frac{1}{6.28 \times 100 \text{ Hz} \times 50 \mu\text{F}}$   
 $X_C = \frac{1}{.0314} \Omega$   
 $X_C = 31.8 \Omega \text{ or } 32 \Omega$

*Q9. What is the term for the opposition that a capacitor presents to ac?*

*Q10. What is the formula used to compute this opposition?*

*Q11. What happens to the value of  $X_C$  as frequency decreases?*

*Q12. What happens to the value of  $X_C$  as capacitance increases?*

## **REACTANCE, IMPEDANCE, AND POWER RELATIONSHIPS IN AC CIRCUITS**

Up to this point inductance and capacitance have been explained individually in ac circuits. The rest of this chapter will concern the combination of inductance, capacitance, and resistance in ac circuits.

To explain the various properties that exist within ac circuits, the series RLC circuit will be used. Figure 4-4 is the schematic diagram of the series RLC circuit. The symbol shown in figure 4-4 that is marked E is the general symbol used to indicate an ac voltage source.

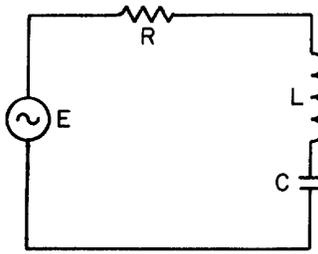


Figure 4-4.—Series RLC circuit.

## REACTANCE

The effect of inductive reactance is to cause the current to lag the voltage, while that of capacitive reactance is to cause the current to lead the voltage. Therefore, since inductive reactance and capacitive reactance are exactly opposite in their effects, what will be the result when the two are combined? It is not hard to see that the net effect is a tendency to cancel each other, with the combined effect then equal to the difference between their values. This resultant is called REACTANCE; it is represented by the symbol  $X$ ; and expressed by the equation  $X = X_L - X_C$  or  $X = X_C - X_L$ . Thus, if a circuit contains 50 ohms of inductive reactance and 25 ohms of capacitive reactance in series, the net reactance, or  $X$ , is 50 ohms – 25 ohms, or 25 ohms of inductive reactance.

For a practical example, suppose you have a circuit containing an inductor of 100  $\mu\text{H}$  in series with a capacitor of .001  $\mu\text{F}$ , and operating at a frequency of 4 MHz. What is the value of net reactance, or  $X$ ?

Given:  $f = 4 \text{ MHz}$   
 $L = 100 \mu\text{H}$   
 $C = .001 \mu\text{F}$

Solution:  $X_L = 2\pi fL$   
 $X_L = 6.28 \times 4 \text{ MHz} \times 100 \mu\text{H}$   
 $X_L = 2512 \Omega$   
 $X_C = \frac{1}{2\pi fC}$   
 $X_C = \frac{1}{6.28 \times 4 \text{ MHz} \times .001 \mu\text{F}}$   
 $X_C = \frac{1}{.02512} \Omega$   
 $X_C = 39.8 \Omega$   
 $X = X_L - X_C$   
 $X = 2512 \Omega - 39.8 \Omega$   
 $X = 2472.2 \Omega$  (inductive)

Now assume you have a circuit containing a 100 -  $\mu\text{H}$  inductor in series with a .0002- $\mu\text{F}$  capacitor, and operating at a frequency of 1 MHz. What is the value of the resultant reactance in this case?

Given:  $f = 1 \text{ MHz}$   
 $L = 100 \mu\text{H}$   
 $C = .0002 \mu\text{F}$

Solution:  $X_L = 2\pi fL$   
 $X_L = 6.28 \times 1 \text{ MHz} \times 100 \mu\text{H}$   
 $X_L = 628 \Omega$   
 $X_C = \frac{1}{2\pi fC}$   
 $X_C = \frac{1}{6.28 \times 1 \text{ MHz} \times .0002 \mu\text{F}}$   
 $X_C = \frac{1}{.001256} \Omega$   
 $X_C = 796 \Omega$   
 $X = X_C - X_L$   
 $X = 796 \Omega - 628 \Omega$   
 $X = 168 \Omega \text{ (capacitive)}$

You will notice that in this case the inductive reactance is smaller than the capacitive reactance and is therefore subtracted from the capacitive reactance.

These two examples serve to illustrate an important point: when capacitive and inductive reactance are combined in series, the smaller is always subtracted from the larger and the resultant reactance always takes the characteristics of the larger.

- Q13. What is the formula for determining total reactance in a series circuit where the values of  $X_C$  and  $X_L$  are known?*
- Q14. What is the total amount of reactance ( $X$ ) in a series circuit which contains an  $X_L$  of 20 ohms and an  $X_C$  of 50 ohms? (Indicate whether  $X$  is capacitive or inductive)*

## IMPEDANCE

From your study of inductance and capacitance you know how inductive reactance and capacitive reactance act to oppose the flow of current in an ac circuit. However, there is another factor, the resistance, which also opposes the flow of the current. Since in practice ac circuits containing reactance also contain resistance, the two combine to oppose the flow of current. This combined opposition by the resistance and the reactance is called the IMPEDANCE, and is represented by the symbol  $Z$ .

Since the values of resistance and reactance are both given in ohms, it might at first seem possible to determine the value of the impedance by simply adding them together. It cannot be done so easily, however. You know that in an ac circuit which contains only resistance, the current and the voltage will be in step (that is, in phase), and will reach their maximum values at the same instant. You also know that in an ac circuit containing only reactance the current will either lead or lag the voltage by one-quarter of a cycle or 90 degrees. Therefore, the voltage in a purely reactive circuit will differ in phase by 90 degrees from that in a purely resistive circuit and for this reason reactance and resistance are not combined by simply adding them.

When reactance and resistance are combined, the value of the impedance will be greater than either. It is also true that the current will not be in step with the voltage nor will it differ in phase by exactly 90 degrees from the voltage, but it will be somewhere between the in-step and the 90-degree out-of-step conditions. The larger the reactance compared with the resistance, the more nearly the phase difference will approach 90°. The larger the resistance compared to the reactance, the more nearly the phase difference will approach zero degrees.

If the value of resistance and reactance cannot simply be added together to find the impedance, or Z, how is it determined? Because the current through a resistor is in step with the voltage across it and the current in a reactance differs by 90 degrees from the voltage across it, the two are at right angles to each other. They can therefore be combined by means of the same method used in the construction of a right-angle triangle.

Assume you want to find the impedance of a series combination of 8 ohms resistance and 5 ohms inductive reactance. Start by drawing a horizontal line, R, representing 8 ohms resistance, as the base of the triangle. Then, since the effect of the reactance is always at right angles, or 90 degrees, to that of the resistance, draw the line X<sub>L</sub>, representing 5 ohms inductive reactance, as the altitude of the triangle. This is shown in figure 4-5. Now, complete the hypotenuse (longest side) of the triangle. Then, the hypotenuse represents the impedance of the circuit.

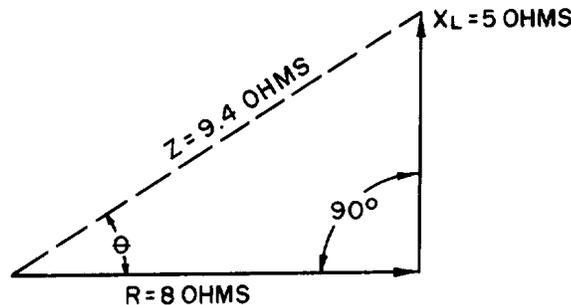


Figure 4-5.—Vector diagram showing relationship of resistance, inductive reactance, and impedance in a series circuit.

One of the properties of a right triangle is:

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$$

or,

$$\text{hypotenuse} = \sqrt{(\text{base})^2 + (\text{altitude})^2}$$

Applied to impedance, this becomes,

$$(\text{impedance})^2 = (\text{resistance})^2 + (\text{reactance})^2$$

or,

$$\text{impedance} = \sqrt{(\text{resistance})^2 + (\text{reactance})^2}$$

or,

$$Z = \sqrt{R^2 + X^2}$$

Now suppose you apply this equation to check your results in the example given above.

Given:  $R = 8 \Omega$   
 $X_L = 5 \Omega$

Solution:  $Z = \sqrt{R^2 + X_L^2}$   
 $Z = \sqrt{(8 \Omega)^2 + (5 \Omega)^2}$   
 $Z = \sqrt{64 + 25 \Omega}$   
 $Z = \sqrt{89 \Omega}$  (See the Appendix III  
for a square Root  
Table.)  
 $Z = 9.4 \Omega$

When you have a capacitive reactance to deal with instead of inductive reactance as in the previous example, it is customary to draw the line representing the capacitive reactance in a downward direction. This is shown in figure 4-6. The line is drawn downward for capacitive reactance to indicate that it acts in a direction opposite to inductive reactance which is drawn upward. In a series circuit containing capacitive reactance the equation for finding the impedance becomes:

$$Z = \sqrt{R^2 + X_C^2}$$

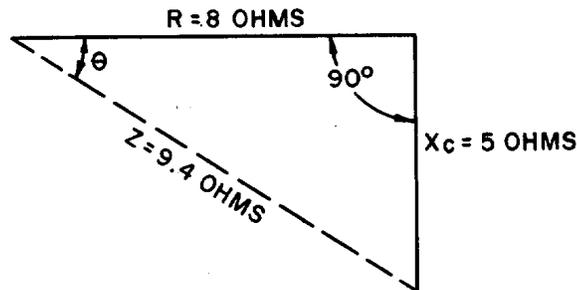


Figure 4-6.—Vector diagram showing relationship of resistance, capacitive reactance, and impedance in a series circuit.

In many series circuits you will find resistance combined with both inductive reactance and capacitive reactance. Since you know that the value of the reactance, X, is equal to the difference between the values of the inductive reactance,  $X_L$ , and the capacitive reactance,  $X_C$ , the equation for the impedance in a series circuit containing R,  $X_L$ , and  $X_C$  then becomes:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or,

$$Z = \sqrt{R^2 + X^2}$$

(Note: The formulas  $Z = \sqrt{R^2 + X_L^2}$ ,  
 $Z = \sqrt{R^2 + X_C^2}$ , and  $Z = \sqrt{R^2 + X^2}$  can be  
 used to calculate Z only if the resistance and  
 reactance are connected in series.)

In figure 4-7 you will see the method which may be used to determine the impedance in a series circuit consisting of resistance, inductance, and capacitance.

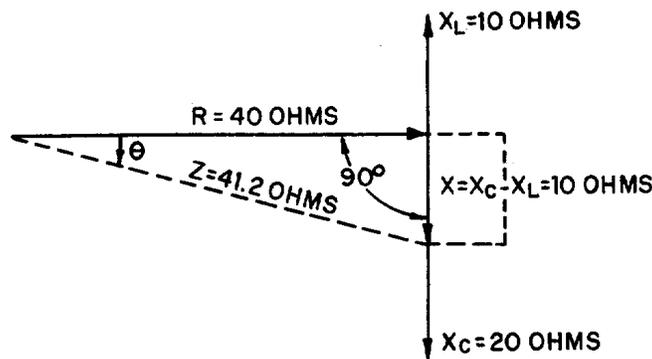


Figure 4-7.—Vector diagram showing relationship of resistance, reactance (capacitive and inductive), and impedance in a series circuit.

Assume that 10 ohms inductive reactance and 20 ohms capacitive reactance are connected in series with 40 ohms resistance. Let the horizontal line represent the resistance R. The line drawn upward from the end of R, represents the inductive reactance,  $X_L$ . Represent the capacitive reactance by a line drawn downward at right angles from the same end of R. The resultant of  $X_L$  and  $X_C$  is found by subtracting  $X_L$  from  $X_C$ . This resultant represents the value of X.

Thus:

$$X = X_C - X_L$$

$$X = 10 \text{ ohms}$$

The line, Z, will then represent the resultant of R and X. The value of Z can be calculated as follows:

Given:  $X_L = 10 \Omega$   
 $X_C = 20 \Omega$   
 $R = 40 \Omega$

$$\begin{aligned}
\text{Solution: } X &= X_C - X_L \\
X &= 20 \Omega - 10 \Omega \\
X &= 10 \Omega \\
Z &= \sqrt{R^2 + X^2} \\
Z &= \sqrt{(40 \Omega)^2 + (10 \Omega)^2} \\
Z &= \sqrt{1600 + 100} \Omega \\
Z &= \sqrt{1700} \Omega \\
Z &= 41.2 \Omega
\end{aligned}$$

Q15. What term is given to total opposition to ac in a circuit?

Q16. What formula is used to calculate the amount of this opposition in a series circuit?

Q17. What is the value of Z in a series ac circuit where  $X_L = 6$  ohms,  $X_C = 3$  ohms, and  $R = 4$  ohms?

### OHMS LAW FOR AC

In general, Ohm's law cannot be applied to alternating-current circuits since it does not consider the reactance which is always present in such circuits. However, by a modification of Ohm's law which does take into consideration the effect of reactance we obtain a general law which is applicable to ac circuits. Because the impedance, Z, represents the combined opposition of all the reactances and resistances, this general law for ac is,

$$I = \frac{E}{Z}$$

this general modification applies to alternating current flowing in any circuit, and any one of the values may be found from the equation if the others are known.

For example, suppose a series circuit contains an inductor having 5 ohms resistance and 25 ohms inductive reactance in series with a capacitor having 15 ohms capacitive reactance. If the voltage is 50 volts, what is the current? This circuit can be drawn as shown in figure 4-8.

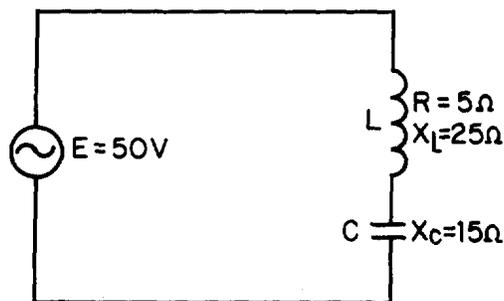


Figure 4-8.—Series LC circuit.

Given:  $R = 5 \Omega$   
 $X_L = 25 \Omega$   
 $X_C = 15 \Omega$   
 $E = 50 \text{ V}$

Solution:  $X = X_L - X_C$   
 $X = 25 \Omega - 15 \Omega$   
 $X = 10 \Omega$   
 $Z = \sqrt{R^2 + X^2}$   
 $Z = \sqrt{(5 \Omega)^2 + (10 \Omega)^2}$   
 $Z = \sqrt{25 + 100 \Omega}$   
 $Z = \sqrt{125 \Omega}$   
 $Z = 11.2 \Omega$   
 $I = \frac{E}{Z}$   
 $I = \frac{50 \text{ V}}{11.2 \Omega}$   
 $I = 4.46 \text{ A}$

Now suppose the circuit contains an inductor having 5 ohms resistance and 15 ohms inductive reactance in series with a capacitor having 10 ohms capacitive reactance. If the current is 5 amperes, what is the voltage?

Given:  $R = 5 \Omega$   
 $X_L = 15 \Omega$   
 $X_C = 10 \Omega$   
 $I = 5 \text{ A}$

Solution:  $X = X_L - X_C$   
 $X = 15 \Omega - 10 \Omega$   
 $X = 5 \Omega$   
 $Z = \sqrt{R^2 + X^2}$   
 $Z = \sqrt{(5 \Omega)^2 + (5 \Omega)^2}$   
 $Z = \sqrt{25 + 25} \Omega$   
 $Z = \sqrt{50} \Omega$   
 $Z = 7.07 \Omega$   
 $E = IZ$   
 $E = 5 \text{ A} \times 7.07 \Omega$   
 $E = 35.35 \text{ V}$

*Q18. What are the Ohm's law formulas used in an ac circuit to determine voltage and current?*

## POWER IN AC CIRCUITS

You know that in a direct current circuit the power is equal to the voltage times the current, or  $P = E \times I$ . If a voltage of 100 volts applied to a circuit produces a current of 10 amperes, the power is 1000 watts. This is also true in an ac circuit when the current and voltage are in phase; that is, when the circuit is effectively resistive. But, if the ac circuit contains reactance, the current will lead or lag the voltage by a certain amount (the phase angle). When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives only what is known as the APPARENT POWER. The TRUE POWER depends upon the phase angle between the current and voltage. The symbol for phase angle is  $\theta$  (Theta).

When an alternating voltage is impressed across a capacitor, power is taken from the source and stored in the capacitor as the voltage increases from zero to its maximum value. Then, as the impressed voltage decreases from its maximum value to zero, the capacitor discharges and returns the power to the source. Likewise, as the current through an inductor increases from its zero value to its maximum value the field around the inductor builds up to a maximum, and when the current decreases from maximum to zero the field collapses and returns the power to the source. You can see therefore that no power is used up in either case, since the power alternately flows to and from the source. This power that is returned to the source by the reactive components in the circuit is called REACTIVE POWER.

In a purely resistive circuit all of the power is consumed and none is returned to the source; in a purely reactive circuit no power is consumed and all of the power is returned to the source. It follows that in a circuit which contains both resistance and reactance there must be some power dissipated in the resistance as well as some returned to the source by the reactance. In figure 4-9 you can see the relationship between the voltage, the current, and the power in such a circuit. The part of the power curve which is shown below the horizontal reference line is the result of multiplying a positive instantaneous

value of current by a negative instantaneous value of the voltage, or vice versa. As you know, the product obtained by multiplying a positive value by a negative value will be negative. Therefore the power at that instant must be considered as negative power. In other words, during this time the reactance was returning power to the source.

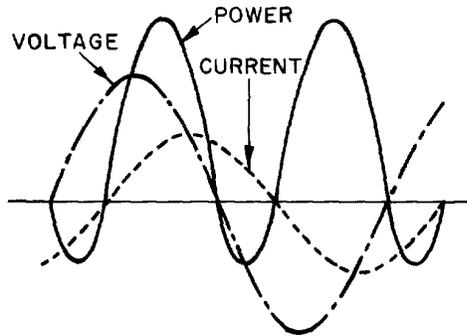


Figure 4-9.—Instantaneous power when current and voltage are out of phase.

The instantaneous power in the circuit is equal to the product of the applied voltage and current through the circuit. When the voltage and current are of the same polarity they are acting together and taking power from the source. When the polarities are unlike they are acting in opposition and power is being returned to the source. Briefly then, in an ac circuit which contains reactance as well as resistance, the apparent power is reduced by the power returned to the source, so that in such a circuit the net power, or true power, is always less than the apparent power.

### Calculating True Power in AC Circuits

As mentioned before, the true power of a circuit is the power actually used in the circuit. This power, measured in watts, is the power associated with the total resistance in the circuit. To calculate true power, the voltage and current associated with the resistance must be used. Since the voltage drop across the resistance is equal to the resistance multiplied by the current through the resistance, true power can be calculated by the formula:

$$\text{True Power} = (I_R)^2 R$$

Where: True Power is measured in watts  
 $I_R$  is resistive current in amperes  
 $R$  is resistance in ohms

For example, find the true power of the circuit shown in figure 4-10.

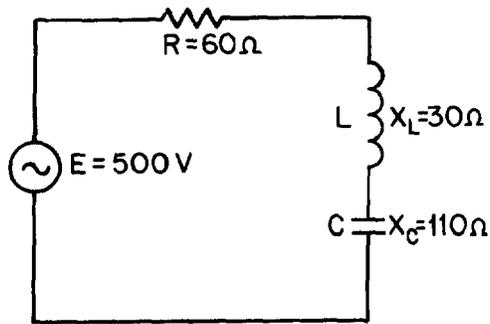


Figure 4-10.—Example circuit for determining power.

Given:  $R = 60 \Omega$   
 $X_L = 30 \Omega$   
 $X_C = 110 \Omega$   
 $E = 500 \text{ V}$

Solution:  $X = X_C - X_L$   
 $X = 110 \Omega - 30 \Omega$   
 $X = 80 \Omega$   
 $Z = \sqrt{R^2 + X^2}$   
 $Z = \sqrt{(60 \Omega)^2 + (80 \Omega)^2}$   
 $Z = \sqrt{3600 + 6400 \Omega}$   
 $Z = \sqrt{10,000 \Omega}$   
 $Z = 100 \Omega$   
 $I = \frac{E}{Z}$   
 $I = \frac{500 \text{ V}}{100 \Omega}$   
 $I = 5 \text{ A}$

Since the current in a series circuit is the same in all parts of the circuit:

$$\begin{aligned} \text{TruePower} &= (I_R)^2 R \\ \text{TruePower} &= (5 \text{ A})^2 \times 60 \Omega \\ \text{True Power} &= 1500 \text{ watts} \end{aligned}$$

Q19. What is the true power in an ac circuit?

Q20. What is the unit of measurement of true power?

Q21. What is the formula for calculating true power?

### Calculating Reactive Power in AC Circuits

The reactive power is the power returned to the source by the reactive components of the circuit. This type of power is measured in Volt-Amperes-Reactive, abbreviated var.

Reactive power is calculated by using the voltage and current associated with the circuit reactance.

Since the voltage of the reactance is equal to the reactance multiplied by the reactive current, reactive power can be calculated by the formula:

$$\text{Reactive Power} = (I_X)^2 X$$

Where: Reactive power is measured in volt-amperes - reactive.

$I_X$  is reactive current in amperes.

$X$  is total reactance in ohms.

Another way to calculate reactive power is to calculate the inductive power and capacitive power and subtract the smaller from the larger.

$$\text{Reactive Power} = (I_L)^2 X_L - (I_C)^2 X_C$$

or

$$(I_C)^2 X_C - (I_L)^2 X_L$$

Where: Reactive power is measured in volt-amperes - reactive.

$I_C$  is capacitive current in amperes.

$X_C$  is capacitive reactance in ohms.

$I_L$  is inductive current in amperes.

$X_L$  is inductive reactance in ohms.

Either one of these formulas will work. The formula you use depends upon the values you are given in a circuit.

For example, find the reactive power of the circuit shown in figure 4-10.

$$\begin{aligned}\text{Given: } X_L &= 30 \Omega \\ X_C &= 110 \Omega \\ X &= 80 \Omega \\ I &= 5 \text{ A}\end{aligned}$$

Since this is a series circuit, current (I) is the same in all parts of the circuit.

$$\begin{aligned}\text{Solution: } \text{Reactive power} &= (I_X)^2 X \\ \text{Reactive power} &= (5 \text{ A})^2 \times 80 \Omega \\ \text{Reactive power} &= 2,000 \text{ var}\end{aligned}$$

$$\begin{aligned}\text{To prove the second formula also works,} \\ \text{Reactive power} &= (I_C)^2 X_C - (I_L)^2 X_L \\ \text{Reactive power} &= (5 \text{ A})^2 \times 110 \Omega - (5 \text{ A})^2 \times 30 \Omega \\ \text{Reactive power} &= 2,750 \text{ var} - 750 \text{ var} \\ \text{Reactive power} &= 2000 \text{ var}\end{aligned}$$

- Q22. What is the reactive power in an ac circuit?*
- Q23. What is the unit of measurement for reactive power?*
- Q24. What is the formula for computing reactive power?*

### **Calculating Apparent Power in AC Circuits.**

Apparent power is the power that appears to the source because of the circuit impedance. Since the impedance is the total opposition to ac, the apparent power is that power the voltage source "sees." Apparent power is the combination of true power and reactive power. Apparent power is not found by simply adding true power and reactive power just as impedance is not found by adding resistance and reactance.

To calculate apparent power, you may use either of the following formulas:

$$\text{Apparent power} = (I_Z)^2 Z$$

Where: Apparent power is measured in  
VA (volt-amperes)

$I_Z$  is impedance current in  
amperes.

$Z$  is impedance in ohms.

or

$$\text{Apparent power} = \sqrt{(\text{True power})^2 + (\text{reactive power})^2}$$

For example, find the apparent power for the circuit shown in figure 4-10.

$$\begin{aligned} \text{Given: } \quad Z &= 100 \, \Omega \\ I &= 5 \, \text{A} \end{aligned}$$

Recall that current in a series circuit is the same in all parts of the circuit.

Solution:

$$\text{Apparent Power} = (I_Z)^2 Z$$

$$\text{Apparent power} = (5 \, \text{A})^2 \times 100 \, \Omega$$

$$\text{Apparent power} = 2500 \, \text{VA}$$

or

Given:

$$\text{True power} = 1500 \, \text{W}$$

$$\text{Reactive power} = 2000 \, \text{var}$$

$$\text{Apparent power} = \sqrt{(\text{True power})^2 + (\text{reactive power})^2}$$

$$\text{Apparent power} = \sqrt{(1500 \, \text{W})^2 + (2000 \, \text{var})^2}$$

$$\text{Apparent power} = \sqrt{625 \times 10^4 \, \text{VA}}$$

$$\text{Apparent power} = 2500 \, \text{VA}$$

Q25. What is apparent power?

Q26. What is the unit of measurement for apparent power?

Q27. What is the formula for apparent power?

## Power Factor

The POWER FACTOR is a number (represented as a decimal or a percentage) that represents the portion of the apparent power dissipated in a circuit.

If you are familiar with trigonometry, the easiest way to find the power factor is to find the cosine of the phase angle ( $\theta$ ). The cosine of the phase angle is equal to the power factor.

You do not need to use trigonometry to find the power factor. Since the power dissipated in a circuit is true power, then:

$$\text{Apparent Power} \times \text{PF} = \text{True Power},$$

$$\text{Therefore,} \quad \text{PF} = \frac{\text{True Power}}{\text{Apparent Power}}$$

If true power and apparent power are known you can use the formula shown above.

Going one step further, another formula for power factor can be developed. By substituting the equations for true power and apparent power in the formula for power factor, you get:

$$\text{PF} = \frac{(I_R)^2 R}{(I_Z)^2 Z}$$

Since current in a series circuit is the same in all parts of the circuit,  $I_R$  equals  $I_Z$ . Therefore, in a series circuit,

$$\text{PF} = \frac{R}{Z}$$

For example, to compute the power factor for the series circuit shown in figure 4-10, any of the above methods may be used.

Given:

$$\begin{aligned}\text{True Power} &= 1500 \text{ W} \\ \text{Apparent Power} &= 2500 \text{ VA}\end{aligned}$$

$$\begin{aligned}\text{Solution:} \quad \text{PF} &= \frac{\text{True Power}}{\text{Apparent Power}} \\ \text{PF} &= \frac{1500 \text{ W}}{2500 \text{ VA}} \\ \text{PF} &= .6\end{aligned}$$

Another method:

Given:  $R = 60 \Omega$   
 $Z = 100 \Omega$

Solution:  $PF = \frac{R}{Z}$   
 $PF = \frac{60 \Omega}{100 \Omega}$   
 $PF = .6$

If you are familiar with trigonometry you can use it to solve for angle  $\theta$  and the power factor by referring to the tables in appendices V and VI.

Given:  $R = 60 \Omega$   
 $X = 80 \Omega$

Solution:  $\tan \theta = \frac{X}{R}$   
 $\tan \theta = \frac{80 \Omega}{60 \Omega}$   
 $\tan \theta = 1.333$   
 $\theta = 53.1^\circ$   
 $PF = \cos \theta$   
 $PF = .6$

NOTE: As stated earlier the power factor can be expressed as a decimal or percentage. In this example the decimal number .6 could also be expressed as 60%.

*Q28. What is the power factor of a circuit?*

*Q29. What is a general formula used to calculate the power factor of a circuit?*

### **Power Factor Correction**

The apparent power in an ac circuit has been described as the power the source "sees". As far as the source is concerned the apparent power is the power that must be provided to the circuit. You also know that the true power is the power actually used in the circuit. The difference between apparent power and true power is wasted because, in reality, only true power is consumed. The ideal situation would be for apparent power and true power to be equal. If this were the case the power factor would be 1 (unity) or 100 percent. There are two ways in which this condition can exist. (1) If the circuit is purely resistive or (2) if the circuit "appears" purely resistive to the source. To make the circuit appear purely resistive there must be no reactance. To have no reactance in the circuit, the inductive reactance ( $X_L$ ) and capacitive reactance ( $X_C$ ) must be equal.

Remember:  $X = X_L - X_C$

Therefore, when

$$X_L = X_C, X = 0$$

The expression "correcting the power factor" refers to reducing the reactance in a circuit.

The ideal situation is to have no reactance in the circuit. This is accomplished by adding capacitive reactance to a circuit which is inductive and inductive reactance to a circuit which is capacitive. For example, the circuit shown in figure 4-10 has a total reactance of 80 ohms capacitive and the power factor was .6 or 60 percent. If 80 ohms of inductive reactance were added to this circuit (by adding another inductor) the circuit would have a total reactance of zero ohms and a power factor of 1 or 100 percent. The apparent and true power of this circuit would then be equal.

*Q30. An ac circuit has a total reactance of 10 ohms inductive and a total resistance of 20 ohms. The power factor is .89. What would be necessary to correct the power factor to unity?*

### **SERIES RLC CIRCUITS**

The principles and formulas that have been presented in this chapter are used in all ac circuits. The examples given have been series circuits.

This section of the chapter will not present any new material, but will be an example of using all the principles presented so far. You should follow each example problem step by step to see how each formula used depends upon the information determined in earlier steps. When an example calls for solving for square root, you can practice using the square-root table by looking up the values given.

The example series RLC circuit shown in figure 4-11 will be used to solve for  $X_L$ ,  $X_C$ ,  $X$ ,  $Z$ ,  $I_T$ , true power, reactive power, apparent power, and power factor.

The values solved for will be rounded off to the nearest whole number.

First solve for  $X_L$  and  $X_C$ .

Given:  $f = 60 \text{ Hz}$   
 $L = 27 \text{ mH}$   
 $C = 380 \mu\text{F}$

Solution:  $X_L = 2\pi fL$   
 $X_L = 6.28 \times 60 \text{ Hz} \times 27 \text{ mH}$   
 $X_L = 10 \Omega$   
 $X_C = \frac{1}{2\pi fC}$   
 $X_C = \frac{1}{6.28 \times 60 \text{ Hz} \times 380 \mu\text{F}}$   
 $X_C = \frac{1}{0.143} \Omega$   
 $X_C = 7 \Omega$

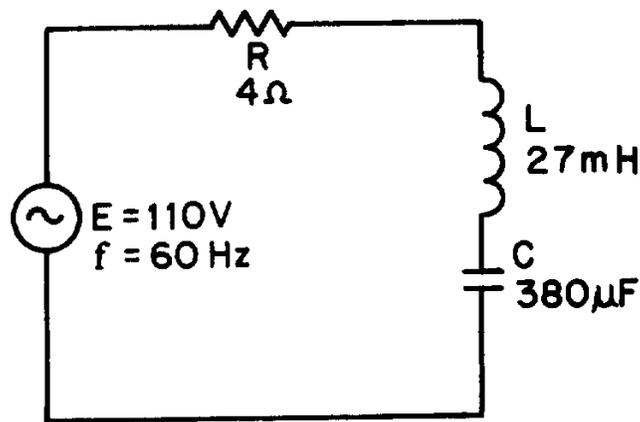


Figure 4-11.—Example series RLC circuit

Now solve for X

Given:  $X_C = 7 \Omega$   
 $X_L = 10 \Omega$

Solution:  $X = X_L - X_C$   
 $X = 10 \Omega - 7 \Omega$   
 $X = 3 \Omega \text{ (Inductive)}$

Use the value of X to solve for Z.

$$\begin{aligned}\text{Given: } X &= 3 \Omega \\ R &= 4 \Omega\end{aligned}$$

$$\begin{aligned}\text{Solution: } Z &= \sqrt{X^2 + R^2} \\ Z &= \sqrt{(3 \Omega)^2 + (4 \Omega)^2} \\ Z &= \sqrt{9 + 16} \Omega \\ Z &= \sqrt{25} \Omega \\ Z &= 5 \Omega\end{aligned}$$

This value of Z can be used to solve for total current ( $I_T$ ).

$$\begin{aligned}\text{Given: } Z &= 5 \Omega \\ E &= 110 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{Solution: } I_T &= \frac{E}{Z} \\ I_T &= \frac{110 \text{ V}}{5 \Omega} \\ I_T &= 22 \text{ A}\end{aligned}$$

Since current is equal in all parts of a series circuit, the value of  $I_T$  can be used to solve for the various values of power.

Given:  $I_T = 22 \text{ A}$   
 $R = 4 \Omega$   
 $X = 3 \Omega$   
 $Z = 5 \Omega$

Solution:

$$\begin{aligned} \text{True Power} &= (I_R)^2 R \\ \text{True Power} &= (22 \text{ A})^2 \times 4 \Omega \\ \text{True Power} &= 1936 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Reactive power} &= (I_X)^2 X \\ \text{Reactive power} &= (22 \text{ A})^2 \times 3 \Omega \\ \text{Reactive power} &= 1452 \text{ var} \end{aligned}$$

$$\begin{aligned} \text{Apparent power} &= (I_Z)^2 Z \\ \text{Apparent Power} &= (22 \text{ A})^2 \times 5 \Omega \\ \text{Apparent Power} &= 2420 \text{ VA} \end{aligned}$$

The power factor can now be found using either apparent power and true power or resistance and impedance. The mathematics in this example is easier if you use impedance and resistance.

Given:  $R = 4 \Omega$   
 $Z = 5 \Omega$

Solution:  $PF = \frac{R}{Z}$   
 $PF = \frac{4 \Omega}{5 \Omega}$   
 $PF = .8 \text{ or } 80\%$

### PARALLEL RLC CIRCUITS

When dealing with a parallel ac circuit, you will find that the concepts presented in this chapter for series ac circuits still apply. There is one major difference between a series circuit and a parallel circuit that must be considered. The difference is that current is the same in all parts of a series circuit, whereas voltage is the same across all branches of a parallel circuit. Because of this difference, the total impedance of a parallel circuit must be computed on the basis of the current in the circuit.

You should remember that in the series RLC circuit the following three formulas were used to find reactance, impedance, and power factor:

$$X = X_L - X_C \text{ or } X = X_C - X_L$$

$$Z = \sqrt{(I_R)^2 + X^2}$$

$$PF = \frac{R}{Z}$$

When working with a parallel circuit you must use the following formulas instead:

$$I_X = I_L - I_C \text{ or } I_X = I_C - I_L$$

$$I_Z = \sqrt{(I_R)^2 + (I_X)^2}$$

$$PF = \frac{I_R}{I_Z}$$

(The impedance of a parallel circuit is found by the formula  $Z = \frac{E}{I_Z}$ )

NOTE: If no value for E is given in a circuit, any value of E can be assumed to find the values of  $I_L$ ,  $I_C$ ,  $I_X$ ,  $I_R$ , and  $I_Z$ . The same value of voltage is then used to find impedance.

For example, find the value of Z in the circuit shown in figure 4-12.

Given:     E = 300 V  
               R = 100 Ω  
                $X_L = 50 \Omega$   
                $X_C = 150 \Omega$

The first step in solving for Z is to calculate the individual branch currents.

$$\begin{aligned} \text{Solution: } I_R &= \frac{E}{R} \\ I_R &= \frac{300 \text{ V}}{100 \Omega} \\ I_R &= 3 \text{ A} \\ I_L &= \frac{E}{X_L} \\ I_L &= \frac{300 \text{ V}}{50 \Omega} \\ I_L &= 6 \text{ A} \\ I_C &= \frac{E}{X_C} \\ I_C &= \frac{300 \text{ V}}{150 \Omega} \\ I_C &= 2 \text{ A} \end{aligned}$$

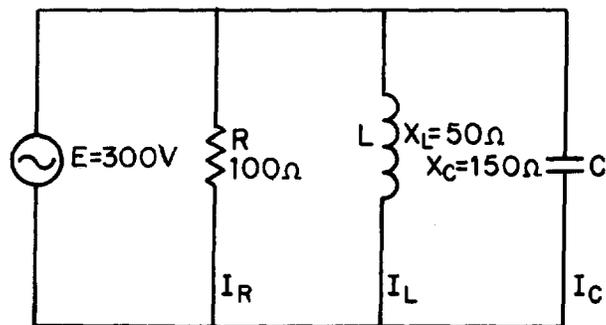


Figure 4-12.—Parallel RLC circuit.

Using the values for  $I_R$ ,  $I_L$ , and  $I_C$ , solve for  $I_X$  and  $I_Z$ .

$$\begin{aligned} I_X &= I_L - I_C \\ I_X &= 6 \text{ A} - 2 \text{ A} \\ I_X &= 4 \text{ A (inductive)} \\ I_Z &= \sqrt{(I_R)^2 + (I_X)^2} \\ I_Z &= \sqrt{(3 \text{ A})^2 + (4 \text{ A})^2} \\ I_Z &= \sqrt{25 \text{ A}} \\ I_Z &= 5 \text{ A} \end{aligned}$$

Using this value of  $I_Z$ , solve for  $Z$ .

$$Z = \frac{E}{I_Z}$$
$$Z = \frac{300 \text{ V}}{5 \text{ A}}$$
$$Z = 60 \Omega$$

If the value for  $E$  were not given and you were asked to solve for  $Z$ , any value of  $E$  could be assumed. If, in the example problem above, you assume a value of 50 volts for  $E$ , the solution would be:

Given:  $R = 100 \Omega$   
 $X_L = 50 \Omega$   
 $X_C = 150 \Omega$   
 $E = 50 \text{ V (assumed)}$

First solve for the values of current in the same manner as before.

Solution:  $I_R = \frac{E}{R}$   
 $I_R = \frac{50 \text{ V}}{100 \Omega}$   
 $I_R = .5 \text{ A}$   
 $I_L = \frac{E}{X_L}$   
 $I_L = \frac{50 \text{ V}}{50 \Omega}$   
 $I_L = 1 \text{ A}$   
 $I_C = \frac{E}{X_C}$   
 $I_C = \frac{50 \text{ V}}{150 \Omega}$   
 $I_C = .33 \text{ A}$

Solve for  $I_X$  and  $I_Z$ .

$$I_X = I_L - I_C$$

$$I_X = 1 \text{ A} - .33 \text{ A}$$

$$I_X = .67 \text{ A (Inductive)}$$

$$I_Z = \sqrt{(I_R)^2 + (I_X)^2}$$

$$I_Z = \sqrt{(0.5 \text{ A})^2 + (0.67 \text{ A})^2}$$

$$I_Z = \sqrt{0.6989 \text{ A}}$$

$$I_Z = 0.836 \text{ A}$$

Solve for  $Z$ .

$$Z = \frac{E}{I_Z}$$

$$Z = \frac{50 \text{ V}}{.836 \text{ A}}$$

$$Z = 60 \Omega \text{ (rounded off)}$$

When the voltage is given, you can use the values of currents,  $I_R$ ,  $I_X$ , and  $I_Z$ , to calculate for the true power, reactive power, apparent power, and power factor. For the circuit shown in figure 4-12, the calculations would be as follows.

To find true power,

$$\text{Given: } R = 100 \Omega$$

$$I_R = 3 \text{ A}$$

Solution:

$$\text{True Power} = (I_R)^2 X$$

$$\text{True Power} = (3 \text{ A})^2 \times 75 \Omega$$

$$\text{True Power} = 900 \text{ W}$$

To find reactive power, first find the value of reactance ( $X$ ).

Given:  $E = 300 \text{ V}$   
 $I_X = 4 \text{ A (Inductive)}$

Solution:  $X = \frac{E}{I_X}$   
 $X = \frac{300 \text{ V}}{4 \text{ A}}$   
 $X = 75 \Omega \text{ (Inductive)}$

$$\text{Reactive power} = (I_X)^2 X$$

$$\text{Reactive power} = (4 \text{ A})^2 \times 75 \Omega$$

$$\text{Reactive power} = 1200 \text{ var}$$

To find apparent power,

Given:  $Z = 60 \Omega$   
 $I_Z = 5 \text{ A}$

Solution:

$$\text{Apparent Power} = (I_Z)^2 Z$$

$$\text{Apparent Power} = (5 \text{ A})^2 \times 60 \Omega$$

$$\text{Apparent Power} = 1500 \text{ VA}$$

The power factor in a parallel circuit is found by either of the following methods.

Given:

$$\text{True Power} = 900 \text{ W}$$

$$\text{Apparent Power} = 1500 \text{ VA}$$

Solution: 
$$\text{PF} = \frac{\text{true power}}{\text{apparent power}}$$

$$\text{PF} = \frac{900 \text{ W}}{1500 \text{ VA}}$$

$$\text{PF} = .6$$

or

Given: 
$$I_R = 3 \text{ A}$$

$$I_Z = 5 \text{ A}$$

Solution: 
$$\text{PF} = \frac{I_R}{I_Z}$$

$$\text{PF} = \frac{3 \text{ A}}{5 \text{ A}}$$

$$\text{PF} = .6$$

*Q31. What is the difference between calculating impedance in a series ac circuit and in a parallel ac circuit?*

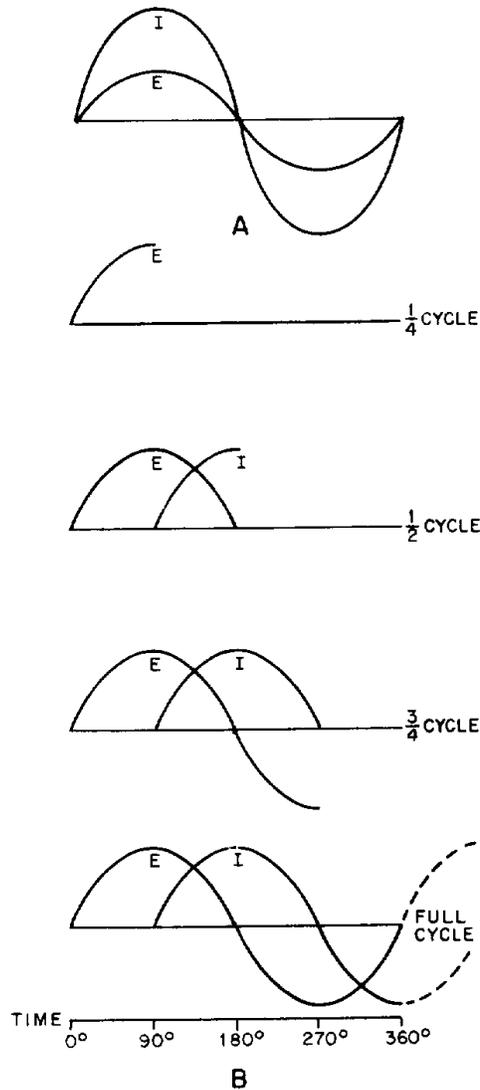
### SUMMARY

With the completion of this chapter you now have all the building blocks for electrical circuits. The subjects covered from this point on will be based upon the concepts and relationships that you have learned. The following summary is a brief review of the subjects covered in this chapter.

**INDUCTANCE IN AC CIRCUITS**—An inductor in an ac circuit opposes any change in current flow just as it does in a dc circuit.

**PHASE RELATIONSHIPS OF AN INDUCTOR**—The current lags the voltage by  $90^\circ$  in an inductor (ELI).

**INDUCTIVE REACTANCE**—The opposition an inductor offers to ac is called inductive reactance. It will increase if there is an increase in frequency or an increase in inductance. The symbol is  $X_L$ , and the formula is  $X_L = 2\pi fL$ .

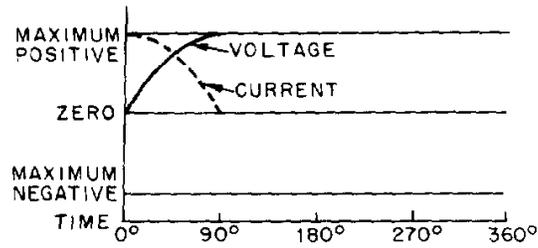


**CAPACITANCE IN AC CIRCUITS**—A capacitor in an ac circuit opposes any change in voltage just as it does in a dc circuit.

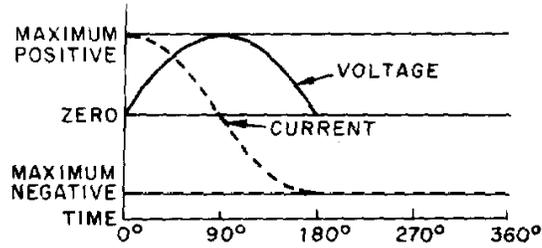
**PHASE RELATIONSHIPS OF A CAPACITOR**—The current leads the voltage by 90° in a capacitor (ICE).

**CAPACITIVE REACTANCE**—The opposition a capacitor offers to ac is called capacitive reactance. Capacitive reactance will decrease if there is an increase in frequency or an increase in capacitance. The symbol is  $X_C$  and the formula is

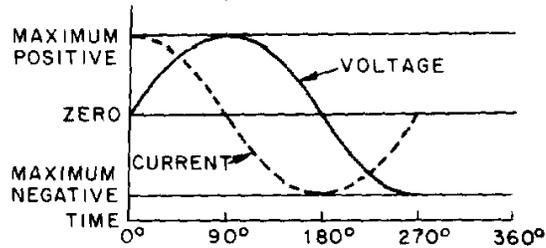
$$X_C = \frac{1}{2\pi fC}$$



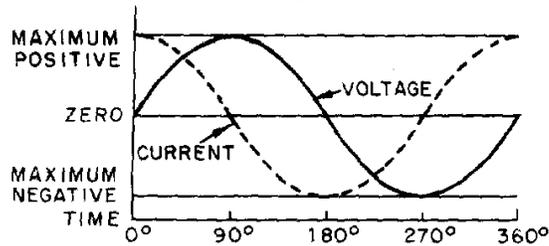
A



B



C



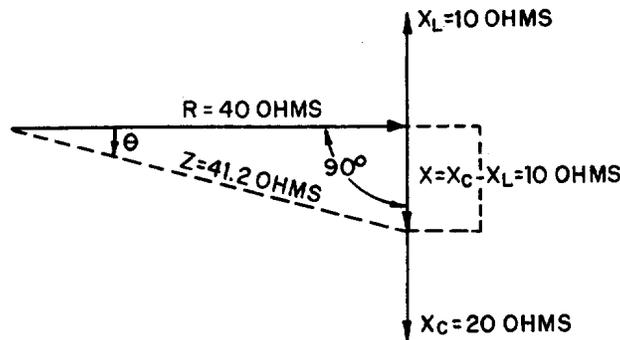
D

**TOTAL REACTANCE**—The total reactance of a series ac circuit is determined by the formula  $X = X_L - X_C$  or  $X = X_C - X_L$ . The total reactance in a series circuit is either capacitive or inductive depending upon the largest value of  $X_C$  and  $X_L$ . In a parallel circuit the reactance is determined by

$$\frac{E}{I_X},$$

where  $I_X = I_C - I_L$  or  $I_X = I_L - I_C$ . The reactance in a parallel circuit is either capacitive or inductive depending upon the largest value of  $I_L$  and  $I_C$ .

IMPEDANCE – The total opposition to a.c. is called impedance. The symbol is  $Z$ . In a series circuit  $Z = \sqrt{R^2 + X^2}$ . In a parallel circuit  $I_Z = \sqrt{(I_R)^2 + (I_X)^2}$  and  $Z = \frac{E}{I_Z}$ .



PHASE ANGLE—The number of degrees that current leads or lags voltage in an ac circuit is called the phase angle. The symbol is  $\theta$ .

OHM'S LAW FORMULAS FOR AC—The formulas derived for Ohm's law used in ac are:  $E = IZ$  and  $I = E/Z$ .

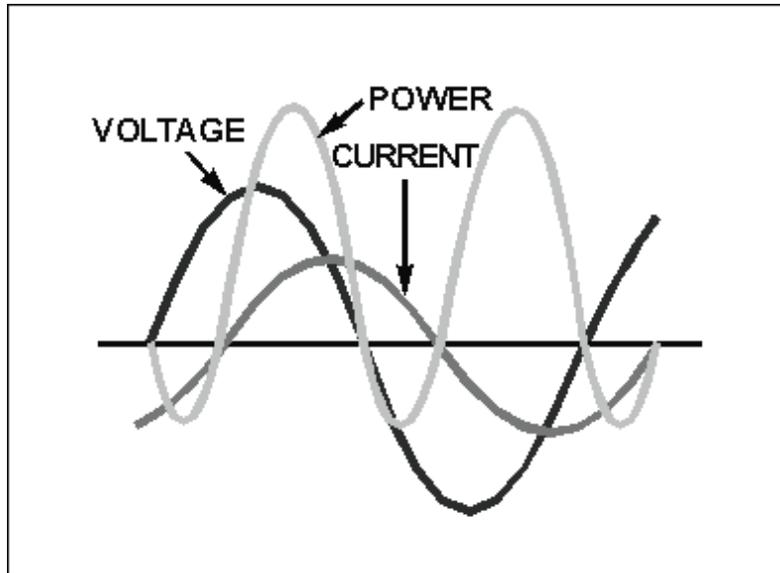
TRUE POWER—The power dissipated across the resistance in an ac circuit is called true power. It is measured in watts and the formula is: True Power =  $(I_R)^2 R$ .

REACTIVE POWER—The power returned to the source by the reactive elements of the circuit is called reactive power. It is measured in volt-amperes reactive (var). The formula is: Reactive Power =  $(I_X)^2 X$ .

APPARENT POWER—The power that appears to the source because of circuit impedance is called apparent power. It is the combination of true power and reactive power and is measured in volt-amperes (VA). The formulas are:

$$\text{Apparent Power} = (I_Z)^2 Z$$

$$\text{Apparent Power} = \sqrt{(\text{true power})^2 + (\text{reactive power})^2}$$



**POWER FACTOR**—The portion of the apparent power dissipated in a circuit is called the power factor of the circuit. It can be expressed as a decimal or a percentage. The formulas for power

factor are  $PF = \frac{\text{true power}}{\text{apparent power}}$  or  $PF = \cos \theta$ . In a

series circuit,  $PF = \frac{R}{Z}$ . In a parallel circuit,  $Pf = \frac{I_R}{I_Z}$ .

**POWER FACTOR CORRECTION**—To reduce losses in a circuit the power factor should be as close to unity or 100% as possible. This is done by adding capacitive reactance to a circuit when the total reactance is inductive. If the total reactance is capacitive, inductive reactance is added in the circuit.

#### ANSWERS TO QUESTIONS Q1. THROUGH Q31.

- A1. An inductor opposes a change in current.
- A2. Current lags voltage by  $90^\circ$  (ELI).
- A3. Inductive reactance.
- A4.  $X_L = 2\pi fL$ .
- A5.  $X_L$  increases.
- A6.  $X_L$  decreases.

A7. *The capacitor opposes any change in voltage.*

A8. *Current leads voltage by 90° (ICE).*

A9. *Capacitive reactance.*

A10.

$$X = \frac{1}{2\pi fC}$$

A11. *X<sub>C</sub> increases.*

A12. *X<sub>C</sub> decreases.*

A13. *X = X<sub>L</sub> - X<sub>C</sub> or X = X<sub>C</sub> - X<sub>L</sub>*

A14. *30 Ω (capacitive).*

A15. *Impedance.*

A16.

$$Z = \sqrt{R^2 + X^2}.$$

A17. *Z = 5Ω*

A18.

$$E = IZ$$

$$I = \frac{E}{Z}.$$

A19. *True power is the power dissipated in the resistance of the circuit or the power actually used in the circuit.*

A20. *Watt.*

A21. *True Power = (I<sub>R</sub>)<sup>2</sup>R.*

A22. *Reactive power is the power returned to the source by the reactive components of the circuit.*

A23. *var.*

A24.

$$\text{Reactive Power} = (I_X)^2 X \text{ or}$$

$$(I_C)^2 X_C - (I_L)^2 X_L \text{ or}$$

$$(I_L)^2 X_L - (I_C)^2 X_C.$$

A25. *The power that appears to the source because of circuit impedance, or the combination of true power and reactive power.*

A26. *VA (volt-amperes).*

A27.

Apparent power =  $(I_Z)^2 Z$  or

$$\sqrt{(\text{true power})^2 + (\text{reactive power})^2}$$

A28. *PF is a number representing the portion of apparent power actually dissipated in a circuit.*

A29.

$$\text{PF} = \frac{\text{true power}}{\text{apparent power}} \text{ or } \text{PF} = \cos \theta.$$

A30. *Add 10 ohms of capacitive reactance to the circuit.*

A31. *In a series circuit impedance is calculated from the values of resistance and reactance. In a parallel circuit, the values of resistive current and reactive current must be used to calculate total current (impedance current) and this value must be divided into the source voltage to calculate the impedance.*