



**PDHonline Course S152 (3 PDH)**

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# **Design of Reinforced Concrete Beams per ACI 318-02**

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## Design of Reinforced Concrete Beams per ACI 318-02

### Course Content

#### A) Flexural Strength of Reinforced Concrete Beams and Slabs

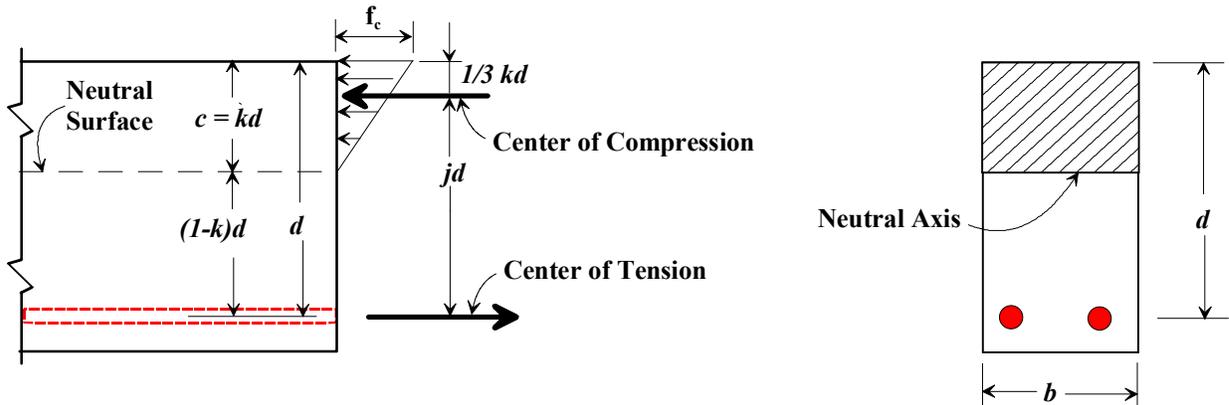
##### 1. Introduction

The design of reinforced concrete structural members may be done by two different methods. One, called *working stress design (WSD)*, is based on the straight-line distribution of compressive stress in the concrete (Fig. 1), covered in Appendix B by ACI 318. This method was the prevalent methodology up until the 1971 edition of the ACI code, and the evaluation is accomplished using service loads.

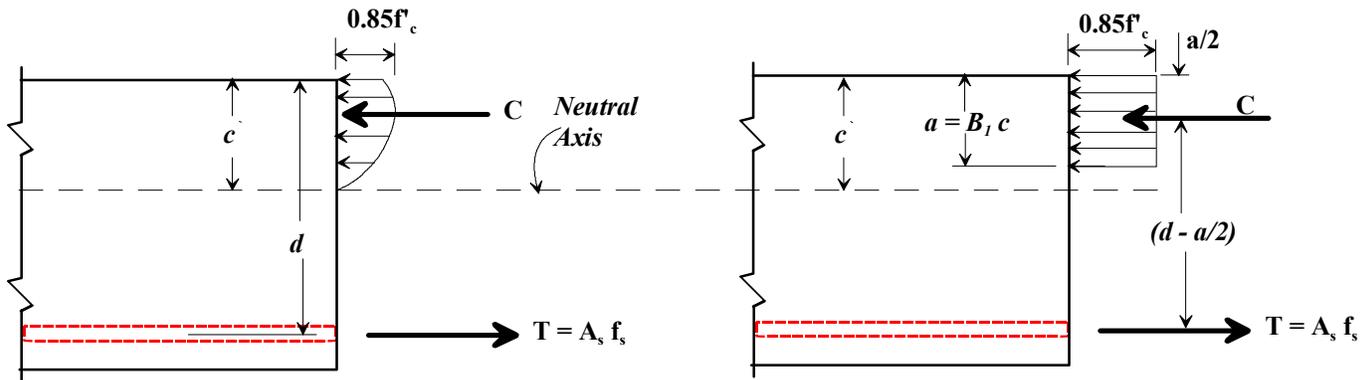
The other is known as the *strength design method or ultimate strength design (USD)*, and is the predominant design method used in the design of reinforced concrete structures. This method will predict with satisfactory accuracy the maximum load that the structural member under consideration will carry. The actual distribution of the compressive stress in a section has the form of a rising parabola (Fig. 2a), and an equivalent rectangular stress block (Whitney block, Fig. 2b) can be used without loss of accuracy to calculate the flexural moment strength of a section. This USD method will be covered solely in this course.

Other important stresses present in beams are shear stresses. These stresses are uniquely significant in concrete beams because when they combined with longitudinal stresses (due to flexure), they produce inclined stresses (diagonal tension) which in turn induce *diagonal cracks*. The basic mechanism of these sloping cracks is the lack of tensile strength inherent in concrete. For practical considerations the intensity of the vertical shear is deemed to be a measure of the intensity of the diagonal tension. The ACI code evaluates the shear strength of concrete section as being made up of two components: a) the shear strength of the uncracked concrete section, and b) the strength of the steel shear reinforcement. Shear is covered in ACI Chapter 11.

The following concrete structural elements are designed using the principles addressed in this course once the internal forces are determined: floor slabs, beams, girders, footings, and retaining walls.



**Figure 1**



**Figure 2a**

**Figure 2b**

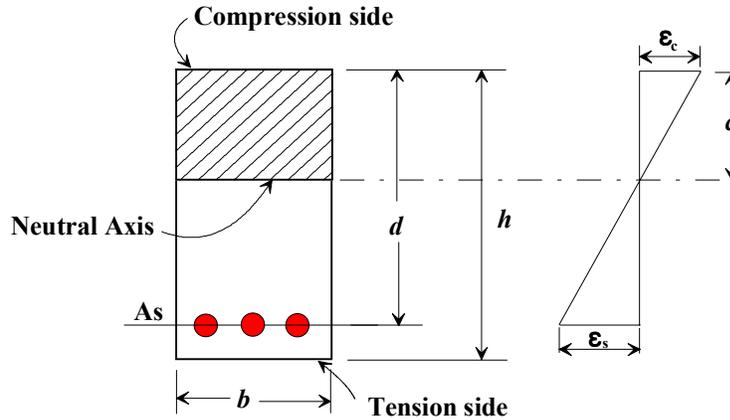
**2. Design Assumptions**

The ACI code covers the design assumptions for flexural strength under Chapter 10, section 10.2.

To compute the strength of a member using the strength design method of the ACI code requires that two basic conditions need be satisfied: (1) static equilibrium, and (2) compatibility of strains.

The following assumptions are made in defining the behavior of a beam member with span-to-depth ( $L_n/h$ ) ratio greater than 4:

1. Strain distribution is assumed to be linear. A cross section which was plane before loading remains plane under load, (Fig. 3).
2. Strain in the steel and the surrounding concrete is the same prior to crushing of the concrete or yielding of the steel.
3. Concrete in the tension zone of the section is neglected in the flexural analysis and design calculations, and the tension reinforcement is assumed to resist the total tensile force (Concrete tensile strength is approximately 10% of its compressive strength).
4. The maximum usable strain at extreme concrete fiber is assumed equal to 0.003.
5. Stress in reinforcement below the specified yield strength,  $f_y$  is taken as  $E_s$  (modulus of elasticity of reinforcement) times steel strain  $\epsilon_s$ . For strength greater than  $f_y$ , the stress in the reinforcement is considered independent of strain and equal to  $f_y$ .
6. The relationship between the concrete compressive stress distribution and concrete strain is assumed to be rectangular, with concrete stress of  $0.85f'_c$  assumed uniformly distributed over an equivalent compression zone (Fig. 2b).
7. The distance  $c$  is measured from the fiber of maximum strain to the neutral axis in a direction perpendicular to that axis.
8. The depth of the rectangular compressive block is taken as  $a = \beta_1 c$  from the fiber of maximum compressive strain.
9. Factor  $\beta_1$  is taken as 0.85 for concrete strengths  $f'_c$  less and equal to 4,000 psi. For strengths above 4,000 psi,  $\beta_1$  is reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4,000 psi, but  $\beta_1$  shall not be taken less than 0.65 (Fig. 4).



**Figure 3**

The value of the stress block depth factor  $\beta_1$ :

$\beta_1 = 0.85$	for $0 < f'_c \leq 4000$ psi
$\beta_1 = 0.85 - 0.05(f'_c - 4000)/1000$	for $4000$ psi $< f'_c \leq 8000$ psi
$\beta_1 = 0.65$	for $f'_c > 8000$ psi

**Figure 4**

Using all the preceding assumptions, one can set the compression force C equal to the steel tension force to satisfy the equation of equilibrium for the forces: See Fig. 2a

$$C = T \quad (\text{Equation 1})$$

C can be written as  $0.85f'_c b a$ , that is the volume of the compressive block at or near the ultimate when the tension steel has yielded,  $\epsilon_s > \epsilon_y$

$$C = 0.85f'_c ba \quad (\text{Equation 2})$$

The tensile force T is then:

$$T = A_s f_y \quad (\text{Equation 3})$$

Since C=T

$$0.85f'_c ba = A_s f_y \quad (\text{Equation 4})$$

the depth of the compression block is obtained from:

$$a = A_s f_y / 0.85f'_c b \quad (\text{Equation 5})$$

The moment of resistance of the section, the nominal strength  $M_n$  is:

$$M_n = (A_s f_y) jd \quad \text{or} \quad M_n = (0.85f'_c ba) jd \quad (\text{Equation 6})$$

Where  $jd$  is the lever arm, the distance between the compression and tensile forces of the internal resisting couple.

Using the Whitney rectangular stress block shown from Fig. 2a, the lever arm,

$$jd = d - a/2 \quad (\text{Equation 7})$$

Then the nominal moment capacity becomes,

$$M_n = A_s f_y (d - a/2) \quad (\text{Equation 8})$$

Because  $C = T$ , the moment can also be written as:

$$M_n = 0.85f'_c ba (d - a/2) \quad (\text{Equation 9})$$

### **3. General principles and requirements**

The ultimate moment at which a beam will fail needs to be calculated in order to determine its ultimate strength. Two modes of failure are identified: (1) tension yielding of the steel, or (2) crushing of the concrete in the outer compression fiber. Depending on the type of failure, analysis of the strain state in the tension reinforcement is the determinant factor in evaluating the ductility of the reinforced section. The percentage of the tension

reinforcement will determine the magnitude of strain, and consequently the type of failure (ductile or brittle).

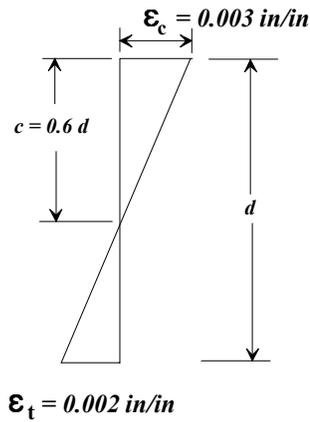
The ACI 318-02 addresses ductility in flexural members by stating the concept of limiting tensile strain, while in previous code editions it was implicit in the maximum tension reinforcement ratio  $\rho_{max}$ , that was given as a fraction of the balanced reinforcement ratio  $\rho_b$ .

A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber reaches 0.003 simultaneously with the yield strain  $\epsilon_y = f_y/E_s$  in the tension reinforcement. The new provisions of the ACI code stated in sections 10.3.3, 10.3.4, and 10.3.5 defines the strain limit for compression-controlled, tension-controlled, and transition sections.

Compression-controlled sections (ACI section 10.3.3) occur when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in the compression zone reaches the assumed strain limit of 0.003 (Fig. 5). The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced condition. For Grade 60 steel, this limit is set at 0.002.

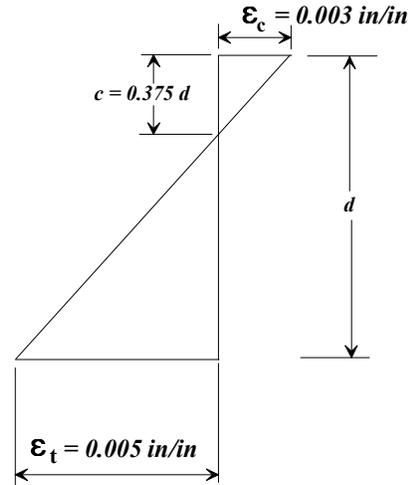
Sections are tension-controlled (ACI section 10.3.4) when the net tensile strain in the tension steel reaches a value of 0.005 or more just as the concrete in compression is equal to its strain limit of 0.003 (Fig. 6). In most cases, the 0.005 limit will provide ductile behavior for acceptable designs. Sections with tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 are considered as part of the transition strain limit zone (Fig. 7).

Furthermore, for nonprestressed flexural members and nonprestressed members with axial load less than  $0.10f'_cA_g$ , the net tensile strain  $\epsilon_t$  is limited to 0.004 (section 10.3.5). This provision covers beams and slabs with small axial forces, and the minimum tensile strain allowed is 0.004. Note that prestressed members are exempted from this limitation.



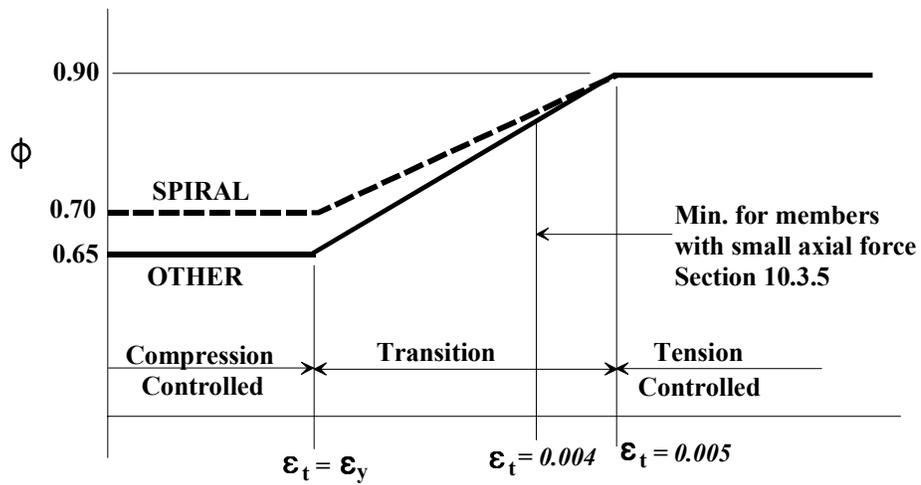
**Compression-Controlled Strain Limit  
Grade 60 Steel**

**Figure 5**



**Tension-Controlled Strain Limit**

**Figure 6**



**Strain Limit Zones and Variation of Strength Reduction Factor  $\phi$   
with the Net Tensile Strain  $\epsilon_t$**

**Figure 7**

From the strain distribution of Fig. 3, the distance to the neutral axis is obtained by evaluating the similar triangles, thus

$$c = \epsilon_c d_t / (\epsilon_s + \epsilon_c) = 0.003d / (f_s / E_s + 0.003) \quad (\text{Equation 10})$$

$E_s$  = Steel Modulus of Elasticity = 29,000,000 psi (ACI section 8.5.2)

If  $f_s = f_y$  then the location of the neutral axis under balanced condition can be computed as:

$$c_b = 87,000 d_t / (87,000 + f_y) \quad (\text{Equation 11})$$

Similarly the strain in the steel reinforcement can be computed from the following expression:

$$\epsilon_s = 0.003(d_t - c) / c \quad (\text{Equation 12})$$

Note:

ACI 318-02 introduced the term  $d_t$  as the distance from extreme compression fiber to extreme tension steel. The effective depth,  $d$  is the distance from extreme compression fiber to centroid of tension reinforcement.

#### **4. Minimum Reinforcement of Flexural Members**

Flexural members requiring tensile reinforcement by analysis should be provided with area of steel  $A_s$  not less than the followings per ACI section 10.5:

$$A_{s,\min} = 3 \sqrt{f'_c} b_w d / f_y \quad \text{but not less than } 200b_w d / f_y \quad (\text{Equation 13})$$

The above requirement is not required provided that the area of tensile reinforcement provided exceeds one-third that required by analysis;

$$A_{s,\text{provided}} > (4/3) A_{s,\text{req'd}} \quad (\text{Equation 14})$$

For structural slabs and footings of uniform thickness the minimum area of tensile reinforcement in the direction of the span should be the same as that required for shrinkage and temperature reinforcement (ACI section 10.5.3). Maximum spacing of this reinforcement must not exceed 18 inches, nor three times the thickness of the section.

Other provisions (section 10.5.2) are given for statically determinate members with a flange in tension, which is beyond the scope of this course.

#### **5. Distribution of Flexural Reinforcement in Beams**

The proper distribution of flexural reinforcement is required to control flexural cracking in beams and slabs (ACI section 10.6). This section replaces the z factor requirements of the 1995 and previous code editions.

The spacing of the reinforcement closest to a surface must not be larger than:

$$S = 540/f_s - 2.5c_c \quad \text{but less than } 12(36/f_s) \quad (\text{Equation 15})$$

Where:

$f_s$  = calculated stress in reinforcement at service load in ksi. The stress can be approximated as 60% of the specified yield strength,  $f_y$

$c_c$  = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in.

For the usual case of beams with Grade 60 steel, and 2 in. clear cover to the main reinforcement, if  $f_s$  is taken as 36 ksi, the maximum bar spacing is 10 in.

Structures subject to very aggressive exposure or designed to be watertight, will require special investigations and precautions.

## **6. Lateral Supports for Flexural Members**

The spacing of lateral supports for a beam shall not exceed 50 times the least width **b** compression flange or face (ACI section 10.4). Laterally unbraced reinforced concrete beams of reasonable dimensions, will not fail prematurely by lateral buckling provided that they are not loaded with lateral eccentricity that induces torsion.

## **7. New Load Factors**

ACI 318-02 has revised the load factor combinations and strength reduction factors of the 1999 code. The old factors have been kept as an alternative and are located in Appendix “C”.

The 1999 combinations have been replaced with those of ASCE 7-98. The basic formulation for the USD method is stated as:

$$\text{Design Strength} \geq \text{Required Strength}$$

$$\phi (\text{Nominal Strength}) \geq U$$

Where:

$\phi$  = strength reduction factor (ACI section 9.3)

$U$  = required strength to resist factored loads or related internal moments and forces

The new  $\phi$  factors are listed in section 9.3.2 and some are listed below:

Tension-controlled sections ..... 0.90

Compression-controlled sections

(a) Members with spiral reinforcement ..... 0.70

(b) Other reinforced members ..... 0.65

Transition members: See Fig. 7

Shear and torsion ..... 0.75

Bearing on concrete ..... 0.65

The load factors were changed with the goal to unify the design profession on one set of load factors and combinations, facilitating the proportioning of concrete building structures that include members of materials other than concrete (i.e. steel).

The required strength  $U$  is given as a set of load combinations in section 9.2, three of these commonly used combinations are listed below with the new load factors:

$$U = 1.2 D + 1.6 L + 0.5 (L_r + S)$$

$$U = 1.2 D + 1.6 (L_r \text{ or } S) + 1.0 L$$

$$U = 1.2 D + 1.6 W + 1.0 L + 0.5 (L_r \text{ or } S)$$

Where:

D = dead loads

L = live loads

$L_r$  = roof live loads

S = snow loads

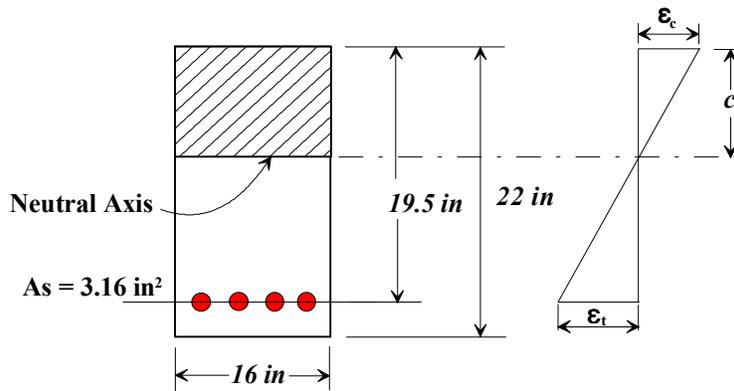
W = wind loads

Note the new load factors for dead loads (1.2 new, 1.4 old), and other cases (1.6 new, 1.7 old).

Finally, the ACI code addresses serviceability or control of deflections in section 9.5. These provisions are concern mainly with deflections or deformations that occur at service load levels. Two methods are given for controlling deflections, one based on a minimum overall thickness determined from Table 9.5(a), for nonprestressed beams, one-way slabs, and for composite members. The other method allows for calculations of deflections by a procedure prescribed in section 9.5.2.3. Computation of deflection is not in the scope of this course.

**Example 1 – Moment Capacity of a given section**

A rectangular singly reinforced concrete beam has a width of 16 in. and an effective depth to the centroid of the reinforcing steel of 19.5 in. It is reinforced with four No. 8 bars in one row. Given (a)  $f'_c = 4,000$  psi and  $f_y = 60,000$  psi what is the moment capacity of the beam to be used in design? (b) If  $f_y = 40,000$  psi what is the moment capacity?



**Solution:**

(a)  $f_y = 60,000$  psi  $f'_c = 4,000$  psi

Actual steel ratio,

$$\rho = 3.16 / (16 \times 19.5) = 0.0101$$

from equation 13, the minimum allowable reinforcement ratio

$$\rho_{\min} = 3 \sqrt{f'_c} / f_y = 3 \sqrt{4000} / 60000 = 0.0032$$

but not less than  $200/f_y = 200/60000 = \underline{0.0033} < 0.0101 \therefore \text{OK}$

Depth of compressive block from equation 5:

$$a = A_s f_y / 0.85 f'_c b = 3.16 \times 60,000 / 0.85 \times 4,000 \times 16 = 3.49 \text{ in}$$

$$\beta_1 = 0.85 \text{ from Fig. 4}$$

The neutral axis location,  $c = a / \beta_1 = 3.49 / 0.85 = 4.11 \text{ in}$

Check the strain in the steel from equation 12:

$$\epsilon_t = 0.003 (d_t - c) / c = 0.003 (19.5 - 4.11) / 4.11 = \underline{0.0112} > 0.005 \therefore \text{OK}$$

thus  $\phi = 0.90$

Equation 8  $\rightarrow \phi M_n = \phi A_s f_y (d - a/2) =$

$$0.90 \times 3.16 \times 60,000 (19.5 - 3.49/2) / 12 = \underline{252,476 \text{ lb-ft}}$$

**(b)  $f_y = 40,000 \text{ psi}$   $f'_c = 4,000 \text{ psi}$**

Actual steel ratio,

$$\rho = 0.0101 \text{ (from part (a))}$$

from equation 13, the minimum allowable reinforcement ratio

$$\rho_{\min} = 3 \sqrt{f'_c} / f_y = 3 \sqrt{4000} / 40000 = \underline{0.0047}$$

but not less than  $200/f_y = 200/40,000 = \underline{0.005} < 0.0101 \therefore \text{OK}$

Depth of compressive block from equation 5:

$$a = A_s f_y / 0.85 f'_c b = 3.16 \times 40,000 / 0.85 \times 4,000 \times 16 = 2.32 \text{ in}$$

$$\beta_1 = 0.85 \text{ from Fig. 4}$$

$$\text{The neutral axis location, } c = a / \beta_1 = 2.32 / 0.85 = 2.73 \text{ in}$$

Check the strain in the steel from equation 12:

$$\epsilon_t = 0.003 (d_t - c) / c = 0.003 (19.5 - 2.73) / 2.73 = \underline{\underline{0.0184 > 0.005 \therefore \text{OK}}}$$

$$\text{thus } \phi = 0.90$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.90 \times 3.16 \times 40,000 (19.5 - 2.32/2) / 12 = \underline{\underline{173,863 \text{ lb-ft}}}$$

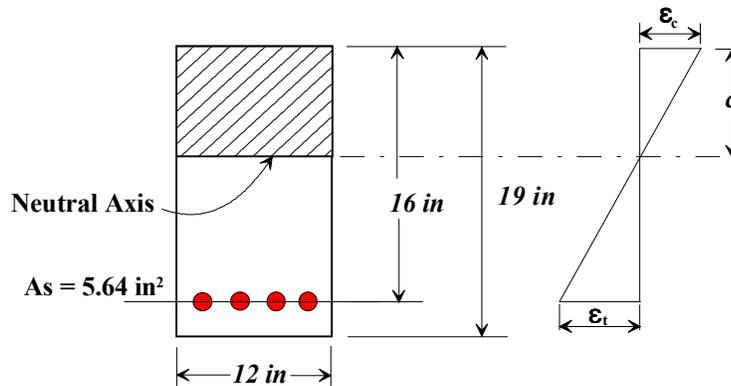
### Example 2 – Tension-controlled or compression-controlled determination of a given section

A reinforced concrete beam has the cross-section shown below. Determine if the beam is tension zone, compression zone or in the transition zone.

Given  $f'_c = 4,000$  psi, find if the beam satisfies ACI 318-02 for:

(a)  $f_y = 60,000$  psi

(b)  $f_y = 40,000$  psi



***Solution:***

$$(a) f_y = 60,000 \text{ psi } f'_c = 4,000 \text{ psi}$$

Actual steel ratio,

$$\rho = 5.64 / (12 \times 16) = 0.0294$$

from equation 13, the minimum allowable reinforcement ratio

$$\rho_{\min} = 3 \sqrt{f'_c} / f_y = 3 \sqrt{4000} / 60000 = 0.0032$$

but not less than  $200/f_y = 200/60000 = \underline{0.0033} < 0.0294 \therefore \text{OK}$

Depth of compressive block from equation 5:

$$a = A_s f_y / 0.85 f'_c b = 5.64 \times 60,000 / 0.85 \times 4,000 \times 12 = 8.29 \text{ in}$$

$$\beta_1 = 0.85 \text{ from Fig. 4}$$

The neutral axis location,  $c = a / \beta_1 = 8.29 / 0.85 = 9.76 \text{ in}$

$$c / d_t = 9.76 / 16 = \underline{0.61} > 0.60 \text{ (see Fig. 5)}$$

Check the strain in the steel from equation 12:

$$\epsilon_t = 0.003 (d_t - c) / c = 0.003 (16 - 9.76) / 9.76 = \underline{0.0019} < \epsilon_y = 0.0021 \therefore \text{NG}$$

Since  $\epsilon_t$  is less than the yield strain  $\epsilon_y$ , brittle behavior governs, and the section is in the compression-controlled zone and does not satisfy the ACI code requirements for flexural beams.

$$(b) f_y = 40,000 \text{ psi } f'_c = 4,000 \text{ psi}$$

Actual steel ratio,

$$\rho = 5.64 / (12 \times 16) = 0.0294$$

from equation 13, the minimum allowable reinforcement ratio

$$\rho_{\min} = 3 \sqrt{f'_c} / f_y = 3 \sqrt{4000} / 40000 = 0.0047$$

but not less than  $200/f_y = 200/40000 = \underline{0.005} < 0.0294 \therefore \text{OK}$

Depth of compressive block from equation 5:

$$a = A_s f_y / 0.85 f'_c b = 5.64 \times 40,000 / 0.85 \times 4,000 \times 12 = 5.53 \text{ in}$$

$$\beta_1 = 0.85 \text{ from Fig. 4}$$

The neutral axis location,  $c = a / \beta_1 = 5.53 / 0.85 = 6.51 \text{ in}$

Check the strain in the steel:

$$\varepsilon_t = 0.003 (d_t - c) / c = 0.003 (16 - 6.51) / 6.51 = \underline{0.0044} > \varepsilon_y = 0.0014$$

Since  $\varepsilon_t$  is smaller than the tension-controlled minimum of 0.005, therefore the section is in the transition zone with  $\varepsilon_t > 0.004$  and does satisfy the ACI code requirements for flexural beams. The strength reduction factor  $\phi$  will be less than 0.90, therefore the section may be considered uneconomical.

### Example 3 – Moment capacity for a section in the transition zone

**Calculate the nominal resisting moment for the beam in example 2 case (b)**

**Solution:**

From example 2, the depth of the compressive block “a” was found  
 $a = 5.53 \text{ in}$

Since  $\varepsilon_t < 0.005$  but  $> \varepsilon_y$  the section is in the transition zone, and  $\phi < 0.90$

Determine  $\phi$  from Fig. 7:

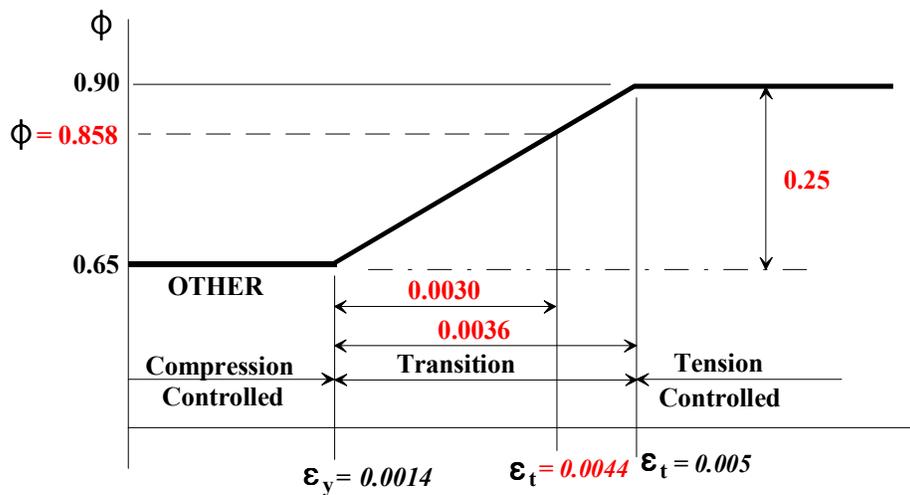
$$\phi = 0.65 \text{ for } \varepsilon_t = \varepsilon_y = 0.0014$$

$$\phi = 0.90 \text{ for } \varepsilon_t = 0.005$$

$$\phi = \{[(0.9 - 0.65) / (0.005 - 0.0014)] \times (0.0044 - 0.0014)\} + 0.65 = 0.858$$

From Equation 8:  $\phi M_n = \phi A_s f_y (d - a/2)$

$$\phi M_n = 0.858 \times 5.64 \times 40,000(16 - 5.53/2)/12 = \underline{\underline{213,486 \text{ lb-ft}}}$$



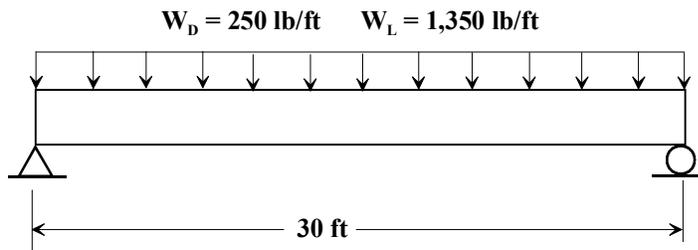
Strength Reduction Factor with the Net Tensile Strain,  $\epsilon_t = 0.0044$

Example 3

### Example 4 – Design of a Singly Reinforced Simply Supported Beam for Flexure

Design a reinforced concrete simply supported beam with a span of 30 ft and is subject to a service uniform dead load of  $W_D = 250$  lb/ft and uniform live load  $W_L = 1,350$  lb/ft. Design a beam section to resist the factored bending moment.

Design parameters:  $f'_c = 4,500$  psi  $f_y = 60,000$  psi



#### Solution:

Assume a minimum beam depth from the ACI Code deflection Table 9.5(a):

$$L_n / 16 = 30 \times 12 / 16 = 22.5 \text{ in}$$

Try a section with  $b = 12$  in,  $d = 23$  in, and  $h = 26$  in

Using a concrete density  $\gamma_c = 150$  pcf

$$\text{Self-weight} = (12 \times 26 / 144) \times 150 = 325 \text{ lb/ft}$$

$$\text{Factored, } w_u = 1.2 \times (325 + 250) + 1.6 \times 1350 = 2,850 \text{ lb/ft}$$

$$\text{Factored moment, } M_u = 2850(30)^2 / 8 = 320,625 \text{ lb-ft}$$

Try a maximum area of tension reinforcement to ensure ductile behavior (see Fig. 6)

$$\text{Assume } c / d_t \sim 0.75 \times 0.375 = 0.281$$

Limit balanced strain state:

From Equation 11:

$$C_b = 23[87000/(87000 + 60000)] = 13.61 \text{ in}$$

$$c = 0.281 \times 23 = 6.46 \text{ in} \quad \text{then } a = 0.85 \times 6.46 = 5.5 \text{ in}$$

Assume tension-controlled member,  $\phi = 0.90$  and solve for  $A_s$ :

$$A_s = M_u / \phi f_y (d - a/2) = 320625 \times 12 / 0.90 \times 60000 (23 - 5.5/2) = 3.52 \text{ in}^2$$

Check :

$$a = A_s f_y / 0.85 f'_c b = 3.52 \times 60000 / .85 \times 4,500 \times 12 = 4.60 \text{ in}$$

Assume  $a = 4.60$  in and try again

$$A_s = M_u / \phi f_y (d - a/2) = 320625 \times 12 / 0.90 \times 60000 (23 - 4.60/2) = 3.44 \text{ in}^2$$

Check :

$$a = A_s f_y / 0.85 f'_c b = 3.44 \times 60000 / .85 \times 4,500 \times 12 = 4.50 \text{ in} \sim 4.60 \text{ say OK}$$

**$A_s = 3.44 \text{ in}^2$  is chosen for design**

**Use 5 - #8 bar reinforcement then  $A_s$  provided = 3.95 in<sup>2</sup>**

Actual depth of compressive block from equation 5:

$$a = A_s f_y / 0.85 f'_c b = 3.95 \times 60,000 / 0.85 \times 4,500 \times 12 = 5.16 \text{ in}$$

$\beta_1 = 0.825$  Interpolating from Fig. 4 since  $f'_c > 4,000$  psi

The neutral axis location,  $c = a / \beta_1 = 5.16 / 0.825 = 6.25$  in

$$c / d_t = 6.25 / 23 = \underline{\underline{0.272}} < \underline{\underline{0.281}} \therefore \text{OK (see Fig. 6)}$$

Check the strain in the steel using equation 12:

$$\epsilon_t = 0.003 (d_t - c) / c = 0.003 (23 - 6.25) / 6.25 = \underline{\underline{0.008 > 0.005}}$$

Since  $\epsilon_t > 0.005$  the section is in the tension-controlled zone, thus assumption of  $\phi = 0.90$  is valid

Check minimum reinforcement requirement:

Actual steel ratio,

$$\rho = 3.95 / (12 \times 23) = 0.0143$$

Using equation 13, the minimum allowable reinforcement ratio

$$\rho_{\min} = 3 \sqrt{f'c} / f_y = 3 \sqrt{4500} / 60,000 = 0.0034$$

$$\text{but not less than } 200/f_y = 200/60,000 = \underline{\underline{0.0034 < 0.0143 \therefore \text{OK}}}$$

$$M_n = A_s f_y (d - a/2) = 3.95 \times 60,000 (23 - 5.16/2) / 12 = 403,295 \text{ lb-ft}$$

Actual Moment Capacity of beam:

$$\underline{\underline{\phi M_n = 0.90 \times 403,295 = 362,966 \text{ lb-ft} > M_u = 320,625 \text{ lb-ft} \therefore \text{OK}}}$$

***The design is adequate***

### **Conclusion:**

This course has covered the basic principles related to the design of nonprestressed singly reinforced concrete rectangular beams using the latest edition of the Building Code Requirements for Structural Concrete ACI 318-02.

The items discussed in this lecture included the analysis and design of singly reinforced rectangular sections, design assumptions, general requirements, variation of strength reduction factor, reinforcement strain limits, minimum reinforcement, and new load factors. Other issues covered were distribution of flexural reinforcement, and distance between lateral supports.

**References:**

1. American Concrete Institute, Building Code Requirements for Structural Concrete ACI 318-02
2. American Society of Civil Engineers, Minimum Design Loads for Buildings and Other Structures, ASCE 7-98
3. George Winter and Arthur H. Nilson, Design of Concrete Structures, 8<sup>th</sup> Edition
4. Edward G. Nawy, Reinforced Concrete, 5<sup>th</sup> Edition
5. Harry Parker, Simplified Design of Reinforced Concrete, 4<sup>th</sup> Edition