



**PDHonline Course S165 (4 PDH)**

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**Design of Beams and Other Flexural  
Members per AISC LRFD 3rd Edition  
(2001)**

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## **Design of Beams and Other Flexural Members AISC LRFD 3<sup>rd</sup> Edition (2001)**

*Jose-Miguel Albaine, M.S., P.E.*

### **COURSE CONTENT**

#### **1. Bending Stresses and Plastic Moment**

The stress distribution for a linear elastic material considering small deformations is as shown on Figure No. 1. The orientation of the beam is such that bending is about the x-x axis. From mechanics of materials, the stress at any point can be found as:

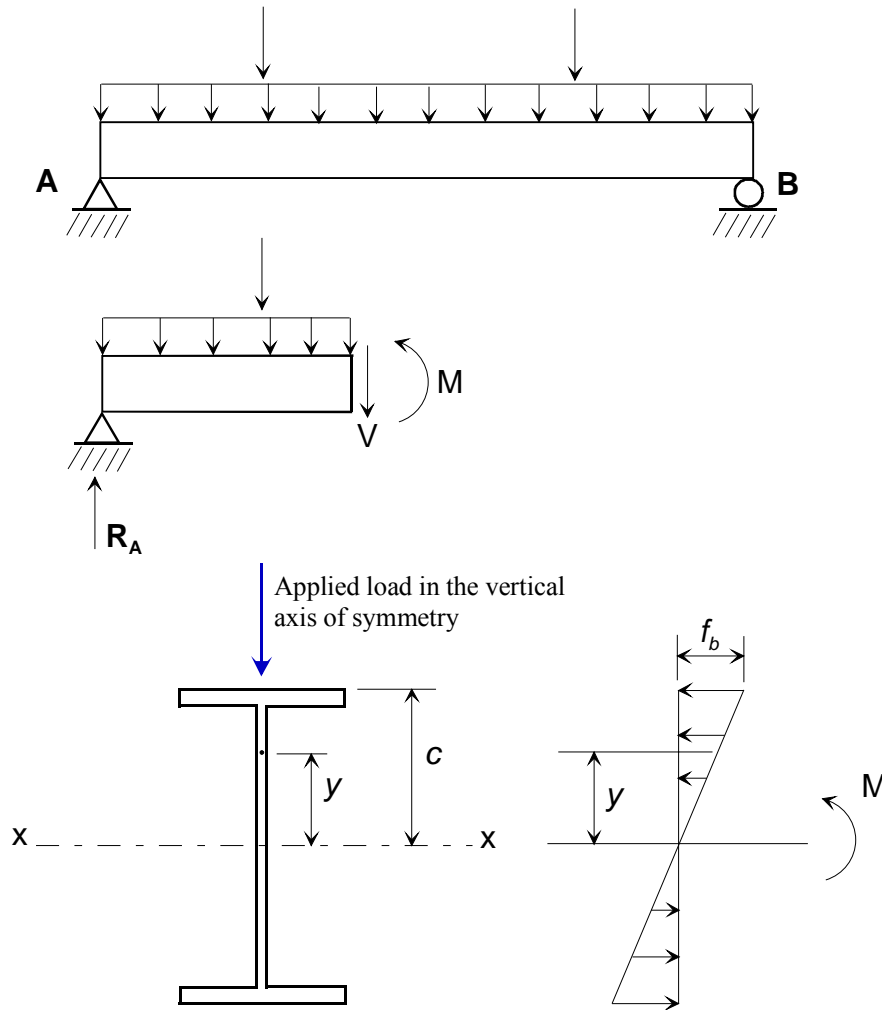
$$f_b = \frac{M y}{I_x} \quad \text{(Eq. 1)}$$

where  $M$  is the bending moment at the cross section,  $y$  is the distance from the neutral axis to the point under consideration, and  $I_x$  is the moment of inertia of the area of the cross section.

Equation 1 is based on the following assumptions:

- 1) Linear distribution of strains from top to bottom
- 2) Cross sections that are plane before bending remain plane after bending
- 3) The beam section must have a vertical axis of symmetry

- 4) The applied loads must be in the longitudinal plane containing the vertical axis of symmetry otherwise a torsional twist will develop along with the bending



**FIGURE 1**

The maximum stress will occur at the extreme fiber, where  $y$  is at a maximum. Therefore there are two maxima: maximum compressive stress in the top fiber and maximum tensile stress in the bottom fiber. If the neutral axis is an axis of symmetry, these two stresses are equal in magnitude.

The maximum stress is then given by the equation:

$$f_{max} = \frac{M c}{I_x} = \frac{M}{I_x / c} = \frac{M}{S_x} \quad (\text{Eq. 2})$$

Where  $c$  is the distance from the neutral axis to the extreme fiber, and  $S_x$  is the elastic section modulus of the cross section. Equations 1 and 2 are valid as long as the loads are small enough that the material remains within the elastic range, or that  $f_{\max}$  does not exceed  $F_y$ , the yield strength of the beam.

The bending moment that brings the beam to the point of yielding is given by:

$$M_y = F_y S_x \quad (\text{Eq. 3})$$

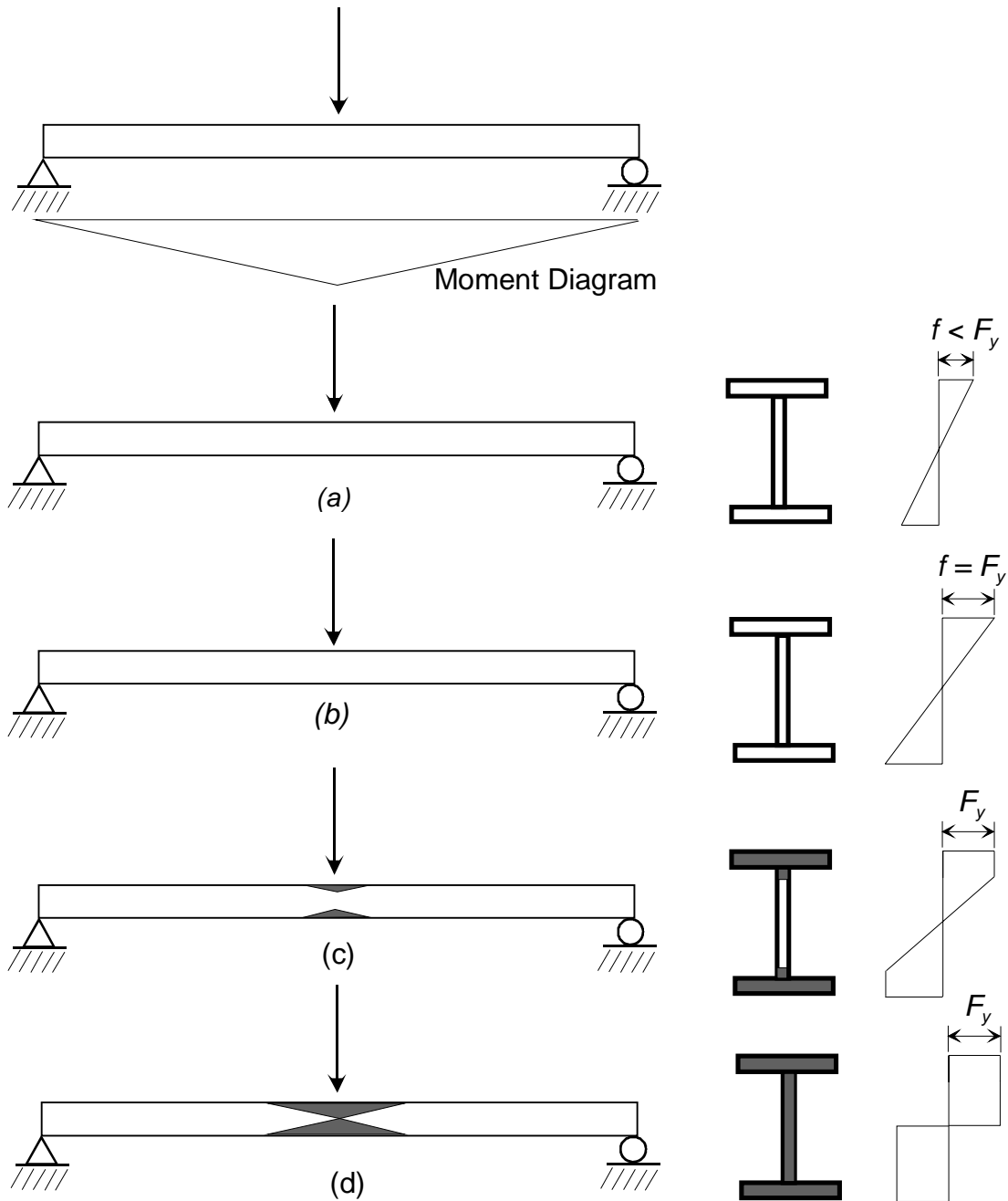
In Figure No. 2, a simply supported beam with a concentrated load at midspan is shown at successive stages of loading. Once yielding begins, the distribution of stress on the cross section is no longer linear, and yielding progresses from the extreme fiber toward the neutral axis. The yielding region also extends longitudinally from the center of the beam as the bending moment reaches  $M_y$  at more locations.

In Figure 2b yielding has just begun, in Figure 2c, yielding has progressed to the web, and in Figure 2d the entire section has reached the yield point. The additional moment to bring the beam from stage b to d is, on average, about 12% of the yield moment,  $M_y$ , for W-shapes. After stage d is reached, any further load increase will cause collapse. A plastic hinge has been formed at the center of the beam.

The plastic moment which is the moment required to form the plastic hinge is computed as:

$$M_p = F_y Z_x \quad (\text{Eq. 4})$$

where  $Z_x$  is the plastic section modulus and is defined as shown on Figure No. 3.

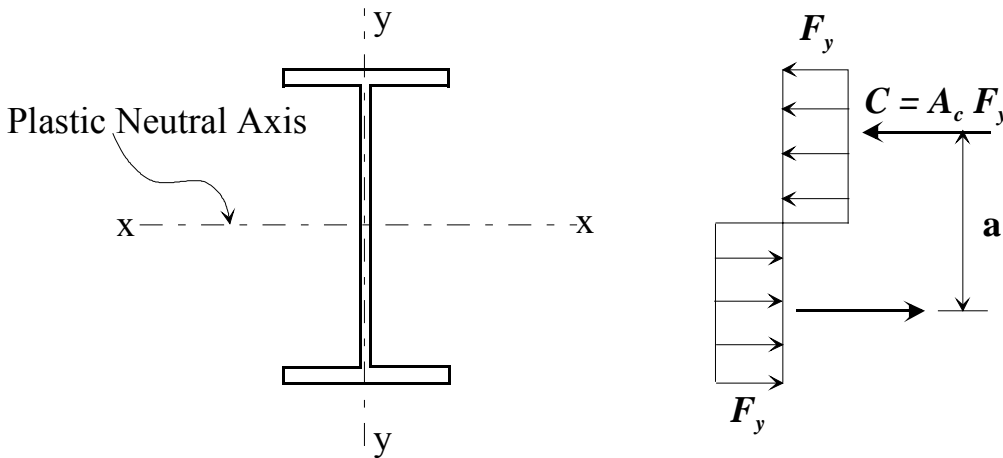


**Figure 2**

The tensile and compressive stress resultants are depicted, showing that  $A_c$  has to be equal to  $A_t$  for the section to be in equilibrium. Therefore, for a symmetrical W-shape,  $A_c = A_t = A / 2$ , and  $A$  is the total cross sectional area of the section, and the plastic section modulus can be found as:

$$M_p = F_y (A_c) a = F_y (A_t) a = F_y (A/2) a = F_y Z_x \quad \text{(Eq. 5)}$$

$$Z_x = \left( \frac{A}{2} \right) a$$



**Figure 3**

## 2. AISC LRFD 3<sup>rd</sup> Edition – November 2001

Load and resistance factor design (LRFD) is based on a consideration of failure conditions rather than working load conditions. Members and its connections are selected by using the criterion that the structure will fail at loads substantially higher than the working loads. Failure means either collapse or extremely large deformations.

Load factors are applied to the service loads, and members with their connections are designed with enough strength to resist the factored loads. Furthermore, the theoretical strength of the element is reduced by the application of a resistance factor.

The equation format for the LRFD method is stated as:

$$\Sigma \gamma_i Q_i = \phi R_n \quad \text{(Eq. 6)}$$

Where:

$Q_i$  = a load (force or moment)

$\gamma_i$  = a load factor (LRFD section A4 Part 16, Specification)

$R_n$  = the nominal resistance, or strength, of the component under consideration

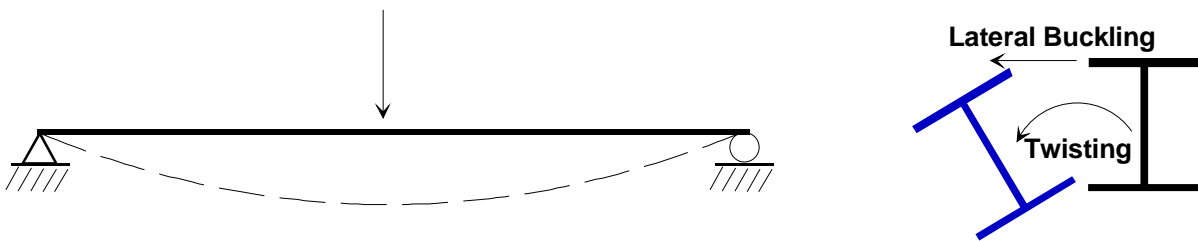
$\phi$  = resistance factor (for beams given in LRFD Part 16, Chapter F)

The LRFD manual also provides extensive information and design tables for the design of beams and other flexural members.

### 3. Stability of Beam Sections

As long as a beam remain stable up to the fully plastic condition as depicted on Figure 2, the nominal moment strength can be taken as the plastic moment capacity as given in Equations 4 and 5.

Instability in beams subject to moment arises from the buckling tendency of the thin steel elements resisting the compression component of the internal resistance moment. Buckling can be of a local or global nature. Overall buckling (or global buckling) is illustrated in Figure 4.

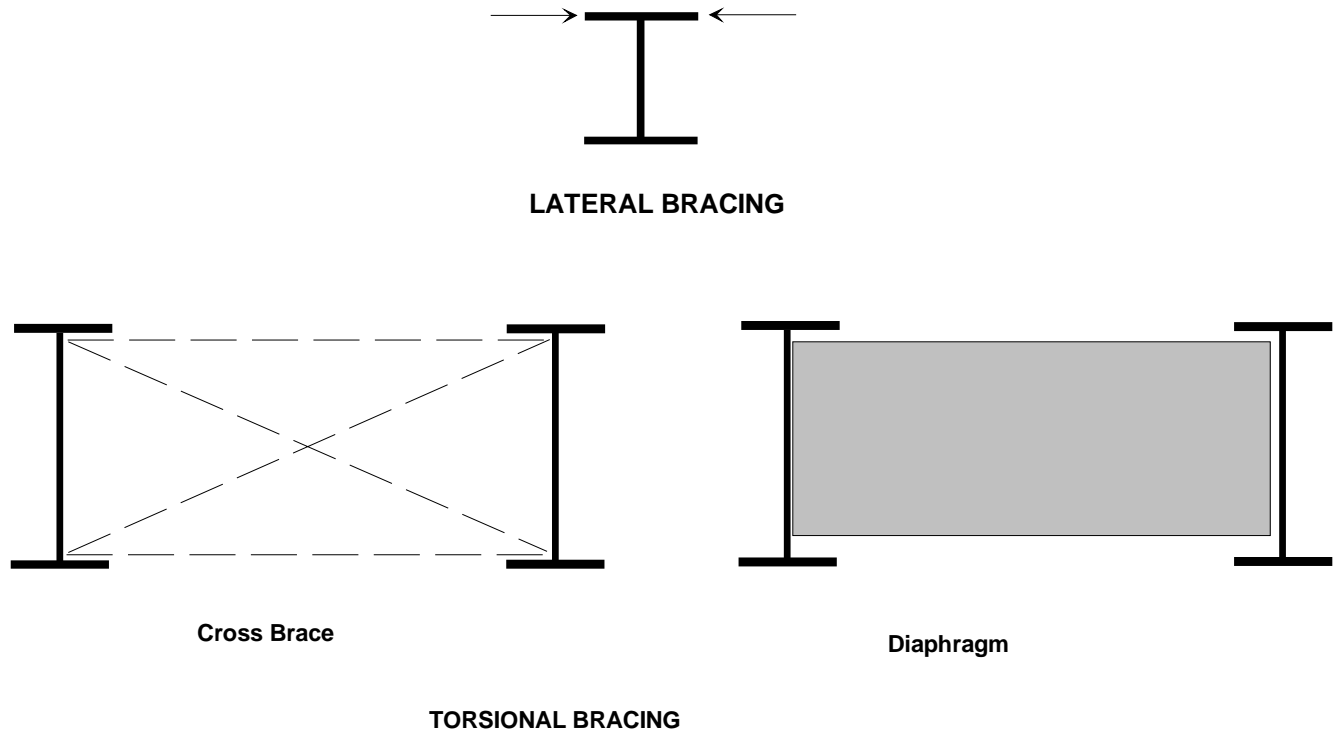


**Figure 4**

When a beam bends, the compression zone (above the neutral axis) is similar to a column and it will buckle if the member is slender enough. Since the web is connected to the compression flange, the tension zone provides some restraint, and the outward deflection (lateral buckling) is accompanied by

twisting (torsion). This mode of failure is called *lateral-torsional buckling* (LTB).

Lateral-torsional buckling is prevented by bracing the beam against twisting at sufficient intervals as shown on Figure 5.



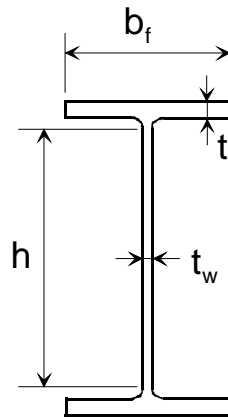
**Figure 5**

The capacity of a beam to sustain a moment large enough to reach the fully plastic moment also depends on whether the cross-sectional integrity is maintained. This *local instability* can be either compression flange buckling, called *flange local buckling* (FLB), or buckling of the compression part of the web, called *web local buckling* (WLB). The local buckling will depend on the width-thickness ratio of the compressed elements of the cross section.

#### **4. Compact, Noncompact and Slender Sections**

The classification of cross-sectional shapes is found on AISC Section B5 of the Specification, “Local Buckling”, in Table B.5.1. For I- and H-shapes, the ratio of the projecting flange (an *unstiffened* element) is  $b_f / 2t_f$ , and the ratio for the web (a *stiffened* element) is  $h / t_w$ , see Figure 6.





### Width-Thickness Dimensions

**Figure 6**

Defining,

$\lambda$  = width-thickness ratio

$\lambda_p$  = upper limit for compact sections

$\lambda_r$  = upper limit for noncompact sections

Then,

If  $\lambda \leq \lambda_p$  and the flange is continuously connected to the web, the shape is compact

If  $\lambda_p < \lambda \leq \lambda_r$ , the shape is noncompact

$\lambda > \lambda_r$ , the shape is slender

The following Table summarizes the criteria of local buckling for hot-rolled I- and H-shapes in flexure.

**TABLE 1**

Element	$\lambda$	$\lambda_p$	$\lambda_r$
Flange	$\frac{b_f}{2 t_f}$	$0.38 \sqrt{\frac{E}{F_y}}$	$0.83 \sqrt{\frac{E}{F_y - 10}}$
Web	$\frac{h}{t_w}$	$3.76 \sqrt{\frac{E}{F_y}}$	$5.70 \sqrt{\frac{E}{F_y}}$

### 5. Bending Strength of Compact Shapes

A beam can fail by reaching the plastic moment  $M_p$  and becoming fully plastic, or it can fail by:

- Lateral-Torsional buckling (LTB), either elastically or inelastically;
- Flange local buckling (FLB), elastically or inelastically;
- Web local buckling (WBL), elastically or inelastically.

When the maximum bending stress is less than the proportional limit, the failure is elastic. If the maximum bending stress is larger than the proportional limit, then the failure is said to be inelastic.

The discussion in this course will be limited to only hot-rolled I- and H-shapes. The same principles discussed here apply to channels bent about the strong axis and loaded through the shear center (or restrained against twisting).

*Compact shapes* are those shapes whose webs are continuously connected to the flanges and that meet the following width-thickness ratio requirement for both flanges and web:

$$\frac{b_f}{2 t_f} \leq 0.38 \sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}}$$

Note that web criteria is satisfied by all standard I- and C-shapes listed in the Manual of Steel Construction, and only the flange ratio need to be checked. Most shapes will also meet the flange requirement and thus will be classified as compact. If the beam is compact and has continuous lateral support (or the unbraced length is very short), the nominal moment strength  $M_n$  is equal to the full plastic moment capacity of the section,  $M_p$ . For members with inadequate lateral support, the moment capacity is limited by the lateral-torsional buckling strength, either elastic or inelastic.

Therefore, the nominal moment strength of lateral laterally supported compact sections is given by

$$M_n = M_p \quad (\text{Eq. 7; AISC F1-1})$$

Where  $M_p = F_y Z_x \leq 1.5 M_y$

$M_p$  is limited to  $1.5 M_y$  to avoid excessive working-load deformations and

$$F_y Z_x \leq 1.5 F_y S_x \quad \text{or} \quad Z_x / S_x = 1.5$$

Where  $S$  = elastic section modulus and for channels and I- and H-shapes bent about the strong axis,  $Z_x / S_x$  will always be  $\leq 1.5$ .

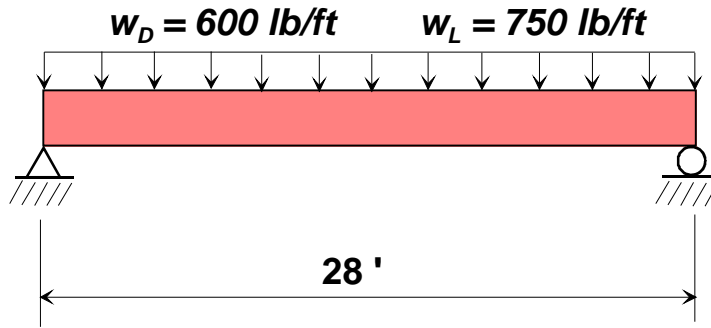
The flexural design strength of compact beams, laterally supported is given by:

$$\phi_b M_n = \phi_b F_y Z_x \leq \phi_b 1.5 F_y S_x \quad (\text{Eq. 8})$$

and  $\phi_b = 0.90$

### **Example 1**

A W 16 x 36 beam of A992 steel ( $F_y = 50$  ksi) supports a concrete floor slab that provides continuous lateral support to the compression flange. The service dead load is 600 lb/ft, and the service live load is 750 lb/ft. Find the design moment strength of the beam?



### ***Solution***

Cross-sectional properties of the beam (LRFD Part1, Table 1-1):

$$b_f = 6.99 \text{ in.} \quad t_f = 0.43 \text{ in.} \quad d = 15.9 \text{ in.} \quad t_w = 0.295 \text{ in.}$$

$$Z_x = 64.0 \text{ in}^3 \quad S_x = 56.5 \text{ in}^3$$

The total service dead load, including the beam weight is

$$W_D = 600 + 36 = 636 \text{ lb/ft}$$

The maximum bending moment for a simply supported beam, loaded with a uniformly distributed load,

$$M_{MAX} = w L^2 / 8$$

The factored applied load,  $W_u = 1.2 (636) + 1.6 (750) = 1,963 \text{ lb/ft}$

$$\text{and } M_u = 1.963 (28)^2 / 8 = 192.4 \text{ k-ft}$$

Check for compactness:

$$\frac{b_f}{2 t_f} = 8.13 \leq 0.38 \sqrt{\frac{29000}{50}} = 9.15 \quad \therefore \text{the flange is compact}$$

$$\frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} \quad \text{for all shapes in the AISC Manual}$$

∴ W 16 x 36 is compact for  $F_y = 50$  ksi

Since the beam is compact and laterally supported,

$$M_n = M_p = F_y Z_x = 50 \times 64.0 = 3,200 \text{ in.-kips} = 266.7 \text{ ft-kips}$$

Check for  $M_p \leq 1.5 M_y$

$$\frac{Z_x}{S_x} = \frac{64}{56.5} = 1.13 < 1.5$$

$$\phi_b M_n = 0.90 (266.7) = 240.0 \text{ ft-kips} > 192.4 \text{ ft-kips} \quad (\text{OK})$$

## **1. Bending Strength of Beams Subject to Lateral-Torsional Buckling**

When the unbraced length,  $L_b$  (the distance between points of lateral support for the compression flange), of a beam is less than  $L_p$ , the beam is considered fully lateral supported, and  $M_n = M_p$  as described in the preceding section. The limiting unbraced length,  $L_p$ , is given for I-shaped members by equation (9) below:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} \quad (\text{Eq. 9 ; AISC F1-4})$$

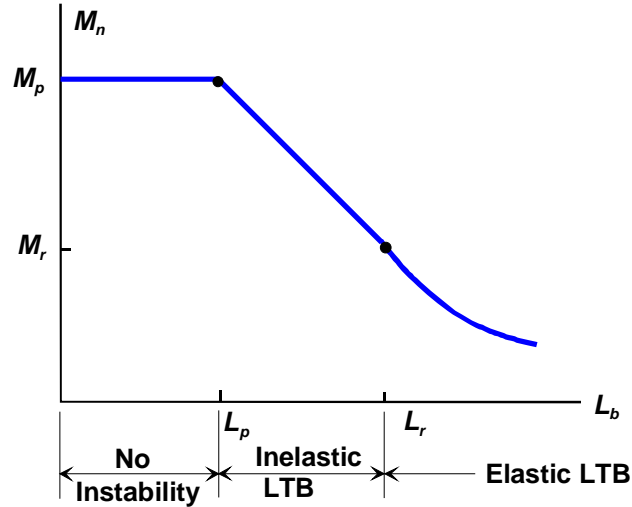
where,

$r_y$  = radius of gyration about the axis parallel to web, y-axis

$E$  = Modulus of Elasticity, ksi

$F_{yf}$  = Yield stress of the flanges, ksi

If  $L_b$  is greater than  $L_p$  but less than or equal to  $L_r$ , the bending strength of the beam is based on inelastic lateral-torsional buckling (LTB). If  $L_b$  is greater than  $L_r$ , the bending strength is based on elastic lateral-torsional buckling (see Figure 7).



**Figure 7**

For Doubly Symmetric I-shapes and Channels with  $L_b \leq L_r$ :

The nominal flexural strength is obtained from;

$$M_n = C_b \left[ M_p - (M_p - M_r) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{Eq. 10 ; AISC F1-2})$$

$C_b$  is a modification factor for non-uniform moment diagrams, and permitted to be conservatively taken as 1.0 for all cases (see AISC LRFD manual equation F1-3 for actual value of  $C_b$ ).

The terms  $L_r$  and  $M_r$  are defined as:

$$L_r = \frac{r_y X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}} \quad (\text{Eq. 11 ; AISC F1-6})$$

$$M_r = F_L S_x \quad (\text{Eq. 12 ; AISC F1-7})$$

For Doubly Symmetric I-shapes and Channels with  $L_b > L_r$ :

The nominal flexural strength is obtained from;

$$M_n = M_{cr} \leq M_p \quad (\text{Eq. 13 ; AISC F1-12})$$

and

$$M_{cr} = \frac{C_b \pi}{L_b} \sqrt{EI_y GJ + \left[ \frac{\pi E}{L_b} \right]^2 I_y C_w} \quad (\text{Eq. 14 ; AISC F1-13})$$

Written also as:

$$M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{L_b / r_y} \sqrt{1 + \frac{X_1^2 X_2}{2 (L_b / r_y)^2}}$$

where,

$$X_1 = \frac{\pi}{S_x} \sqrt{\frac{EGJA}{2}} \quad (\text{Eq. 15 ; AISC F1-8})$$

$$X_2 = 4 \frac{C_w}{I_y} \left[ \frac{S_x}{GJ} \right]^2 \quad (\text{Eq. 16 ; AISC F1-9})$$

$S_x$  = section modulus about major axis, in<sup>3</sup>

$G$  = Shear modulus of elasticity of steel, 11,200 ksi

$F_L$  = smaller of  $(F_{yf} - F_r)$  or  $F_{yw}$ , ksi

$F_r$  = compressive residual stress in flange; 10 ksi for rolled shapes, 16.5 ksi for welded built-up shapes

$F_{yf}$  = yield stress of flange, ksi

$F_w$  = yield stress of web, ksi

$A$  = cross-sectional area, in<sup>2</sup>

$J$  = torsional constant, in<sup>4</sup>

$I_y$  = moment of inertia about y-axis, in<sup>4</sup>

$C_w$  = warping constant, in<sup>6</sup>

Rarely a beam exists with its compression flange entirely free of all restraint. Even when it does not have a positive connection to a floor or roof system, there is friction between the beam flange and the element that it supports.

Figure 8 shows types of definite lateral support, and Fig. 9 illustrates the importance to examine the entire system, not only the individual beam for adequate bracing.

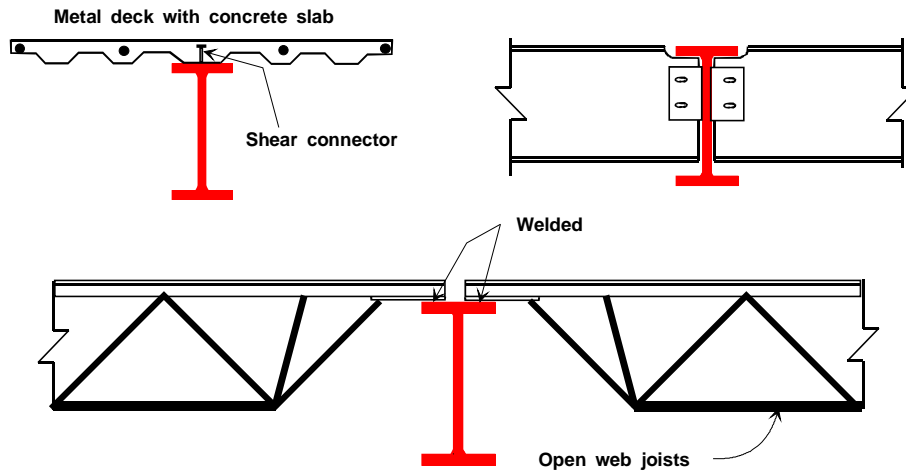


Figure 8

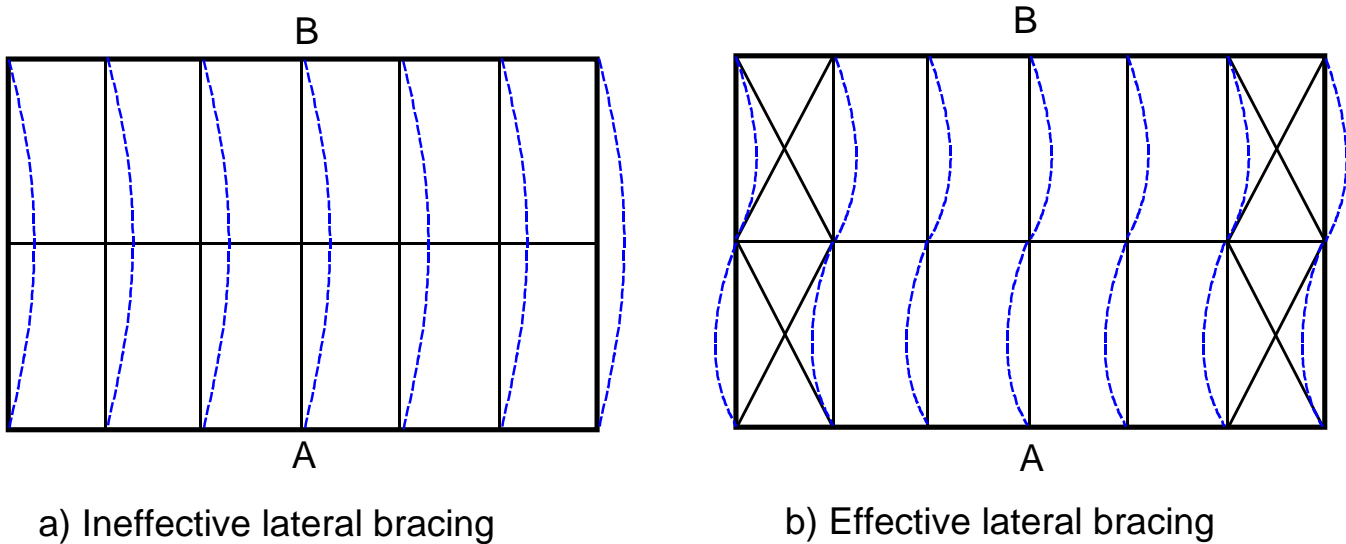


Figure 9

As shown on Figure 9.a, beam AB is laterally supported with a cross beam framing in at midspan, but buckling of the entire system is still possible unless the system is braced as depicted on Fig. 9.b.



## 7. Moment Gradient and Modification Factor $C_b$

The nominal moment strength given by equations 10 and 14 can be taken conservatively using  $C_b = 1.0$ , and it's based on an uniform applied moment over the unbraced length. Otherwise, there is a *moment gradient*, and the modification factor  $C_b$  adjust the moment strength for those situations where the compressive component on the flange element varies along the length.

The factor  $C_b$  is given as:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} \quad (\text{Eq. 17 ; AISC F1-3})$$

where:

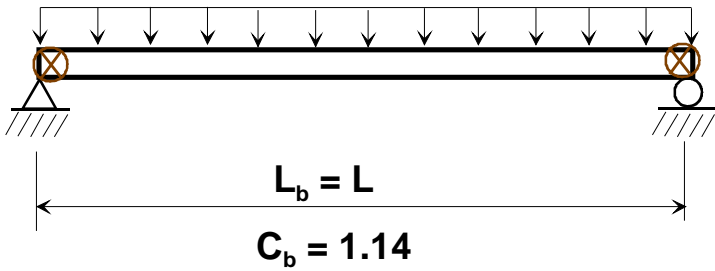
$M_{max}$  = absolute value of the maximum moment within the unbraced length (including the end points)

$M_A$  = absolute value of the moment at the quarter point of the unbraced length

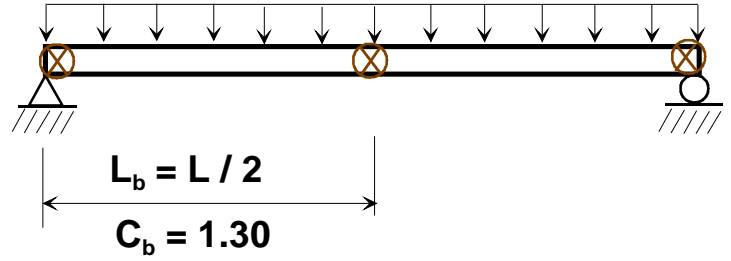
$M_B$  = absolute value of the moment at the midpoint of the unbraced length

$M_C$  = absolute value of the moment at the three-quarter point of the unbraced length

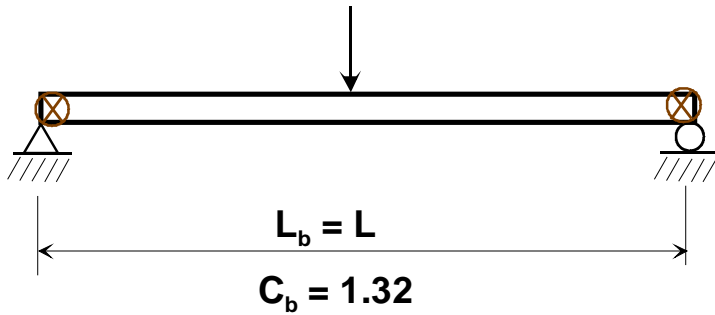
Figure 10 shows typical values for  $C_b$  based on loading conditions and lateral support locations for common conditions. Refer to Table 5-1 in Part 5 of the AISC Manual for additional cases.



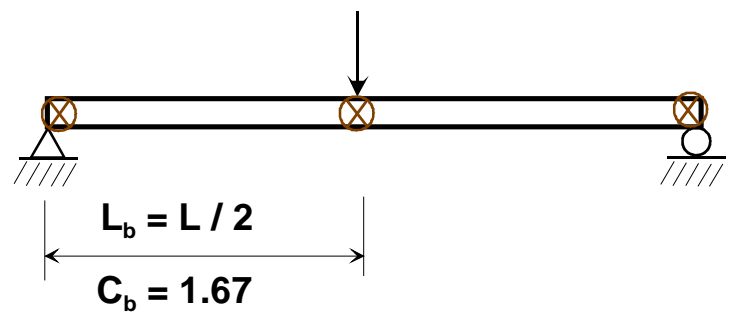
(a)



(b)



(c)



(d)

⊗ Indicate points of lateral support for the compression flange

Figure 10

**Example 2**

Determine the design strength  $\phi_b M_n$  for a W18 x 50 beam, ASTM A992 ( $F_y = 50$  ksi,  $F_u = 65$  ksi).

- a. continuous lateral support
- b. unbraced length = 15 ft.,  $C_b = 1.0$
- c. unbraced length = 15 ft.,  $C_b = 1.32$

Section Properties taken from Part 1 of the AISC Manual (LRFD, 3<sup>rd</sup> Edition):

$$A = 14.7 \text{ in}^2 \quad d = 18.0 \text{ in} \quad t_w = 0.355 \text{ in} \quad b_f = 7.50 \text{ in} \quad t_f = 0.57 \text{ in}$$

$$b_f / 2 t_f = 6.57 \quad S_x = 88.9 \text{ in}^3 \quad Z_x = 101 \text{ in}^3 \quad r_y = 1.65 \text{ in.}$$

$$X_1 = 1920 \quad X_2 = 12400 \times 10^6$$

**Solution**

a. Check whether this shape is compact, non-compact, or slender:

$$\frac{b_f}{2 t_f} = 6.57 \leq 0.38 \sqrt{\frac{29000}{50}} = 9.15$$

This shape is compact and as stated previously all shapes in the Manual meet web compactness.

$$\text{Thus, } M_n = M_p = F_y Z_x = 50(101) = 5,050 \text{ in.-kips} = 420.8 \text{ ft-kips}$$

$$\text{Answer: } \underline{\phi_b M_n = 0.90(420.8) = 378.8 \text{ ft-kips}}$$

b.  $L_b = 15 \text{ ft.}$  and  $C_b = 1.0$

Compute  $L_p$  and  $L_r$ , using equations 9 and 11 below:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} \quad L_r = \frac{r_y X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}}$$

Or, both of these values are given in Tables 5-2 and 5-3, Part 5 of the ASIC Manual:

$$L_p = 5.83 \text{ ft.} \quad \text{and} \quad L_r = 15.6 \text{ ft}$$

Since  $L_p = 5.83 \text{ ft.} < L_b = 15 \text{ ft}$  and  $L_b = 15 \text{ ft.} < L_r = 15.6 \text{ ft}$ , the moment strength is based on inelastic Lateral-Torsional Buckling,

$$M_r = (F_y - F_r) S_x = (50 - 10) 88.9 / 12 = 296.3 \text{ ft.-kips}$$

$$M_n = C_b \left[ M_p - (M_p - M_r) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$

$$M_n = 1.0 \left[ 420.8 - (420.8 - 296.3) \left( \frac{15 - 5.83}{15.6 - 5.83} \right) \right] = 303.9 \text{ ft-kips} \leq 420.8 \text{ ft-kips}$$

$$\text{Answer: } \underline{\phi_b M_n = 0.90(303.9) = 273.6 \text{ ft-kips}}$$

c.  $L_b = 15 \text{ ft. and } C_b = 1.32$

The design strength for  $C_b = 1.32$  is 1.32 times the strength for  $C_b = 1.0$ , then:

$$M_n = 1.32(303.9) = 361.1 \text{ ft-kips} \leq M_p = 420.8 \text{ ft-kips}$$

$$\text{Answer: } \underline{\phi_b M_n = 0.90(361.1) = 325.0 \text{ ft-kips}}$$

Part 5 of the Manual of Steel Construction, "Design of Flexural Members" contains many useful graphs, and tables for the analysis and design of beams. For example, the following value for a listed shape is given in Tables 5-2 and 5-3, for a W18 x 50:

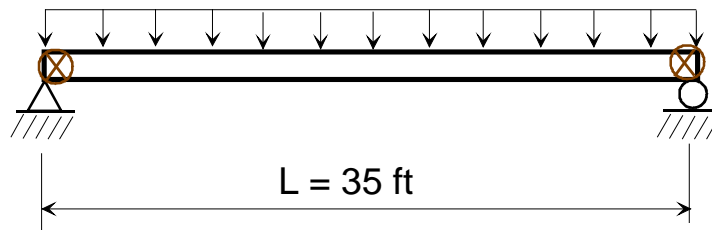
$$\phi_b BF = \phi_b \left( \frac{M_p - M_r}{L_r - L_p} \right) = 11.5$$

thus,  $\phi_b M_n$  can be written as:

$$\phi_b M_n = C_b \left[ \phi_b M_p - BF (L_b - L_p) \right] \leq \phi_b M_p \quad (\text{Eq. 18})$$

### **Example 3**

A simply supported beam with a span length of 35 feet is laterally supported at its ends only. The service dead load = 450 lb/ ft (including the weight of the beam), and the live load is 900 lb/ft. Determine if a W12 x 65 shape is adequate. Use ASTM A992 ( $F_y = 50 \text{ ksi}$ ,  $F_u = 65 \text{ ksi}$ ).



The factored load and moment are:

$$W_u = 1.2 (450) + 1.6 (900) = 1,980 \text{ lb/ft}$$

$$M_u = w_u L^2 / 8 = 1.98 (35)^2 / 8 = 303.2 \text{ ft-kips}$$

W 12 x 65 - Section Properties taken from Part 1 of the AISC Manual (LRFD, 3<sup>rd</sup> Edition):

$$A = 19.1 \text{ in}^2 \quad d = 12.1 \text{ in} \quad t_w = 0.39 \text{ in} \quad b_f = 12.0 \text{ in} \quad t_f = 0.605 \text{ in}$$

$$b_f / 2 t_f = 9.92 \quad S_x = 87.9 \text{ in}^3 \quad Z_x = 96.8 \text{ in}^3 \quad r_y = 3.02 \text{ in.}$$

$$X_1 = 2940 \quad X_2 = 1720 \times 10^6$$

### **Solution**

Check whether this shape is compact, non-compact, or slender:

$$\lambda = \frac{b_f}{2 t_f} = 9.92$$

$$\lambda_p = 0.38 \sqrt{\frac{29000}{50}} = 9.15$$

$$\lambda_r = 0.83 \sqrt{\frac{E}{F_y - F_r}} = 0.83 \sqrt{\frac{29,000}{50 - 10}} = 22.3$$

Since,  $\lambda_p < \lambda < \lambda_r$

This shape is noncompact. Check the capacity based on the limit state of flange local buckling:

$$M_p = F_y Z_x = 50(96.8) / 12 = 403.3 \text{ ft.-kips}$$

$$M_r = (F_y - F_r) S_x = (50 - 10) 87.9 / 12 = 293.0 \text{ ft-kips}$$

$$M_n = \left[ M_p - (M_p - M_r) \left( \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right]$$

$$M_n = 403.3 - (403.3 - 293.0) \left[ \frac{9.92 - 9.15}{22.3 - 9.15} \right] = 396.8 \text{ ft.-kips}$$

Check the capacity based on the limit state of lateral-torsional buckling:

Obtain  $L_p$  and  $L_r$ , using equations 9 and 11 (on pages 13 & 14) or from Tables 5-2 or 5-3 from the LRFD manual Part 5:

$$L_p = 11.9 \text{ ft} \quad \text{and} \quad L_r = 31.7 \text{ ft}$$

$$L_b = 35 \text{ ft} > L_r = 31.7 \text{ ft}$$

$\therefore$  Beam limit state is elastic lateral-torsional buckling

From Part 1 of the manual, for a W12 x 65:

$$I_y = 174 \text{ in}^4 \quad J = 2.18 \text{ in}^4 \quad C_w = 5,770 \text{ in}^6$$

For a uniformly distributed load, simply supported beam with lateral support at the ends,  $C_b = 1.14$  (see Fig. 10)

From equation 14 (AISC F1-13):

$$M_{cr} = \frac{C_b \pi}{L_b} \sqrt{E I_y G J + \left( \frac{\pi E}{L_b} \right)^2 I_y C_w} \leq M_p$$

$$M_{cr} = 1.14 \left[ \frac{\pi}{35(12)} \sqrt{29,000(174)(11,200)(2.18) + \left( \frac{\pi \times 29,000}{35 \times 12} \right)^2 (174)(5,770)} \right]$$

$$M_{cr} = 1.14 (3,088) = 3,520 \text{ in.-kips} = 293 \text{ ft.-kips}$$

$$M_p = 403.3 \text{ ft.-kips} > 293 \text{ ft.-kips} \quad \therefore \text{OK}$$

Answer:

$$\phi_b M_n = 0.90(293) = 264 \text{ ft.-kips}$$

As  $\phi_b M_n = 264 \text{ ft.-kips} < M_u = 303.2 \text{ ft.-kips}$ , this shape is not adequate for the given loading and support condition.

Note: Tables 5-2 and 5-3 in the AISC manual Part 5, facilitates the identification of noncompact shapes with marks on the shapes that leads to the footnotes.

## **8. Shear Design for Rolled Beams**

The shear strength requirement in the LRFD is covered in Part 16, section F2, and it applies to unstiffened webs of singly or doubly symmetric beams, including hybrid beams, and channels subject to shear in the plane of the web.

The design shear strength shall be larger than factored service shear load, applicable to all beams with unstiffened webs, with  $h / t_w \leq 260$  (see figure 6)

$$\phi_v V_n \geq V_u \quad (\text{Equation 19})$$

The three basic equations for nominal shear strength  $V_n$  are given in LRFD as follow:

For unstiffened webs, with  $h / t_w \leq 260$  the design shear strength is  $\phi_v V_n$  where:

$$\phi_v = 0.90$$

and  $V_n$  is given as:

a) No web instability; For  $h / t_w \leq 2.45 \sqrt{E / F_{yw}}$

$$V_n = 0.6 F_{yw} A_w \quad (\text{Eq. 20 ; AISC F2-1})$$

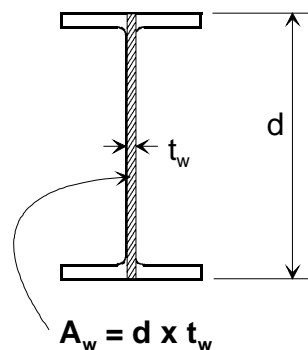
b) Inelastic web buckling; For  $2.45 \sqrt{E / F_{yw}} < h / t_w \leq 3.07 \sqrt{E / F_{yw}}$

$$V_n = 0.6 F_{yw} A_w \left[ \frac{2.45 \sqrt{E / F_{yw}}}{h / t_w} \right] \quad (\text{Eq. 21 ; AISC F2-2})$$

c) For  $3.07 \sqrt{E / F_{yw}} < h / t_w \leq 260$  the limit state is elastic web buckling

$$V_n = A_w \left[ \frac{4.52 E}{(h / t_w)^2} \right] \quad (\text{Eq. 22 ; AISC F2-3})$$

The web area  $A_w$  is taken as the overall depth  $d$  times the web thickness,  $t_w$ ;



The general design shear strength of webs with or without stiffeners is covered in the AISC LRFD, Appendix F2.2.

Shear is rarely a problem in rolled steel beam used in ordinary steel construction. The design of beams usually starts with determining the flexural strength, and then to check it for shear.

#### Example 4



A simply supported beam with a span length of 40 feet has the following service loads: dead load = 600 lb/ ft (including the weight of the beam), and the live load is 1200 lb/ft. Using a S18 x 54.7 rolled shape, will the beam be adequate in shear?

Material specification: ASTM A36 ( $F_y = 36$  ksi,  $F_u = 58$  ksi).

### ***Solution***

The factored load and shear are:

$$W_u = 1.2 (600) + 1.6 (1,200) = 2,640 \text{ lb/ft}$$

$$V_u = w_u L / 2 = 2.64 (40) / 2 = 52.8 \text{ kips}$$

S 18 x 54.7 - Section Properties taken from Part 1 of the AISC Manual (LRFD, 3<sup>rd</sup> Edition):

$$d = 18 \text{ in} \quad t_w = 0.461 \text{ in} \quad h / t_w = 33.2$$

$$2.45 \sqrt{E / F_{yw}} = 2.45 \sqrt{29,000 / 36} = 69.54$$

Since  $h / t_w = 33.2$  is  $< 69.54$ , the shear strength is governed by shear yielding of the web

$$V_n = 0.60 F_{yw} A_w = 0.6(36)(18)(0.461) = 179.2 \text{ kips}$$

$$\phi_v V_n = 0.90(179.2) = 161.3 > 52.8 \text{ kips (OK)}$$

The section S 18 x 54.7 is adequate in resisting the design shear

## **9. Deflection Considerations in Design of Steel Beams**

In many occasions the flexibility of a beam will dictate the final design of such a beam. The reason is that the deflection (vertical sag) should be limited in order for the beam to function without causing any discomfort or perceptions of unsafety for the occupants of the building. Deflection is a serviceability limit state, so service loads (unfactored loads) should be used to check for beam deflections.

The AISC specification provides little guidance regarding the appropriate limit for the maximum deflection, and these limits are usually found in the

governing building code, expressed as a fraction of the beam span length  $L$ , such as  $L/240$ . The appropriate limit for maximum deflection depends on the function of the beam and the possibility of damage resulting from excessive deflections.

The AISC manual provides deflection formulas for a variety of beams and loading conditions, in Part 5, “Design of Flexural Members”.

### **Course Summary:**

This course has presented the basic principles related to the design of flexural members (beams) using the latest edition of the AISC, Manual of Steel Construction, Load Resistance Factor Design, 3<sup>rd</sup> Edition.

The items discussed in this course included: general requirements for flexural strength, bending stress and plastic moment, nominal flexural strength for doubly symmetric shapes and channels, compact and non-compact sections criteria, elastic and inelastic lateral-torsional buckling bent about their major axis, and shear strength of beams.

The complete design of a beam includes items such as bending strength, shear resistance, deflection, lateral support, web crippling and yielding, and support details. We have covered the major issues in the design of rolled shape beams, such as bending, shear and deflection.

### References:

1. American Institute of Steel Construction, Manual of Steel Construction, Load Resistance Factor Design, 3<sup>rd</sup> Edition, November 2001
2. American Society of Civil Engineers, Minimum Design Loads for Buildings and Other Structures, ASCE 7-98
3. Charles G. Salmon and John E. Johnson, Design and Behavior of Steel Structures, 3<sup>rd</sup> Edition
4. William T. Segui, LRFD Steel Design, 3<sup>rd</sup> Edition